Counting non-destructively photons in a cavity, reconstructing Schrödinger cat states of light & realizing movies of their decoherence

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International Workshop on Fundamentals of Light-Matter Interactions, Recife, Brazil
October 20th 2008

Light as « an object of investigation », trapped for long times, manipulated and observed non-destructively for fundamental tests and quantum information purposes

The context: Cavity Quantum Electrodynamics: physics of a qubit coupled to a harmonic oscillator
Instead of trapping atoms...

...we trap light and manipulate it with a beam of atoms

Trapping photons for a long time in a very high-Q cavity and counting them non-destructively with a stream of atoms realizes a new way to look at light, opening many perspectives in quantum optics.

From the observation of individual field quantum trajectories to the generation and reconstruction of «strange» non-classical states....
Outline

1. Our set-up: a photon trap inside a Rydberg atom clock
2. QND counting of photons & the quantum jumps of light
3. Back action of QND photon counting on the field’s phase & the quantum Zeno effect of light
4. Reconstruction of trapped field quantum states by QND photon counting
5. Preparing and reconstructing Schrödinger cat states of light: a movie of decoherence
6. Conclusion and perspectives
Microwave photons in a box

- Superconducting mirrors
- Resonance @ $\nu_{\text{cav}} = 51$ GHz
- Lifetime of photons $T_{\text{cav}} = 130$ ms
- Q factor $= \omega T_{\text{cav}} = 4.2 \cdot 10^{10}$
- Finesse $F = 4.6 \cdot 10^9$

- best mirrors ever
- 1.5 billion photon bounces
- Light travels 40 000 km (Earth circumference)

S. Kuhr et al, APL 90, 164101 (2007)
Special detectors: Circular Rydberg Atoms

\[ n = 51 \quad |e\rangle \]
\[ \nu = 51.099 \text{ GHz} \]
\[ n = 50 \quad |g\rangle \]

- \( n \) large, \( l = |m| = n - 1 \)
- life time: 30 ms \( \Rightarrow \) weak dissipation
- huge electric dipole \( \Rightarrow \) very sensitive to microwave
- Two-level atom behaves as «spin»

But:
- complex preparation
- requires a «directing» \( E \) field \( \rightarrow \) cavity must be open

Raimond, Brune and Haroche, RMP, 73, 565 (2001)
Bloch sphere representation of the two-level Rydberg atom

Equatorial plane of Bloch sphere is the dial and the 'spin' is the hand of an atomic clock.

Atoms are off-resonant and cannot absorb light, but spins are delayed by light-shift effect. One photon can make the «spin hand» miss half a turn while atom crosses cavity (π phase shift per photon).
An artist’s view of the set-up...

Classical pulses
(Ramsey interferometer)

An atomic clock delayed by photons trapped inside
Cold region (at bottom of helium cryostat):
- 40 cm side box
- 40 kg copper and Niobium
- 0.8 K base temperature
- 24 hours cooling time
- below 2K for two years

...and the real thing...
2.

QND counting of photons &
the quantum jumps of light

S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deléglise, U. Busk Hoff, M. Brune, J.-M. Raimond and S. Haroche,
Nature 446, 297 (2007)

C. Guerlin, S. Deléglise, C. Sayrin, J. Bernu, S. Gleyzes, S. Kuhr, M. Brune, J.-M. Raimond and S. Haroche,
Each atom is a clock whose rate is affected by light

1. Reset the “stopwatch” (1\textsuperscript{st} Ramsey pulse).
2. Precession of the spin through the cavity: clock ticks.

The clock’s shift is proportional to \( n \): non-demolition photon counting by measuring spin direction (using 2\textsuperscript{nd} Ramsey pulse)
Detecting 0 or 1 photon

Strong dispersive coupling:

$\Phi_0 = \pi$

One atom = one bit of information (+ or - spin along y) perfectly correlated with the photon number.
Detecting 0 or 1 photon

Strong dispersive coupling:

\[ \Phi_0 = \pi \]

\[ e \rightarrow \text{field projected onto } |1\rangle \]

\[ g \rightarrow \text{field projected onto } |0\rangle \]

\[ n = 0 \rightarrow \text{field projected onto } |0\rangle \]

\[ n = 1 \rightarrow \text{field projected onto } |1\rangle \]

2nd Ramsey pulse

Detection e or g
Birth and death of a photon
(thermal field at 0.8K)
Birth and death of a photon

Quantum jump

Hundreds of atoms see same photon: a Quantum Non-Demolition (QND) measurement

\( n_B = 0.05 \) at \( T = 0.8K \)
QND measurement of arbitrary photon numbers: progressive collapse of field state

A small coherent state with Poissonian uncertainty and $0 \leq n \leq 7$ is initially injected in the cavity and its photon number is progressively pinned-down by QND atoms.

Experiment illustrates on light quanta the three postulates of measurement: state collapse, statistics of results, repeatability.

A coherent field (Glauber state) has uncertain photon number: $\Delta n \Delta \phi \geq 1/2$ Heisenberg relation.
Counting larger photon numbers: 1st atom effect on inferred photon distribution

Chose $\Phi_0 = \pi/4$

If «spin» found in state + $(j=0)$ (along $n=2$)

If «spin» found in state - $(j=1)$ (along $n=6$)

2nd Ramsey pulse maps a direction in equatorial plane back into Oz before detection

Random decimation of photon number projection postulate (or Bayes law)

Probability multiplied by a cosine function of $n$

Phase shift per photon
A step-by-step acquisition of information

To pin down photon number, send a sequence of atoms one by one....

...and change direction of spin detection to decimate different numbers

\[
P^{(N)}(n) = \frac{P^{(0)}(n)}{2Z} \prod_{k=1}^{N} \left[ 1 + \cos(n\Phi_0 - \phi(k) - j(k)\pi) \right] / 2
\]

Spin reading 000101101010001011001°K
Direction abdcadb cbadcaabcbacdb°K

\[P^{(N)}(n) \rightarrow \delta(n - n_0)\]

Progressive collapse!
A progressive collapse: *which number wins the race?*
Statistical analysis of 2000 sequences: histogram of the Fock states obtained after collapse

Coherent field with $<n> = 3.43$

Illustrates quantum measurement postulate about statistics
Evolution of the photon number probability distribution in a long measuring sequence

Field state collapse
Repeated measurement
Quantum jumps (field decay)

Single realization of field trajectory: real Monte Carlo
Two trajectories following collapses into $n=5$ and $7$.

An inherently random process (durations of steps widely fluctuate and only their statistics can be predicted).

Four trajectories following collapse into $n=4$.

Photon number trajectories

Similar QND trajectories observed between oscillator-like cyclotron states of an electron (Peil and Gabrielse, PRL 83, 1287 (1999).
Relaxing Fock states: quantitative analysis

Decay of $|n_0\rangle$ Fock state:
Sort out ensemble of realizations passing through $|n_0\rangle$ (at random times redefined as $t=0$) & reconstruct the photon number distribution of these ensembles at subsequent times.

Rate equations:

$$\frac{dP(n,t)}{dt} = \sum_{n'} K_{nn'} P(n',t)$$

Jump rates ($\kappa=1/T_c$)

$$K_{n-1,n} = \kappa (1+n_b)n$$
$$K_{n,n-1} = \kappa n_b n$$
$$K_{n,n} = -K_{n-1,n} - K_{n+1,n} = -\kappa [(1+n_b)n+n_b(n+1)]$$
Decay rate of $|n\rangle$ state is linear in $n$

$-K_{nn} = n(1+2n_b)+n_b$

Alternative view of QND photon counting: the «meter» is a N-atom sample entangled with field

QND procedure does not depend upon order or timing of individual spin measurements: instead of detecting N atoms one by one, we could store them before detecting their collective spin at once. Result would be the same.

$$\Psi = \sum_n C_n \{ spin_n \}^N \otimes |n\rangle$$

Photon number coded in a sample of N atoms, all pointing in a direction correlated to photon number: mesoscopic entanglement!
Decoding photon number by spin tomography

Measuring ensemble average of two spin-components:

\[ \langle S_x \rangle \]
\[ \langle S_y \rangle \]

N/2 atoms: measure \( \langle S_x \rangle \)
N/2 atoms: measure \( \langle S_y \rangle \)

\[ N \sim 100 \text{ is a large enough sample to distinguish between different } n \text{ values} \]

In practice, measurement is done one spin at a time, but global N-atom tomography can be extracted from data for each field trajectory.
Measuring a coherent field by atomic tomography

Sample of $N = 110$ first atoms crossing cavity in $T_{\text{meas}} = 26\, ms$

One measurement

$n = 4$
Statistics of measurements performed on many realizations of same coherent field

Spins point in discrete directions:

Each peak corresponds to a well-defined photon number

\[ \langle n \rangle = 2.4 \text{ photons} \]

⇒ A kind of Stern-Gerlach experiment giving visceral evidence of field quantization
3.

Back action of QND photon counting on the field’s phase & the quantum Zeno effect of light

Initial coherent state (phase 0)

Back action of QND photon counting on field's phase distribution ($\Delta n \Delta \Phi \geq 1/2$)

Field state after 1st atom: a « cat »

Field state after ~50 atoms: Fock state (here $n=3$)

Progressive phase blurring observed on the reconstructed Wigner functions of field (more on this later)
Freezing the field in vacuum state by repeated measurement

Cavity is resonantly coupled to a repeatedly pulsed source of coherent light.

If field is **not measured** until end of experiment, amplitude builds up linearly with number of pulses and mean photon number increases quadratically as $t^2$.

If photons are **QND counted** between pulses, phase is randomized by measurement and amplitude undergoes **Brownian motion** near phase space origin: amplitude grows as $\sqrt{t}$ and photon number as $t$. Rate of intensity increase goes to 0 as number of injections goes to infinity.

Equivalently, each measurement projects field back into vacuum.
The Zeno effect of light

After 0 pulse

After 20 pulses

After 50 pulses

Atomic spin tomography

QND measurements between pulse injections

The Zeno effect of light

Coherent injection pulses

QND Measurement

Expanded vertical scale

Observed only on coherent process!
4.

Reconstruction of trapped field quantum states by QND photon counting

Repeated QND photon counting on copies of field determines the diagonal $\rho_{nn}$ elements of the density matrix, but leaves the off-diagonal coherences $\rho_{nn'}$ unknown.

Recipe to determine the off-diagonal elements and completely reconstruct $\rho$:

- translate the field in phase space by homodyning it with coherent fields of different complex amplitudes and count (on many copies) the photon number in the translated fields.

Tomography of trapped light
Reconstructing field state by homodyning and QND photon counting

\[ \rho \rightarrow \rho^{(\alpha)} = D(\alpha) \rho D(-\alpha) \]

Field translation operator (Glauber):
\[ D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \]

The homodyning translation in phase space admixes field coherences \( \rho_{n'n''} \) into the diagonal matrix elements \( \rho^{(\alpha)}_{nn} \) of the translated field:

\[
\text{measured } \rho^{(\alpha)}_{nn} = \sum_{n'n''} D_{nn'}^{(\alpha)} \rho_{n'n''} D_{n''n}^{(-\alpha)}
\]

We determine \( \rho^{(\alpha)}_{nn} \) by QND photon counting on translated fields, for many \( \alpha \)'s, and get a set of linear equations constraining all the \( \rho_{n'n''} \) s. Using the Max. Ent. principle helps.

Requires many copies: quantum state is a statistical concept
From the density operator $\rho$ to the Wigner function $W$

$W$ is a real distribution of the field’s complex amplitude in phase space, defined as:

$$W(\alpha) = \frac{1}{\pi} \int e^{\alpha \lambda^* - \alpha^* \lambda} \text{Tr} \left[ \hat{\rho} e^{-i(\lambda^* \hat{a} - \lambda \hat{a}^\dagger)} \right] d\lambda$$

Once $\rho$ is known, the Wigner function $W(\alpha)$ is obtained by an invertible mathematical formula: $\rho$ and $W(\alpha)$ contain the same information, which completely defines the state.

Classical fields (such as coherent laser fields or thermal fields) have Gaussian Wigner functions.

Non-classical fields (Fock or Schrödinger cats) exhibit oscillating features with negative values which are signatures of quantum interferences. These features are very sensitive to coupling with environment (decoherence).
Reconstructing a coherent state

Fired twice (inject field, then translate it)

Fidelity $F=0.98$  Requires subpicometer mirror stability
Reconstructing Fock states

1) Prepare coherent state in $C$

2) Turn it into a Fock state by (random) projective QND measurement of photon number with first sequence of atoms

3) Reconstruct the Fock state density operator by field translations followed by QND photon counting with second sequence of atoms.
   Statistics performed on many copies

4) Compute $W$ from the reconstructed $\rho$

$\rho_{00} = 0.89$

$N = 0$
Reconstructing Fock states

1) Prepare coherent state in \( C \)

2) Turn it into a Fock state by (random) projective QND measurement of photon number

3) Reconstruct the Fock state density operator by field translations followed by (new) QND photon counting on many copies

4) Compute \( W \) from the reconstructed \( \rho \)

\[ \rho_{11} = 0.98 \]

\[ N = 1 \]
Reconstructing Fock states

1) Prepare coherent state in $C$

2) Turn it into a Fock state by (random) projective QND measurement of photon number

3) Reconstruct the Fock state density operator by field translations followed by (new) QND photon counting on many copies

4) Compute $W$ from the reconstructed $\rho$

$\rho_{22} = 0.92$

$N = 2$
Reconstructing Fock states

1) Prepare coherent state in $C$

2) Turn it into a Fock state by (random) projective QND measurement of photon number

3) Reconstruct the Fock state density operator by field translations followed by (new) QND photon counting on many copies

4) Compute $W$ from the reconstructed $\rho$

$\rho_{33} = 0.82$

$N = 3$
Reconstructing Fock states

1) Prepare coherent state in $C$

2) Turn it into a Fock state by (random) projective QND measurement of photon number.

3) Reconstruct the Fock state density operator by field translations followed by (new) QND photon counting on many copies.

4) Compute $W$ from the reconstructed $\rho$

The 1,2,3 steps must be realized before 1 photon is lost!
5.

Preparing and reconstructing Schrödinger cat states of light: a movie of decoherence

Recipe to prepare and reconstruct the cat

Coherent field prepared by first field injection

First QND atom generates cat state

\[ |\Psi_{\text{cat}}\rangle = |\beta\rangle \pm |\beta\rangle \]

\( \rho = |\Psi_{\text{cat}}\rangle \langle_{\text{cat}}\Psi| \)

Sign depends on detected atom state (e or g)

Cat state translated in phase plane by second field injection:

\[ \rho^{(\alpha)} D(\alpha) \Psi_{\text{cat}} \psi_{\text{cat}} D(-\alpha) \]

QND probe atoms measure field translated by different \( \alpha_i \)’s and yield the \( \rho^{(\alpha)} \) from which \( \rho \) is determined.
Reconstructed 3D-Wigner function of cat $|\beta\rangle + |\beta\rangle$

Gaussian components (correlated to atom crossing cavity in e or g)

Quantum interference (cat’s coherence) due to ambiguity of atom’s state in cavity

$D^2 = 8$ photons

Fidelity: 0.72

Non-classical states of freely propagating fields with similar W function (and smaller photon numbers) have been generated in a different way (Ourjoumtsev et al., Nature 448, 784 (2007))
Various brands of cats….

Even cat

$|\beta e^{i\chi}\rangle + |\beta e^{-i\chi}\rangle$

(preparation atom detected in $e$)

Odd cat

$|\beta e^{i\chi}\rangle - |\beta e^{-i\chi}\rangle$

(preparation atom detected in $g$)

Statistical Mixture

$|\beta e^{i\chi}\rangle <\beta e^{i\chi}\rangle | + |\beta e^{-i\chi}\rangle <\beta e^{-i\chi}\rangle$

(preparation atom detected without discriminating $e$ and $g$)

$D^2=11.8$ photons
A JOURNEY FROM QUANTUM TO CLASSICAL
Fifty milliseconds in the life of a Schrödinger cat
(a movie of decoherence)
The cat’s quantumness vanishes (evolution of difference between even and odd cat states)

Supplementary material on line accompanying Nature Letter

QuickTime™ et un décompresseur mpeg4 sont requis pour visionner cette image.
Exponential decay of cat’s quantum interference term yields decoherence time $T_D$

$T_D = 17\pm3$ ms

Theoretical model (T=0K):
$T_D = \frac{2T_c}{D^2} = 22$ ms

Correction at finite temp. (T = 0.8K):
$T_D = \frac{2T_c}{[D^2(2n_B+1)+4n_B]} = 19.5$ ms

Mean number $n_B$ of blackbody photons = 0.05

Good agreement between experiment and theory


$T_D$ was in microsecond range

Theoretical model (T=0K):
$T_D = \frac{2T_c}{D^2} = 22$ ms


Super-mirrors make new ways to look possible: trapped photons become like trapped atoms.

Soon, channelling field towards desired state by quantum feedback..
Experiments extended soon to two cavities: non-locality in mesoscopic field systems

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Japan Science and Technology Agency