Atoms and photons

Chapter 4

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Outline

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Interaction of quantum light with matter

- Quantum field and classical charges
- Quantum field and quantized atom

Quantum field and classical charges

Coupling of a quantum mode with a classical current (model of electronic source)

$$\mathbf{j}(\mathbf{r}, t) = \mathbf{j}_0(\mathbf{r})e^{-i\omega_0 t} \tag{1}$$

Simplified field Hamiltonian

$$H_0' = \sum_\ell \hbar\omega_\ell a_\ell^\dagger a_\ell \tag{2}$$

From classical interaction energy, $-\mathbf{j}\cdot\mathbf{A}$, guess the interaction Hamiltonian

$$H_i = -\int_{\mathcal{V}} \mathbf{j}(\mathbf{r}, t)\cdot\mathbf{A}(\mathbf{r}, t)\, d^3\mathbf{r} \tag{3}$$

where

$$\mathbf{A}(\mathbf{r}, t) = \sum_\ell \sqrt{\frac{\hbar}{2\epsilon_0\omega_\ell\mathcal{V}}} a_\ell \mathbf{f}_\ell(\mathbf{r}) + \text{c.c.} \tag{4}$$

Quantum field and classical charges

Interaction representation

$$\left|\widetilde{\Psi}\right\rangle = U_0^\dagger \left|\Psi\right\rangle \quad (5)$$

with

$$U_0 = e^{-iH_0' t/\hbar} = \prod_\ell e^{-i\omega_\ell t a_\ell^\dagger a_\ell} \quad (6)$$

New Hamiltonian

$$\widetilde{H} = U_0^\dagger H_i U_0 \quad (7)$$

Annihilation operator transformation

$$\begin{aligned} \widetilde{a}_\ell &= e^{i\omega_\ell t a_\ell^\dagger a_\ell} a_\ell e^{-i\omega_\ell t a_\ell^\dagger a_\ell} \\ &= a_\ell - i\omega_\ell t a_\ell + \frac{(i\omega_\ell t)^2}{2} a_\ell + \dots \\ &= a_\ell e^{-i\omega_\ell t} \end{aligned} \quad (8)$$

using Baker Hausdorff and $[N_\ell, a_\ell] = -a_\ell$.

Quantum field and classical charges

Interaction representation

$$\widetilde{H} = -\int d^3\mathbf{r}\ \left[\mathbf{j}_0(\mathbf{r})e^{-i\omega_0 t} + \mathbf{j}_0^*(\mathbf{r})e^{i\omega_0 t}\right] \cdot \left[\sum_\ell \sqrt{\frac{\hbar}{2\epsilon_0\omega_\ell\mathcal{V}}}\left(a_\ell e^{-i\omega_\ell t}\mathbf{f}_\ell(\mathbf{r}) + a_\ell^\dagger e^{i\omega_\ell t}\mathbf{f}_\ell^*(\mathbf{r})\right)\right] \quad (9)$$

Rotating wave approximation for $\omega_\ell \approx \omega_0$:

$$\widetilde{H} = -\sum_\ell \sqrt{\frac{\hbar\mathcal{V}}{2\epsilon_0\omega_\ell}} J_0 a_\ell^\dagger e^{-i(\omega_0-\omega_\ell)t} + \text{h.c.} \quad (10)$$

where the complex scalar J_0 is defined as:

$$J_0 = \frac{1}{\mathcal{V}}\int d^3\mathbf{r}\,\mathbf{j}_0(\mathbf{r})\cdot\mathbf{f}_\ell^*(\mathbf{r}) \quad (11)$$

Quantum field and classical charges

Single mode evolution

Setting

$$K_0 = \sqrt{\frac{\hbar\mathcal{V}}{2\epsilon_0\omega}} J_0 \tag{12}$$

we can write the Hamiltonian in the simpler form

$$\widetilde{H} = -K_0 e^{-i\delta t} a^\dagger + \text{h.c.} \tag{13}$$

where

$$\delta = \omega_0 - \omega \tag{14}$$

Note that the Hamiltonians at different times do not commute. Evolution operator not simple.

Quantum field and classical charges

Single mode evolution

From t to $t + dt$:

$$\widetilde{H} = -K_0 e^{-i\Phi} a^\dagger + \text{h.c.} \tag{15}$$

where

$$\Phi = \delta t \tag{16}$$

The evolution operator is then a displacement:

$$U(t, t + dt) = D(d\alpha) \tag{17}$$

with

$$d\alpha = \frac{iK_0}{\hbar} e^{-i\Phi} dt \tag{18}$$

Quantum field and classical charges

Single mode evolution

By adding up the amplitudes and within a global phase, the final state is a coherent state with amplitude

$$\beta = \int_0^t \frac{iK_0}{\hbar} e^{-i\delta t'}\, dt' = -\frac{K_0}{\hbar\delta}\left[e^{-i\delta t} - 1\right] \tag{19}$$

- For $\delta \neq 0$, periodic variation of the amplitude
- For $\delta = 0$

$$\beta = \frac{iK_0}{\hbar} t \tag{20}$$

Linear amplitude and quadratic photon number growth

Quantum field and quantized atom

Hamiltonians

$$H_{ap} = -\frac{q}{m}\mathbf{P}\cdot\mathbf{A}(0) \quad (21)$$

or

$$H_{de} = -\mathbf{D}\cdot\mathbf{E}(0) \quad (22)$$

Quantum field and quantized atom

Electric dipole interaction

Dipole

$$\mathbf{D} = d\boldsymbol{\epsilon}_d \left|g\right\rangle \left\langle e\right| + \text{h.c.} \tag{23}$$

Electric field in the plane mode basis

$$\mathbf{E}(0) = i \sum_{\ell} \sqrt{\frac{\hbar\omega_\ell}{2\epsilon_0 \mathcal{V}}} a_\ell \boldsymbol{\epsilon}_\ell + \text{h.c.} \tag{24}$$

For nearly resonant modes (dominant effect in general), two of the four terms in $\mathbf{D} \cdot \mathbf{E}(0)$ can be neglected (RWA approximation)

Spontaneous emission

Coupling an atom to the continuum of modes in free space. Decay of the excited states and (diverging) shifts of the energy levels.

- Fermi Golden Rule argument
- Wigner Weisskopf calculation

Fermi Golden rule

Initial state $|e,0\rangle$. Continuum of final states $|g,1_\ell\rangle$. Compute separately the rate of photon emission in all directions:

$$\Gamma = \int d\Gamma d\Omega \tag{25}$$

$$d\Gamma = \sum_{\boldsymbol{\epsilon}_\ell} \frac{2\pi}{\hbar} |W|^2 d\rho(E = \hbar\omega_0, d\Omega) \tag{26}$$

Density of states $d\rho = \rho d\Omega/4\pi$ where

$$\rho(\nu) = \frac{8\pi}{2c^3} \mathcal{V} \nu^2 d\nu \tag{27}$$

With $\rho(E)dE = \rho(\nu)d\nu$ for $E = h\nu = \hbar\omega_0$

$$\rho(E) = \frac{\mathcal{V}}{2\pi^2 c^3} \frac{1}{\hbar} \left(\frac{E}{\hbar}\right)^2 \tag{28}$$

Fermi Golden rule

Finally

$$d\rho(E = \hbar\omega_0, d\Omega) = \frac{\mathcal{V}}{8\pi^3}\frac{\omega_0^2}{\hbar c^3}d\Omega \quad (29)$$

Coupling

$$|W|^2 = |\langle g, 1_\ell|\, \mathbf{D}\cdot\mathbf{E}\,|e, 0\rangle|^2 \quad (30)$$

Without loss of generality

$$\boldsymbol{\epsilon}_d = \mathbf{u}_z \quad (31)$$

$$|W|^2 = \left| d\mathbf{u}_z \cdot \boldsymbol{\epsilon}_\ell^* \sqrt{\frac{\hbar\omega_\ell}{2\epsilon_0\mathcal{V}}}\right|^2 \quad (32)$$

We can now evaluate the rate

$$d\Gamma = \sum_{\boldsymbol{\epsilon}_\ell} \frac{1}{8\pi^2\epsilon_0}\frac{\omega_0^3}{c^3}\frac{|d|^2}{\hbar}|\mathbf{u}_z\cdot\boldsymbol{\epsilon}_\ell^*|^2 d\Omega \quad (33)$$

Fermi Golden rule

Expand $\mathbf{u}_z$ on the basis of $\mathbf{u}_\mathbf{k}$ (propagation direction) and two orthogonal linear polarizations $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$:

$$(\mathbf{u}_z \cdot \boldsymbol{\epsilon}_1^*)^2 + (\mathbf{u}_z \cdot \boldsymbol{\epsilon}_2^*)^2 = 1 - (\mathbf{u}_z \cdot \mathbf{u}_\mathbf{k})^2 = 1 - \cos^2\theta = \sin^2\theta \qquad (34)$$

Integration over solid angle:

$$\Gamma = \frac{1}{8\pi^2\epsilon_0}\frac{\omega_0^3}{c^3}\frac{|d|^2}{\hbar}\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, d\theta d\phi \qquad (35)$$

and finally

$$\Gamma = \frac{\omega_0^3|d|^2}{3\pi\omega_0\hbar c^3} \qquad (36)$$

Already used many times in these lectures!

Wigner-Weisskopf

A more detailed insight. Atom-field state at time t:

$$|\Psi(t)\rangle = c_0(t)\,|e,0\rangle + \sum_\ell c_\ell(t)\,|g,1_\ell\rangle \tag{37}$$

Schrödinger equation:

$$i\hbar\frac{dc_0}{dt} = \hbar\omega_0 c_0 + \sum_\ell V_\ell c_\ell \tag{38}$$

$$i\hbar\frac{dc_\ell}{dt} = \hbar\omega_\ell c_\ell + V_\ell^* c_0 \tag{39}$$

with

$$V_\ell = -\langle e,0|\,\mathbf{D}\cdot\mathbf{E}\,|g,1_\ell\rangle \tag{40}$$

Interaction representation:

$$b_\ell = c_\ell e^{i\omega_\ell t} \tag{41}$$

$$i\hbar\frac{db_\ell}{dt} = e^{i\omega_\ell t} V_\ell^* c_0 \tag{42}$$

Wigner-Weisskopf

Formal integration

$$b_\ell(t) = \frac{V_\ell^*}{i\hbar}\int_0^t c_0(t')e^{i\omega_\ell t'}\,dt' \tag{43}$$

or

$$c_\ell(t) = \frac{V_\ell^*}{i\hbar}\int_0^t c_0(t')e^{i\omega_\ell (t'-t)}\,dt' \tag{44}$$

Setting

$$c_0 = e^{-i\omega_0 t}\alpha_0(t) \tag{45}$$

We get

$$\frac{d\alpha_0}{dt} = -\sum_\ell \frac{|V_\ell|^2}{\hbar^2}e^{i\omega_0 t}\int_0^t e^{i\omega_\ell(t'-t)}e^{-i\omega_0 t'}\alpha_0\,dt' \tag{46}$$

Wigner-Weisskopf

Changing for the variable $\tau = t - t'$, we get

$$\frac{d\alpha_0}{dt} = -\int_0^t \mathcal{N}(\tau)\alpha_0(t-\tau)\,d\tau \tag{47}$$

where the integral kernel $\mathcal{N}$ is:

$$\mathcal{N}(\tau) = \frac{1}{\hbar^2}\sum_\ell |V_\ell|^2 e^{i(\omega_0-\omega_\ell)\tau} \tag{48}$$

$$\mathcal{N}(\tau) = \frac{|d|^2}{\hbar^2}\left[\sum_\ell |\mathbf{u}_z\cdot\boldsymbol{\epsilon}_\ell|^2 \frac{\hbar\omega_\ell}{2\epsilon_0\mathcal{V}} e^{-i\omega_\ell\tau}\right] e^{i\omega_0\tau} \tag{49}$$

In a time of the order of $1/\omega_0$, $\mathcal{N}$ practically vanishes

Wigner-Weisskopf

Thus:

$$\int_0^t \mathcal{N}(\tau)\alpha(t-\tau)\,d\tau \approx \alpha_0(t)\int_0^\infty \mathcal{N}(\tau)\,d\tau = \left(\frac{\Gamma}{2}+i\Delta\right)\alpha_0(t) \quad (50)$$

$$\frac{d\alpha_0}{dt} = -\left(\frac{\Gamma}{2}+i\Delta\right)\alpha_0 \quad (51)$$

- Γ spontaneous emission rate
- Δ level shift

Wigner-Weisskopf

Final solution

$$c_0(t) = e^{-\Gamma t/2} e^{-i\omega_0 t} e^{-i\Delta t} \tag{52}$$

$$c_\ell(t) = \frac{V_\ell}{i\hbar} \frac{1 - e^{-\Gamma t/2} e^{i(\omega_\ell - \omega_0 - \Delta)t}}{(\Gamma/2) - i(\omega_\ell - \omega_0 - \Delta)} \tag{53}$$

$$|c_\ell(\infty)|^2 = \frac{|V_\ell|^2}{\hbar^2} \frac{1}{(\Gamma^2/4) + (\omega_\ell - \omega_0 - \Delta)^2} \tag{54}$$

a lorentzian profile for the spontaneous emission line.

Wigner-Weisskopf

Explicit integration of the kernel:

$$\left(\frac{\Gamma}{2}+i\Delta\right)=\frac{|d[^2}{\hbar^2}\sum_\ell(\mathbf{u}_z\cdot\boldsymbol{\epsilon}_\ell)^2\frac{\hbar\omega_\ell}{2\epsilon_0\mathcal{V}}\int_0^\infty e^{i(\omega_0-\omega_\ell)\tau}\,d\tau \tag{55}$$

Using

$$\int_0^\infty e^{i\omega t}\,dt=\pi\delta(t)+i\mathcal{P}\mathcal{P}\frac{1}{\omega} \tag{56}$$

get for the real part:

$$\Gamma=\frac{2\pi|d|^2}{\hbar^2}\sum_\ell(\mathbf{u}_z\cdot\boldsymbol{\epsilon}_\ell)^2\frac{\hbar\omega_\ell}{2\epsilon_0\mathcal{V}}\delta(\omega_0-\omega_\ell) \tag{57}$$

same result as the Fermi Golden Rule.

Wigner-Weisskopf

Level shift

A severe problem

Δ is divergent

A (not so simple) solution

Renormalization...

Photodetector model

A simple single system photodetector. A ground state $|g\rangle$ and a continuum of excited states $|e_i\rangle$. Transition to excited state is a click.
Detector Hamiltonian

$$H_d = \sum_i \hbar\omega_i |e_i\rangle \langle e_i| \tag{58}$$

Detector-field interaction $-\mathbf{D} \cdot \mathbf{E}$ with

$$\mathbf{D} = \sum_i d_i(\epsilon_i |g\rangle \langle e_i| + \epsilon_i^* |e_i\rangle \langle g|) \tag{59}$$

Hence, within irrelevant factors

$$H_i = \sum_i \kappa_i |e_i\rangle \langle g| E^+ + \text{h.c.} \tag{60}$$

Photodetector model

Interaction representation $a_\ell \to a_\ell \exp(-i\omega_\ell t)$, $|e_i\rangle\langle g| \to \exp(i\omega_i t)|e_i\rangle\langle g|$

$$\widetilde{H}_i = \sum_i \kappa_i e^{i\omega_i t}|e_i\rangle\langle g| E^+(t) + \text{h.c.} \tag{61}$$

Initial condition

$$|\Psi(0)\rangle = |g\rangle \otimes |\Psi_f\rangle \tag{62}$$

State at time t

$$|\Psi(t)\rangle = |g\rangle \otimes |\Psi_f\rangle + \frac{1}{i\hbar}\int_0^t \widetilde{H}_i(t')\,|\Psi(t')\rangle\, dt' \tag{63}$$

First-order perturbative solution by replacing in the r.h.s. $|\Psi(t')\rangle$ by $|\Psi(0)\rangle = |g\rangle \otimes |\Psi_f\rangle$.

Photodetector model

Noting that, in $\widetilde{H}_i$, $|g\rangle\langle e_i| E^-$ gives zero on the initial state

$$|\Psi(t)\rangle = |g\rangle \otimes |\Psi_f\rangle + \frac{1}{i\hbar}\sum_i \kappa_i \left[\int_0^t dt'\, e^{i\omega_i t'} E^+(t')\,|\Psi_f\rangle\right] \otimes |e_i\rangle \quad (64)$$

Probability for having a count at time t

$$p_e = \sum_i |\langle e_i|\Psi\rangle|^2 = \sum_i \langle\Psi|e_i\rangle\langle e_i|\Psi\rangle \quad (65)$$

$$p_e = \frac{1}{\hbar^2}\sum_i |\kappa_i|^2 \int_0^t dt' \int_0^{t'} dt''\, e^{i\omega_i(t'-t'')}\,\langle\Psi_f| E^-(t'')E^+(t')\,|\Psi_f\rangle \quad (66)$$

Photodetector model

For a high density of final states

$$\sum_i \longrightarrow \int d\omega \rho(\omega) \tag{67}$$

$$\int d\omega e^{i\omega(t'-t'')} = \pi\delta(t'-t'') \tag{68}$$

Hence

$$p_e(t) \propto \int_0^t dt' \, \langle\Psi_f| E^-(t')E^+(t') |\Psi_f\rangle \tag{69}$$

With a large set of photo-detecting systems the ‘photocurrent’ is proportional to

$$I(t) = \langle\Psi_f| E^-(t)E^+(t) |\Psi_f\rangle \tag{70}$$

Intensity correlations

Classical Hanbury-Brown and Twiss

A source, a balanced beamsplitter and two detectors. Correlate the photocurrents:

$$G_2(\tau) = \overline{I_1(t)I_2(t+\tau)} \tag{71}$$

At long times, no correlation.

$$G_2(\infty) = (\bar{I})^2 \tag{72}$$

At $\tau = 0$

$$G_2(0) = \overline{I^2} \tag{73}$$

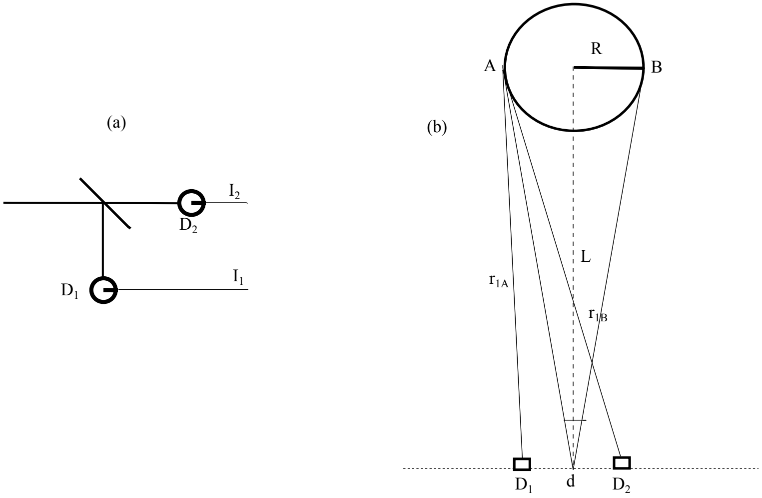
$$\overline{I^2} - (\bar{I})^2 = \overline{(I - \bar{I})^2} \geq 0 \tag{74}$$

$$G_2(0) \geq G_2(\infty) \tag{75}$$

Intensity correlations

HBT stellar interferometry

Nature paper (**178**, 1046): determination of the angular diameter of stars.



Intensity correlations

HBT stellar interferometry

Measure the intensity correlations at zero delay as a function of the distance d. Envelope field received by D_1:

$$E_1 = \alpha e^{ikr_{1A}} e^{i\phi_A} + \beta e^{ikr_{1B}} e^{i\phi_B} \tag{76}$$

with

$$\overline{I_1} = \overline{|E_1|^2} = |\alpha|^2 + |\beta|^2 \tag{77}$$

For D_2

$$E_2 = \alpha e^{ikr_{2A}} e^{i\phi_A} + \beta e^{ikr_{2B}} e^{i\phi_B} \tag{78}$$

After a painful computation and elimination (on the average) of random phases

$$\overline{I_1(t)I_2(t)} = \overline{I_1}\ \overline{I_2} + |\alpha|^2|\beta|^2 \cos k\left[r_{1A} + r_{2B} - r_{1B} - r_{2A}\right] \tag{79}$$

Intensity correlations

HBT stellar interferometry

For a symmetric configuration, when A and B are symmetric with respect to the mediating segment $D_1 - D_2$, $r_{1A} - r_{2A} = r_{2B} - r_{1B}$ and

$$\overline{I_1(t)I_2(t)} = \overline{I_1}\ \overline{I_2} + |\alpha|^2|\beta|^2 \cos 2k(r_{1A} - r_{2A}) = \overline{I_1}\ \overline{I_2} + |\alpha|^2|\beta|^2 \cos 2\frac{\omega}{c}(r_{1A} - r_{2A}) \quad (80)$$

or

$$\overline{I_1(t)I_2(t)} = \overline{I_1}\ \overline{I_2} + |\alpha|^2|\beta|^2 \cos kd\Theta \quad (81)$$

where Θ is the star's angular diameter. Resolution only limited by the distance between the two detectors.

Intensity correlations

Quantum intensity correlations

Admit

$$G_2(\mathbf{r}_1, \mathbf{r}_2, t, \tau) = \langle\Psi_f|\, \hat{G}_2 \,|\Psi_f\rangle \tag{82}$$

where

$$\hat{G}_2 = E^-(\mathbf{r}_1, t)E^-(\mathbf{r}_2, t+\tau)E^+(\mathbf{r}_2, t+\tau)E^+(\mathbf{r}_1, t) \tag{83}$$

Normalized correlation function

$$g_2 = \frac{G_2}{\overline{I_1}\;\overline{I_2}} \tag{84}$$

Intensity correlations

Two field modes

Simple situation: two field modes and two detectors.

$$E_i^+ = \mathcal{E}_i e^{i(\mathbf{k}_1\cdot\mathbf{r}-\omega_i t)} a_i \quad (85)$$

where $i = 1, 2$.

$$E^+ = E_1^+ + E_2^+ \quad (86)$$

Field state: product of Fock states $|\Psi\rangle = |N_1, N_2\rangle$. Complex calculation:

- No interference in the simple photocurrents
- Interferences in the g_2 correlation function

$$\langle\Psi|\, a_1^\dagger a_2^\dagger a_1 a_2 \,|\Psi\rangle + \text{h.c.} =$$
$$2\mathcal{E}_1^2\mathcal{E}_2^2 N_1 N_2 \text{Re}\left[e^{i[(\mathbf{k}_2-\mathbf{k}_1)\cdot(\mathbf{r}_A-\mathbf{r}_B)-(\omega_2-\omega_1)(t_A-t_B))]}\right] \quad (87)$$

Intensity correlations

Single emitter: antibunching

A single atom emitter and compute

$$G_2(\mathbf{r}, 0, \mathbf{r}, \tau) \equiv G_2(\tau) \tag{88}$$

Use Heisenberg picture, E^+ being proportional to the atomic dipole i.e. $\sigma_-(\tau)$

$$G_2(\tau) = \langle \sigma_+(0)\sigma_+(\tau)\sigma_-(\tau)\sigma_-(0) \rangle \tag{89}$$

Initially $\sigma_-(0) = |g\rangle\langle e|$ and $\sigma_+(0) = |e\rangle\langle g|$. At time τ, $\sigma_\pm(\tau) = U^\dagger \sigma_\pm(0) U$ and

$$G_2 = \left\langle |e\rangle\langle g| [U^\dagger |e\rangle\langle g| U][U^\dagger |g\rangle\langle e| U] |g\rangle\langle e| \right\rangle \tag{90}$$

Evaluate average in $|e\rangle$

$$G_2(\tau) = |\langle e| U(\tau) |g\rangle|^2 \tag{91}$$

hence $G_2(0) = 0$

The dressed atom model

A two level atom coupled to a single mode of the radiation field. Coherent coupling larger than dissipative process.

- An atom in an intense laser field
- Cavity quantum electrodynamics

Fruitful to treat atom and mode as a single quantum system. Spontaneous emission and shifts can be added later as a small perturbation.

The dressed atom model

Hamiltonian

$$H = H_a + H_c' + H_{ac} \quad (92)$$

where H_a and $H_c' = \hbar\omega_c N$ are the atom and field Hamiltonians. In the RWA

$$H_{ac} = -i\hbar\frac{\Omega_0}{2}\left[a\sigma_+ - a^\dagger\sigma_-\right] \quad (93)$$

where we introduce the 'vacuum Rabi frequency' Ω_0 (assumed to be real):

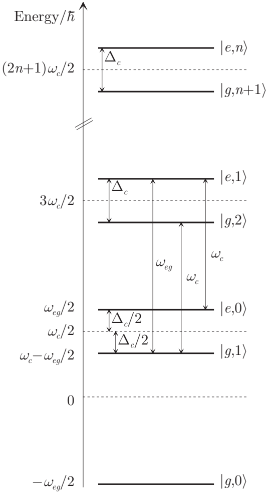
$$\Omega_0 = 2\frac{d\mathcal{E}_0\boldsymbol{\epsilon}_d^*\cdot\boldsymbol{\epsilon}_c}{\hbar} \quad (94)$$

Atom-field detuning

$$\Delta_c = \omega_{eg} - \omega_c \quad (95)$$

The dressed atom model

Uncoupled states



The dressed atom model

Eigenenergies and eigenvectors

In the *n*th doublet

$$H_n = \hbar\omega_c\,(n+1/2)\,\mathbb{1} + V_n \tag{96}$$

with:

$$V_n = \frac{\hbar}{2}\begin{pmatrix}\Delta_c & -i\Omega_n \\ i\Omega_n & -\Delta_c\end{pmatrix} = \frac{\hbar}{2}[\Delta_c\sigma_Z + \Omega_n\sigma_Y] \tag{97}$$

and

$$\Omega_n = \Omega_0\sqrt{n+1} \tag{98}$$

The dressed atom model

Eigenenergies and eigenvectors

Eigenvalues:

$$E_n^{\pm} = (n + 1/2)\,\hbar\omega_c \pm \frac{\hbar}{2}\sqrt{\Delta_c^2 + \Omega_n^2} \tag{99}$$

with

$$\tan\theta_n = \Omega_n/\Delta_c \tag{100}$$

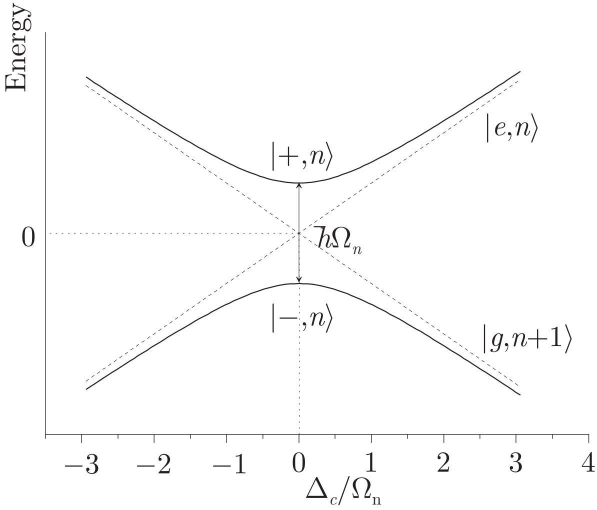
Eigenvectors

$$\begin{aligned} |+, n\rangle &= \cos(\theta_n/2)\,|e, n\rangle + i\sin(\theta_n/2)\,|g, n+1\rangle \\ |-, n\rangle &= \sin(\theta_n/2)\,|e, n\rangle - i\cos(\theta_n/2)\,|g, n+1\rangle \end{aligned} \tag{101}$$

The ‘dressed states’.

The dressed atom model

Dressed states



The dressed atom model

Resonant case: Rabi oscillation

$\theta_n = \pi/2$ for all n values.

$$|\pm, n\rangle = [|e, n\rangle \pm i\,|g, n+1\rangle]\,/\sqrt{2} \tag{102}$$

Initial state $|\Psi_e(0)\rangle = |e, n\rangle$

$$|\Psi_e(0)\rangle = [|+, n\rangle + |-, n\rangle]\,/\sqrt{2} \tag{103}$$

Interaction representation with respect to the constant $\hbar\omega_c(n+1/2)\mathbb{1}$. At time t

$$\left|\widetilde{\Psi}_e(t)\right\rangle = \left[|+, n\rangle\, e^{-i\Omega_n t/2} + |-, n\rangle\, e^{i\Omega_n t/2}\right]/\sqrt{2} \tag{104}$$

In the uncoupled basis

$$\left|\widetilde{\Psi}_e(t)\right\rangle = \cos\frac{\Omega_n t}{2}\,|e, n\rangle + \sin\frac{\Omega_n t}{2}\,|g, n+1\rangle \tag{105}$$

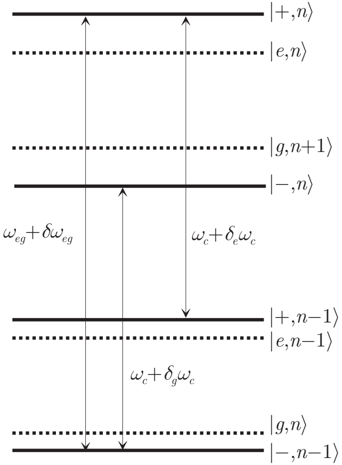
For an atom initially in g

$$\left|\widetilde{\Psi}_g(t)\right\rangle = -\sin\frac{\Omega_n t}{2}\,|e, n\rangle + \cos\frac{\Omega_n t}{2}\,|g, n+1\rangle \tag{106}$$

The dressed atom model

Non-resonant coupling

Large detuning case (dispersive regime) $|\Delta_c| \gg \Omega_n$



The dressed atom model

Non-resonant coupling

$$E_n^{\pm} = (n + 1/2)\,\hbar\omega_c \pm \hbar\left(\frac{\Delta_c}{2} + \frac{\Omega_n^2}{4\Delta_c}\right) \quad (107)$$

$$\Delta_{e,n} = \hbar(n+1)s_0 \;; \qquad \Delta_{g,n} = -\hbar n s_0 \quad (108)$$

with:

$$s_0 = \frac{\Omega_0^2}{4\Delta_c} \quad (109)$$

Two complementary effects

- Atomic frequency change (light shifts and Lamb shift)

$$\delta\omega_{eg} = (2n+1)s_0 \quad (110)$$

- Atomic state-dependent mode shift (index of refraction effect)

$$\delta_e\omega_c = s_0 \quad (111)$$

$$\delta_g\omega_c = -\delta_e\omega_c = -s_0 \quad (112)$$

The dressed atom model

Autler Townes splitting

The large field version of the vacuum Rabi splitting.
Atom driven by an intense laser field resonant on the e/g transition
Two dressed states with a splitting $\Omega\sqrt{n}$ nearly independent of photon number n for a large coherent field. Dressed states are superpositions of e and g with equal weights.
Probe the system on the h to g transition where h is a third level.

- For a negligible laser intensity: a single line.
- For a strong laser: two lines corresponding to the excitation of the two dressed levels, separated by the Rabi splitting.

The dressed atom model

Mollow triplet

Fluorescence of the dressed levels in a strong resonant laser field. Rabi splitting nearly the same for all relevant photon numbers.
Add atomic relaxation by spontaneous emission. Emission possible on all transitions between the levels
A triplet of lines

- Atomic frequency
- Sidebands at the Rabi frequencies