Atoms and photons: cavity quantum electrodynamcis

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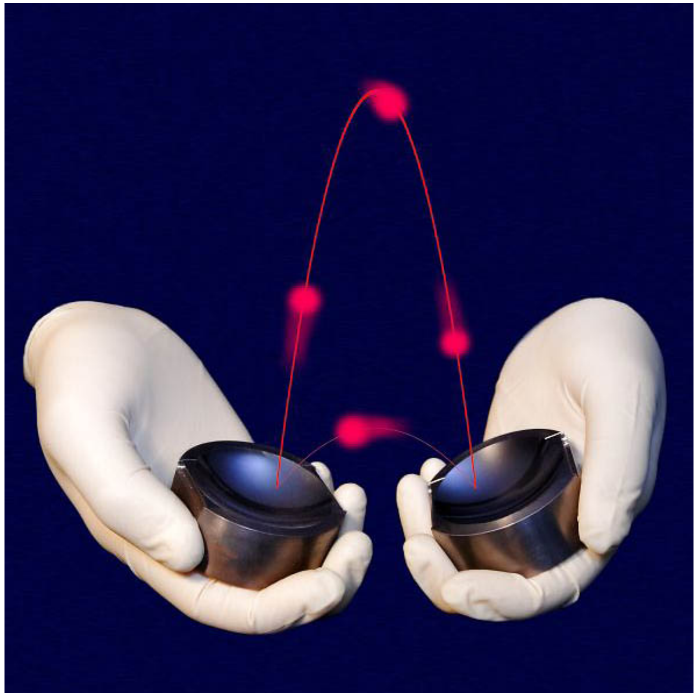
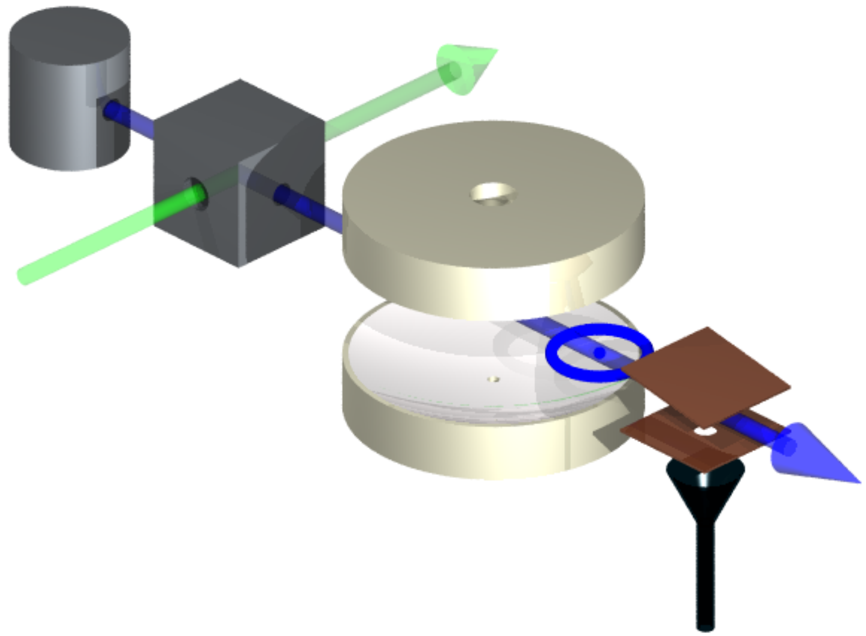
Physique quantique et applications



Cavity Quantum Electrodynamics

- A spin and a spring



  – Realizes the simplest matter-field system: a single atom coherently coupled to a few photons in a single mode of the radiation field.

  – Direct illustrations of quantum postulates

A history of CQED: the origin

- Purcell 1946
  - spontaneous emission rate modification for a spin in a resonant circuit
  - Definition of the 'Purcell factor'
  - Brief but seminal
- Kleppner 81
  - Inhibition of spontaneous emission

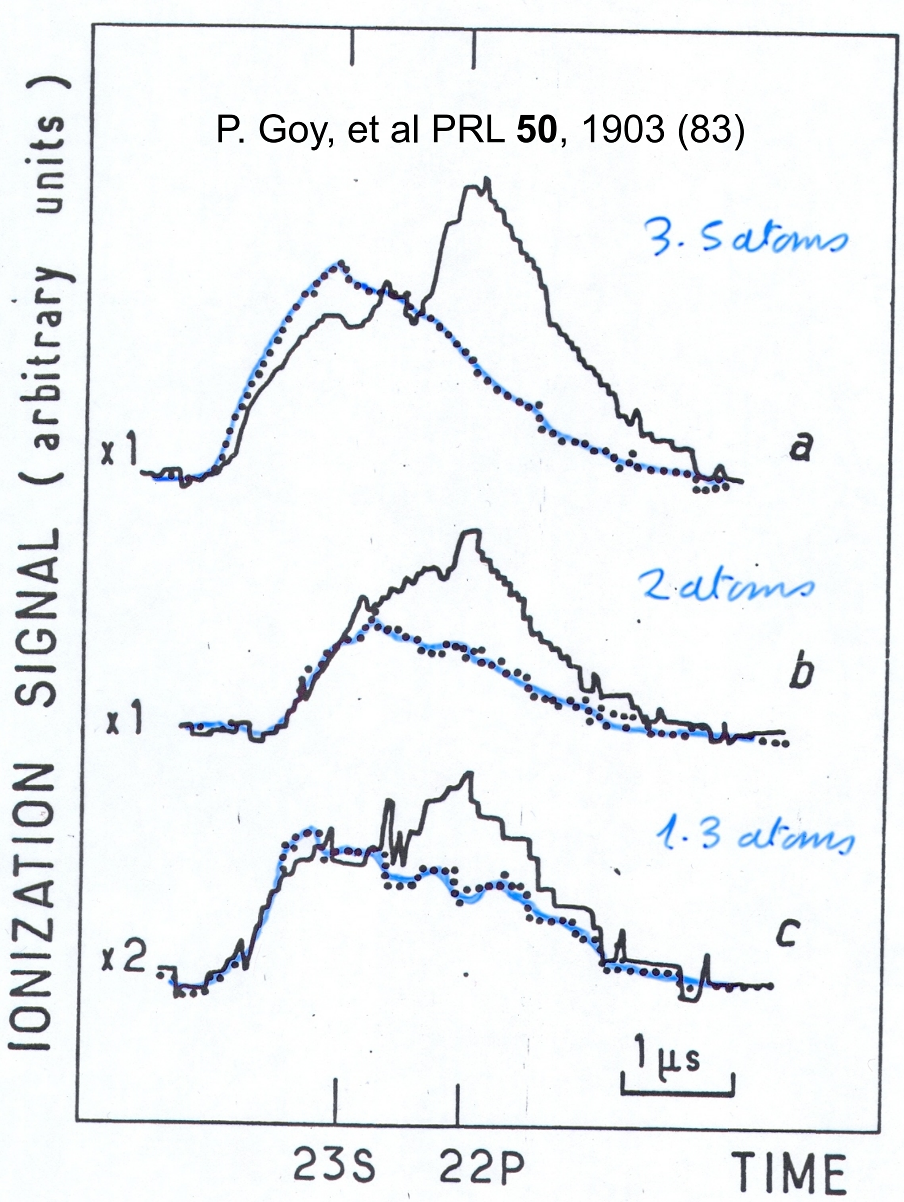
B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University.*—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2) \text{ sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7$ sec.$^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $\boxed{f = 3Q\lambda^3/4\pi^2 V}$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V \sim a^3$, and if δ is the skin-depth at frequency ν, $f \sim \lambda^3/a^2\delta$. For a non-resonant circuit $f \sim \lambda^3/a^3$, and for $a < \delta$ it can be shown that $f \sim \lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7$ sec.$^{-1}$.

First single-atom experiments

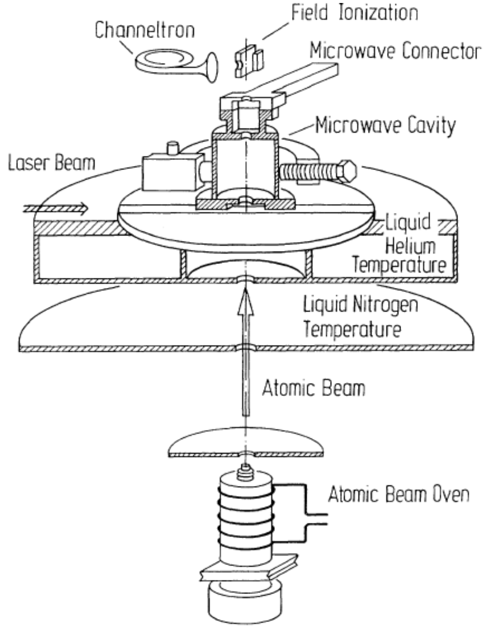
- Spontaneous emission enhancement
  - Superconducting FP cavity
    - Q α 10^6
    - 340 GHz transition
  - Acceleration x 530
  - First experimental evidence of Purcell effect
- Spontaneous emission inhibition
  - Gabrielse and Dehmelt (85)
  - Hulet, Hilfer and Kleppner (85)

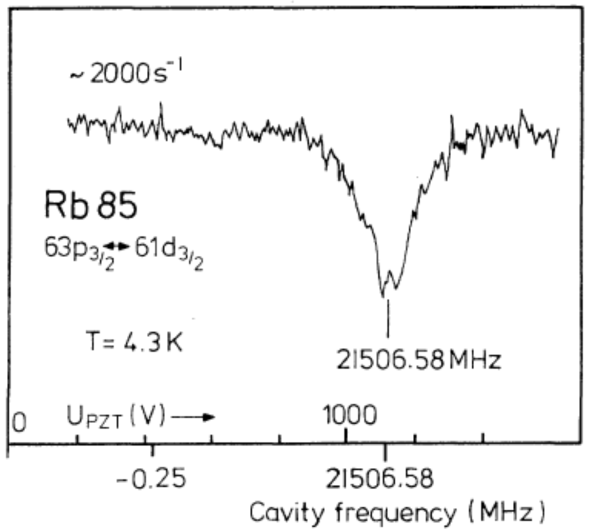


- Spontaneous emission can be altered at will by imposing limiting conditions to the field

The Micromaser

- H. Walther and D. Meschede, 85
  - Cumulative emissions in the cavity in the strong coupling regime





  - A maser with less than one atom at a time in the cavity
  - A new type of quantum oscillator. Role of quantum fluctuations
  - Strong coupling regime
    - Single-Atom-cavity coupling overwhelms dissipation

The two regimes of cavity QED

- Weak coupling regime

  - Atom-field coupling small compared to dissipation
    - No qualitative modifications of the atomic radiative properties
      - Modification of the spontaneous emission rate
      - Modification of the atomic energies

- Strong coupling regime

  - Atom-cavity interaction overwhelms dissipative processes
    - The simplest matter-field coupling situation

      - Radical modification of the atomic radiative properties
      - Creates and manipulates atom/field entangled state

The four time scales of CQED

- Atomic levels lifetime

$$T_{at} = 1/\Gamma$$

- Cavity lifetime

$$T_c = 1/\kappa$$

- Atom-cavity coupling

$$\Omega_0 = 2g = 1/T_{res}$$

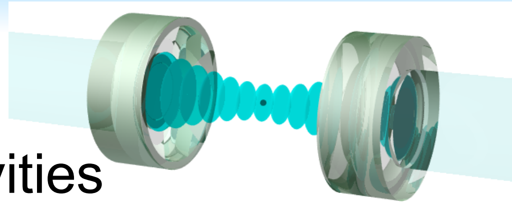
- Atom-cavity interaction time

$$T_{\text{int}}$$

- Strong coupling conditions

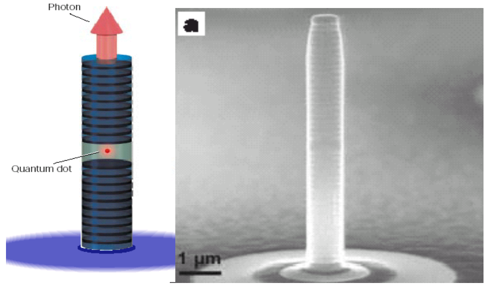
$$T_{\text{int}}\Omega_0 \approx 1; \quad T_{res}, T_{\text{int}} << T_{at}, T_c$$

The four flavours of modern CQED



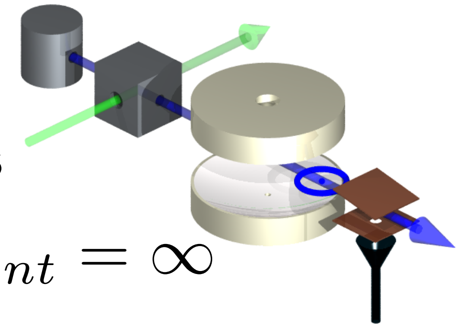
- Optical CQED
  - Ordinary atomic transitions and high finesse FP cavities

$$g \approx 50\ \mathrm{MHz};\ \kappa \approx 100\ \mathrm{kHz};\ \Gamma \approx 10\ \mathrm{MHz};\ T_{\mathrm{int}} \approx 1\mathrm{s}$$



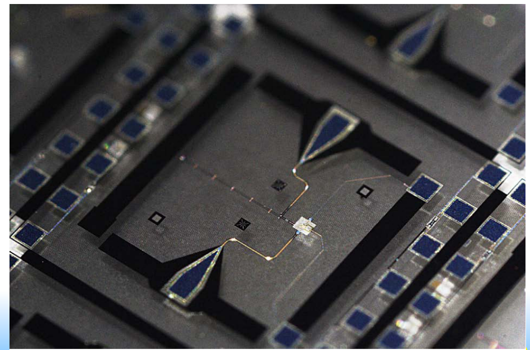
- Solid-state CQED
  - Quantum dots coupled to bragg mirrors/PBG

$$g \approx 10\ \mathrm{GHz};\ \kappa \approx 1\ \mathrm{GHz};\ \Gamma \approx\ 1\mathrm{GHz};\ T_{\mathrm{int}} = \infty$$



- Circuit QED
  - Solid-state qubits and stripline cavities

$$g \simeq 100MHz; \Gamma \ll \kappa \simeq 1MHz; T_{int} = \infty$$



- Microwave CQED
  - (Circular) Rydberg atoms and superconducting cavities

$$g \approx 10\ \mathrm{kHz};\ \kappa \approx 1\ \mathrm{Hz};\ \Gamma \approx 30\ \mathrm{Hz};\ T_{\mathrm{int}} \approx 100\ \mu s$$

These lectures

- Focus on microwave (and circuit) QED
  - Paradigmatic example of CQED
  - Some (hopefully) interesting experiments
    - Quantum Rabi oscillation
    - Entanglement generation
    - Generation and measurement of non-classical mesoscopic states (including Schrödinger cats)
    - Ideal photon number counting…

These lectures

I) Introduction

II) Experimental tools for microwave CQED

III) Theoretical tools for microwave CQED

IV) Resonant microwave CQED

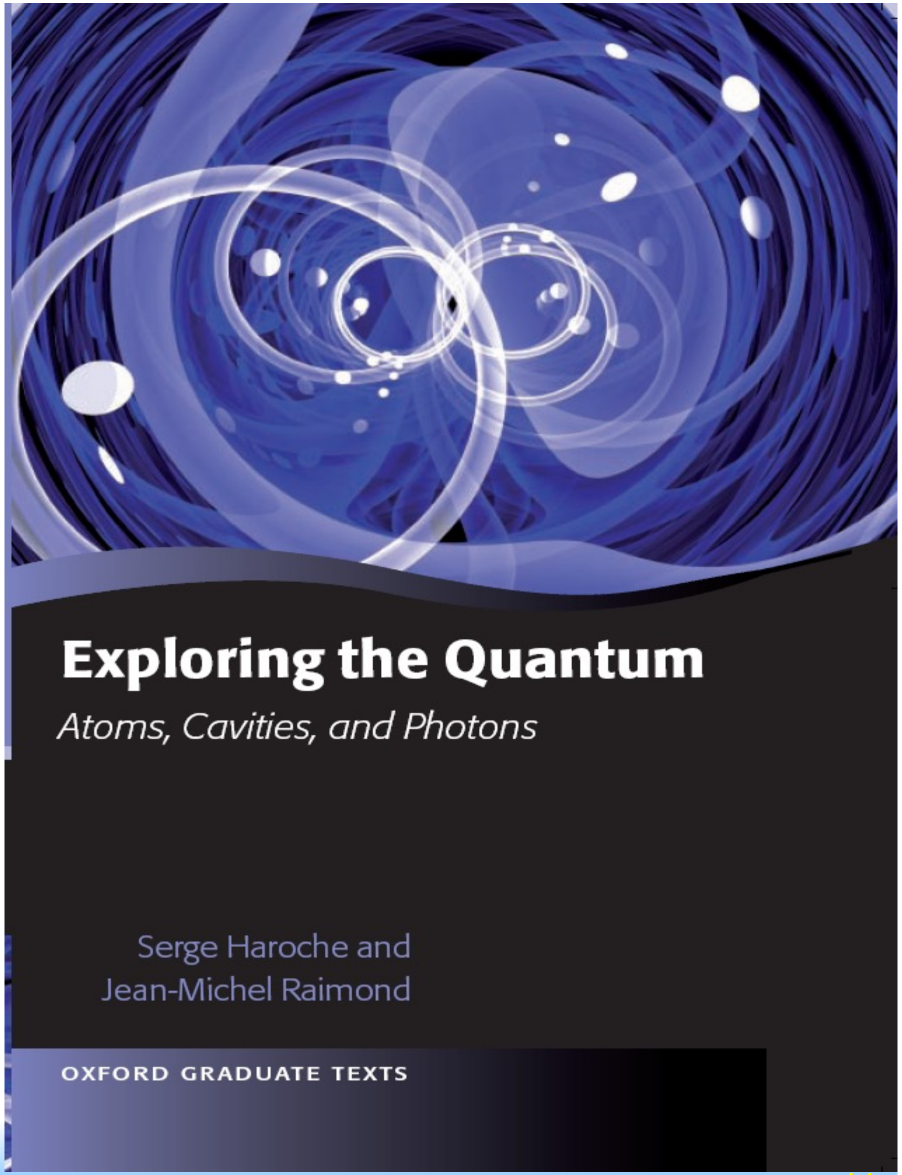
V) Dispersive microwave CQED

VI) Conclusion and perspectives

Bibliography

- S. Haroche and J.M. Raimond
  - Exploring the quantum:
    - atoms, cavities and photons
  - Oxford Univ. Press 2006

  - And many references therein



These lectures

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V) Dispersive microwave CQED

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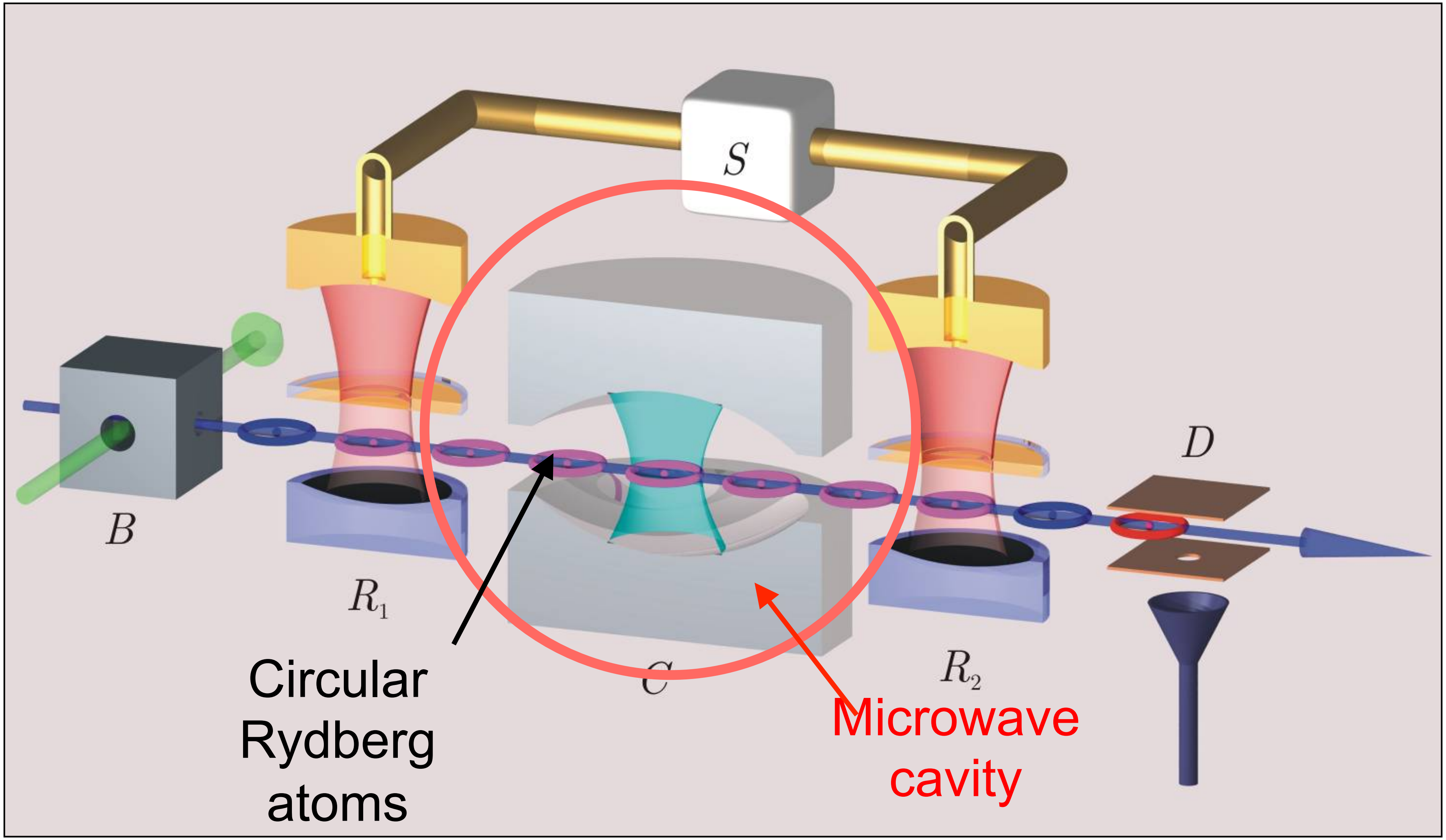
Experimental set-up



RMP **73**, 565

Circular Rydberg atoms

High principal quantum number

Maximal orbital and magnetic quantum numbers

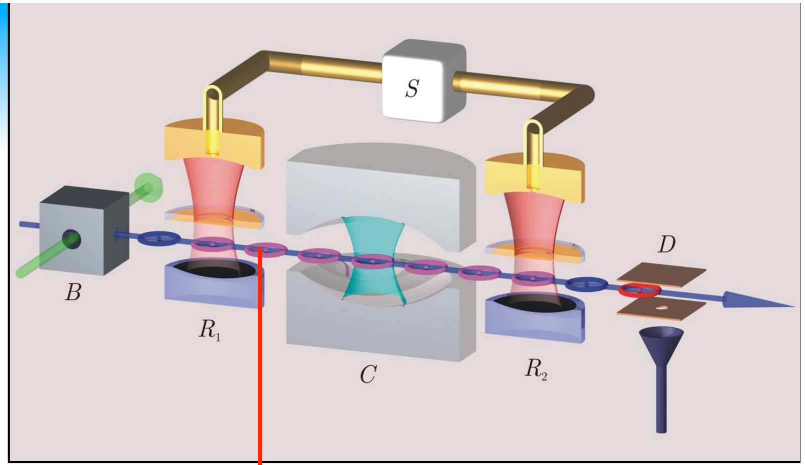
- Long lifetime
- Microwave two-level transition
- Huge dipole matrix element
- Stark tuning
- Field ionization detection
  - selective and sensitive

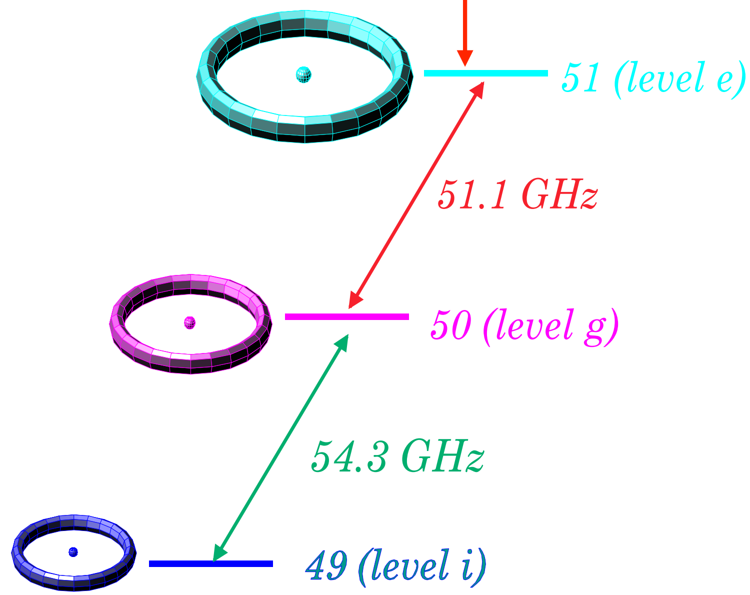
- Velocity selection
  - Controlled interaction time
  - Well known sample position

Atoms individually addressed

(centimeter separation between atoms)

Full control of individual transformations





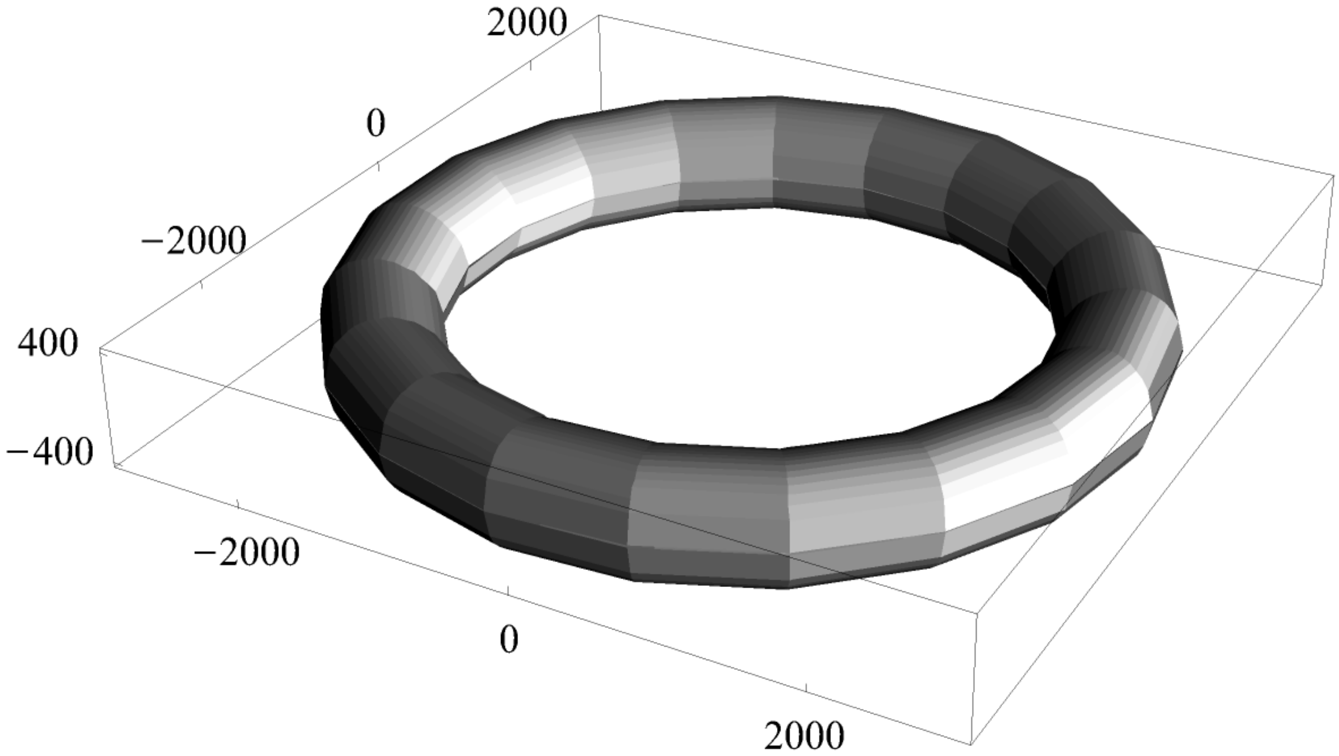
Complex preparation (53 photons !)

Stable in a weak directing electric field

Circular states wavefunction

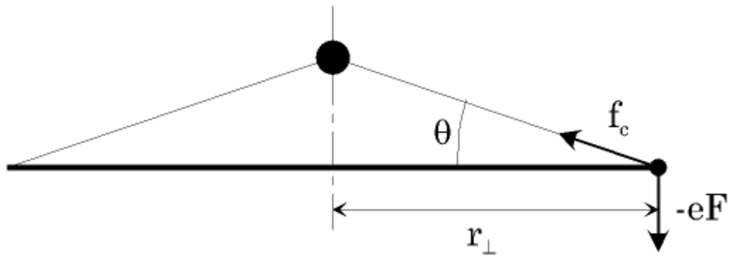
- Simple expression (maximal quantum numbers spherical harmonic)

$$\Psi(r,\theta,\phi) = \frac{1}{(\pi a_0^3)^{1/2}} \frac{1}{n^n n!} \left(-\frac{r}{a_0} \sin\theta e^{i\phi} \right)^{n-1} e^{-r/na_0}$$



A classical atom

- All quantum numbers are large. Most properties can be calculated by classical arguments (correspondence principle)
- Stark polarizability (atomic units used)



  – Using $m_0 \omega r_\perp^2$ is $n\hbar$

$$\cos^2\theta \sin\theta = r_\perp^2 F$$
$$\cos^3\theta = \frac{n^2}{r_\perp},$$

  – Weak field limit $r_\perp \approx n^2$. expand first equation $d_i = n^6 F$

$$E_2 = -\frac{1}{2} n^6 F^2$$

  – In natural units, polarizability of n=50 -2MHz/(V/cm)². -255kHz/(V/cm)² differential on the 50 to 51 transition
    - Easy adjustment of the circular states transition frequency

A classical atom

- Ionization threshold

  - Eliminate radius in the system: closed equation for θ

    $$\cos^8\theta \sin\theta = n^4 F$$

  - First term has a maximum, 0.2, obtained for $\theta = \arcsin(1/3) \approx 19^o$

  - Ionization threshold $F_i \approx 0.2F_0/n^4.$

    $$F_0 = e/4\pi\varepsilon_0 a_0^2 = 5.14\,10^{11}\ \mathrm{V/m}.$$

  - 165 and 152 V/cm for 50 and 51. Good agreement with measured values
  - Easy field ionization detection !

A classical atom

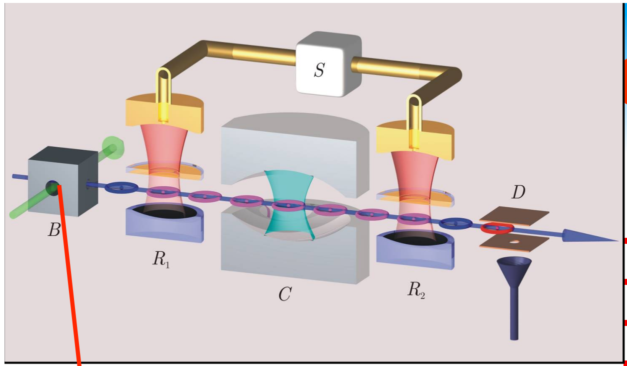
- Spontaneous emission lifetime
  - Radiation reaction force $\mathbf{f}_r = m_0\tau_0 d\boldsymbol{\gamma}/dt, \quad \tau_0 = \frac{1}{6\pi\varepsilon_0}\frac{e^2}{m_0c^3} = \frac{2}{3}\frac{\alpha^3}{\omega_0}$.
  - Angular momentum equation $\frac{d\mathbf{L}}{dt} = \mathbf{r} \times m_0\tau_0\frac{d\boldsymbol{\gamma}}{dt}$
  - Average on long times and note (integration by parts) $\overline{\mathbf{r} \times d\boldsymbol{\gamma}/dt} = -\overline{\mathbf{v} \times \mathbf{a}}$,

$$\mathbf{a} = -\frac{1}{m_0}\frac{dU}{dr}\frac{1}{r}\mathbf{r} \qquad U = -e^2/4\pi\varepsilon_0 r \qquad \overline{\frac{d\mathbf{L}}{dt}} = \frac{\tau_0}{m_0}\overline{\frac{1}{r}\frac{dU}{dr}}\overline{\mathbf{L}}$$

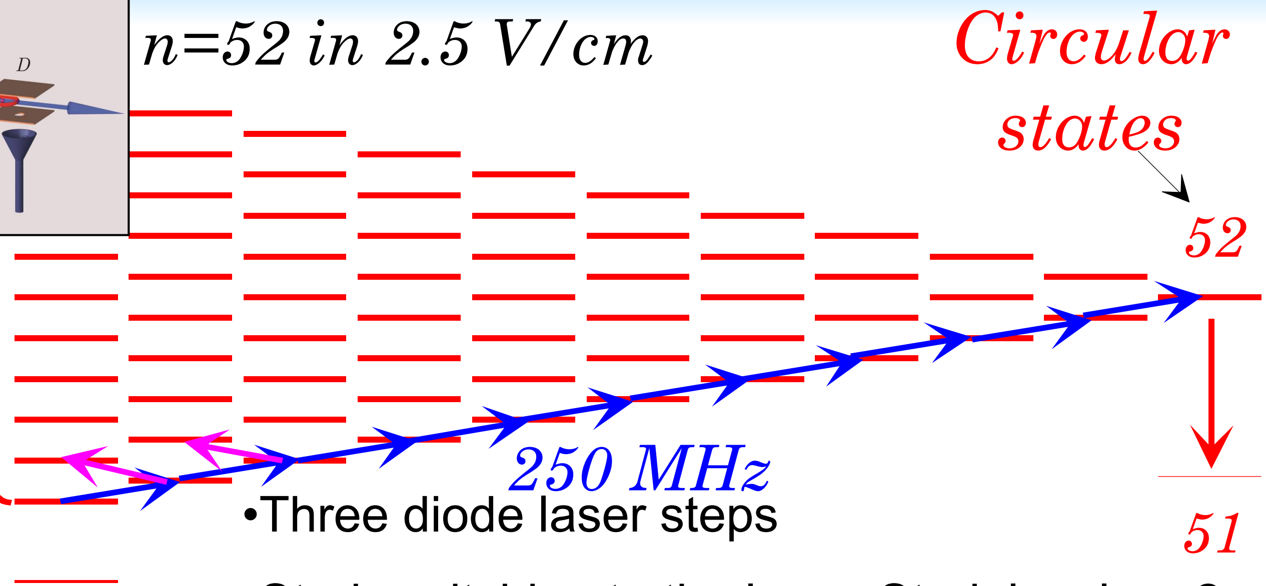
  - Circular to circular transition corresponds to one unit angular momentum $d\mathbf{L}/dt \approx -\hbar\Gamma_n$.

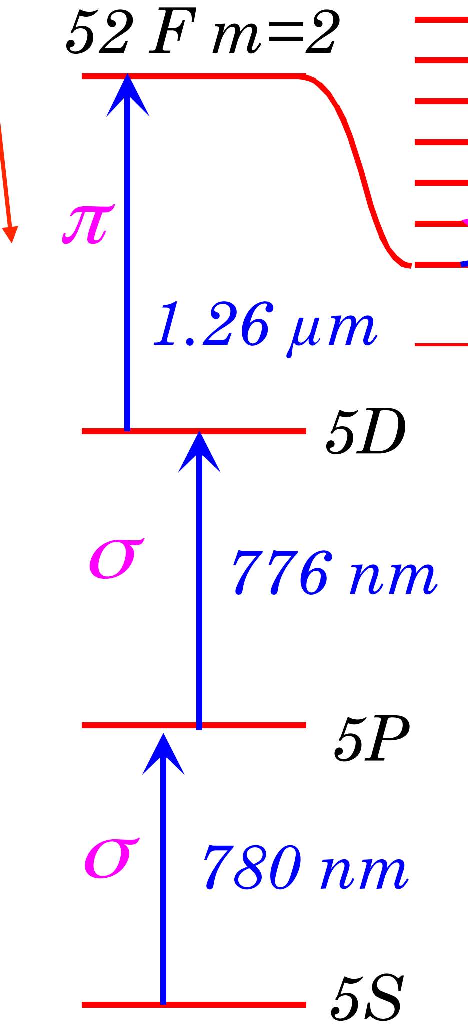
$$\Gamma_n = \frac{2}{3}\omega_0\alpha^3 n^{-5} ,$$

  - 30 ms for n=50: extremely long lifetime
  - Exact agreement with quantum value

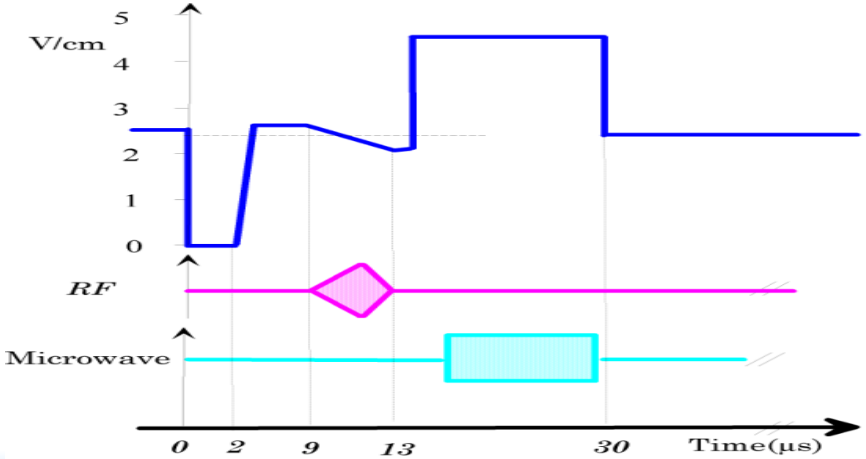


tate preparation





- Three diode laser steps
- Stark switching to the lower Stark level m=2
- Adiabatic 250 MHz transitions to the circular state
- Final microwave transition in a high field: 'purification'



Working with single atoms?

Method

- Weak excitation of the atomic beam: Poisson statistics for the atom number in each sample
- Finite detection efficiency: 40-80%

No deterministic preparation of single atom samples

Brute force approach:

Prepare much less than one (0.1) atom on the average

When an atom is detected, low probability for an undetected second one: single atom samples

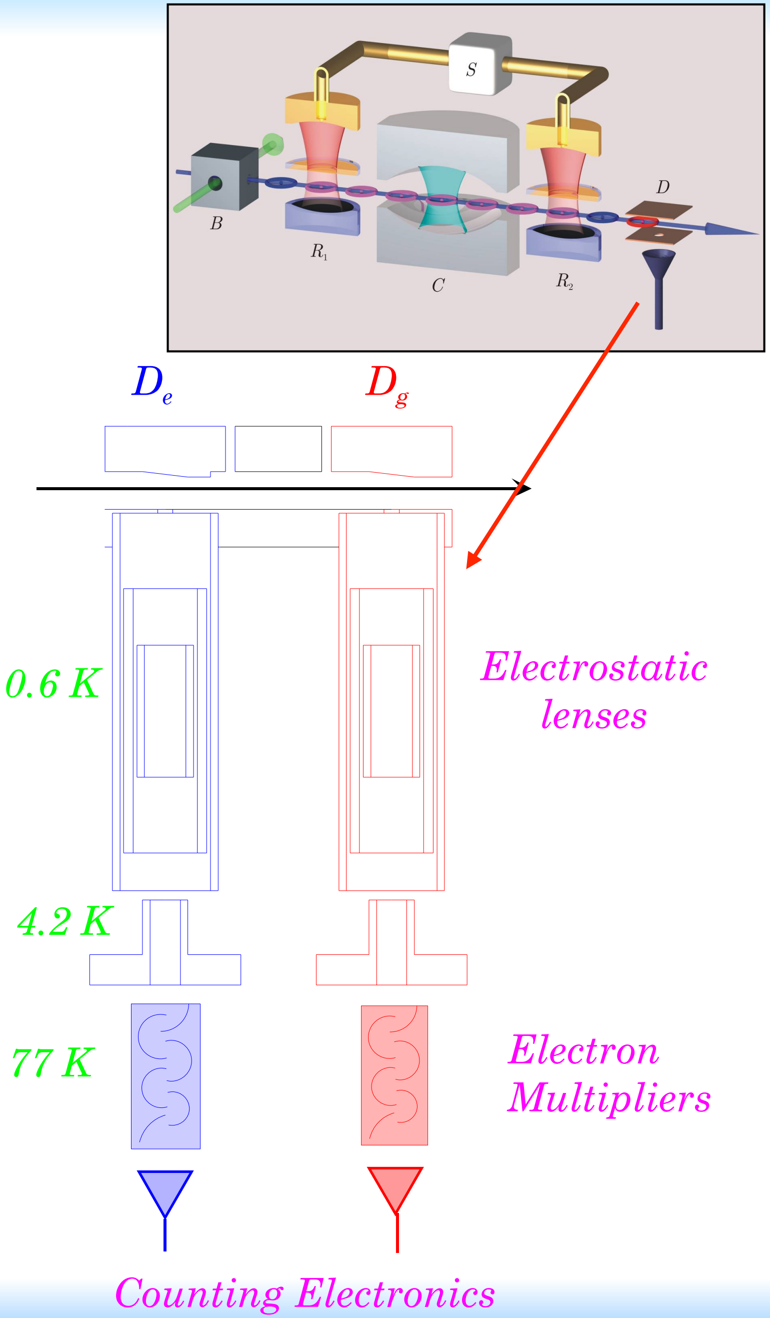
Pros and cons

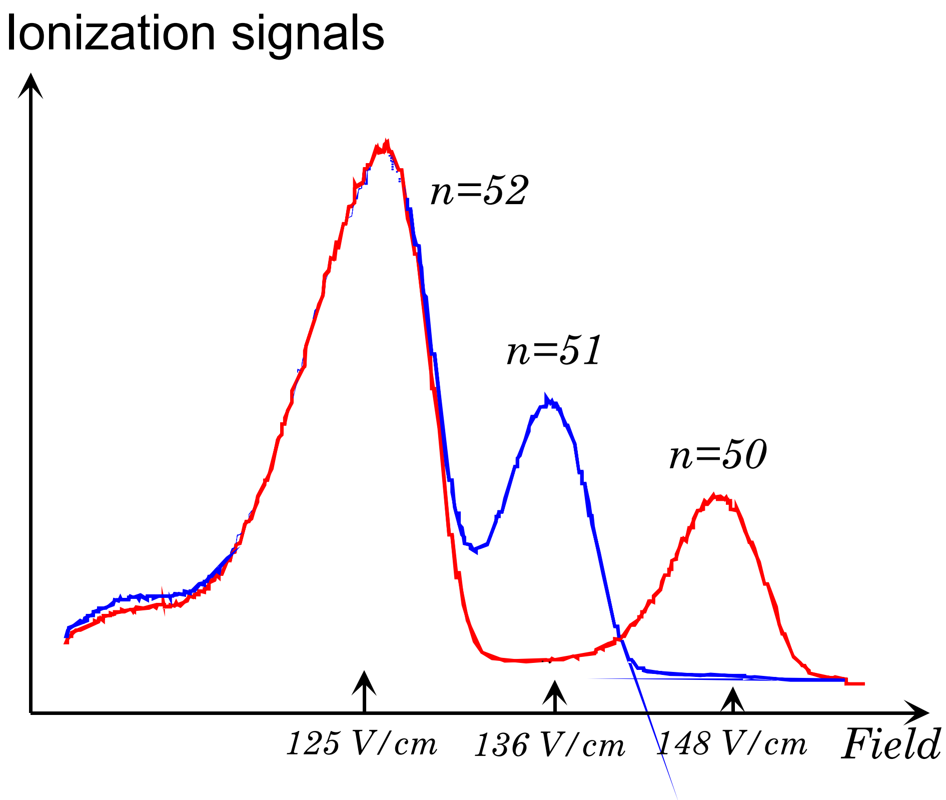
Extremely easy to achieve

Long data taking times, growing exponentially with atomic samples count

- 1 sample (1 atom): 10 minutes
- 2 samples: Hours
- 3 samples: Days
- 4 samples: Weeks (not very practical)

Field ionization detection

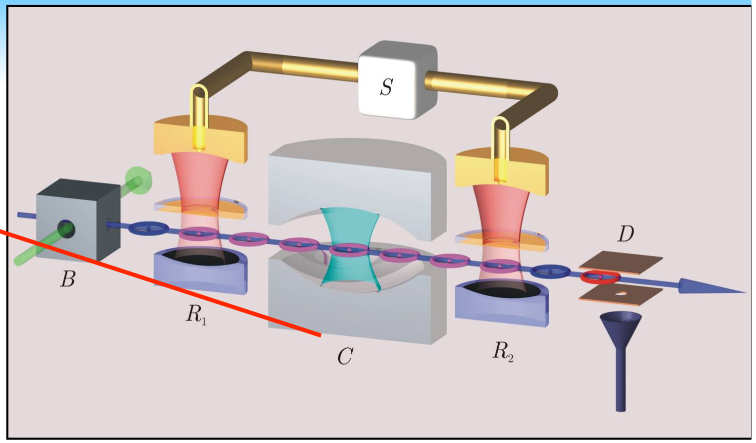




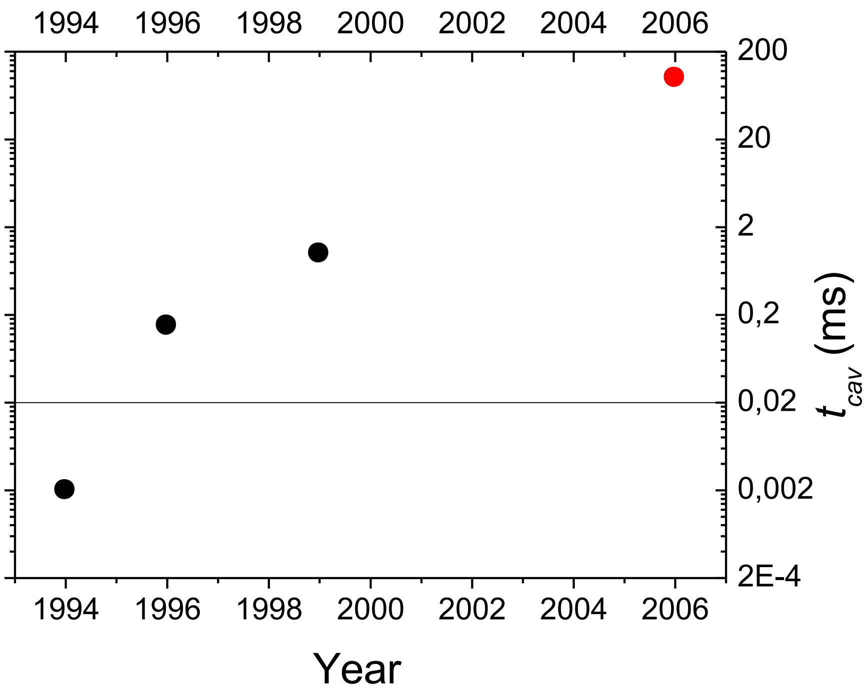
- Detection efficiency >50-80%
- Error rate few %
- Dark counts: negligible

A box for microwave photons

- Superconducting Fabry Perot
  - Optimal conductivity = optimal reflectivity
  - Compatible with a static electric field for circular Rydberg atoms

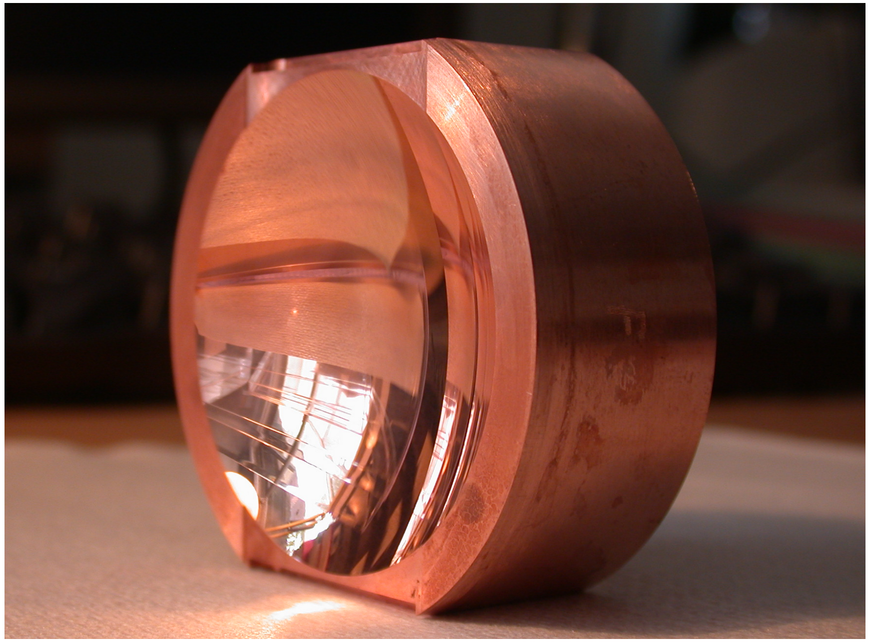


- Two contradictory requirements
  - Excellent superconductor
    - High purity Niobium
  - Excellent surface state
    - $\lambda/10^6$ roughness

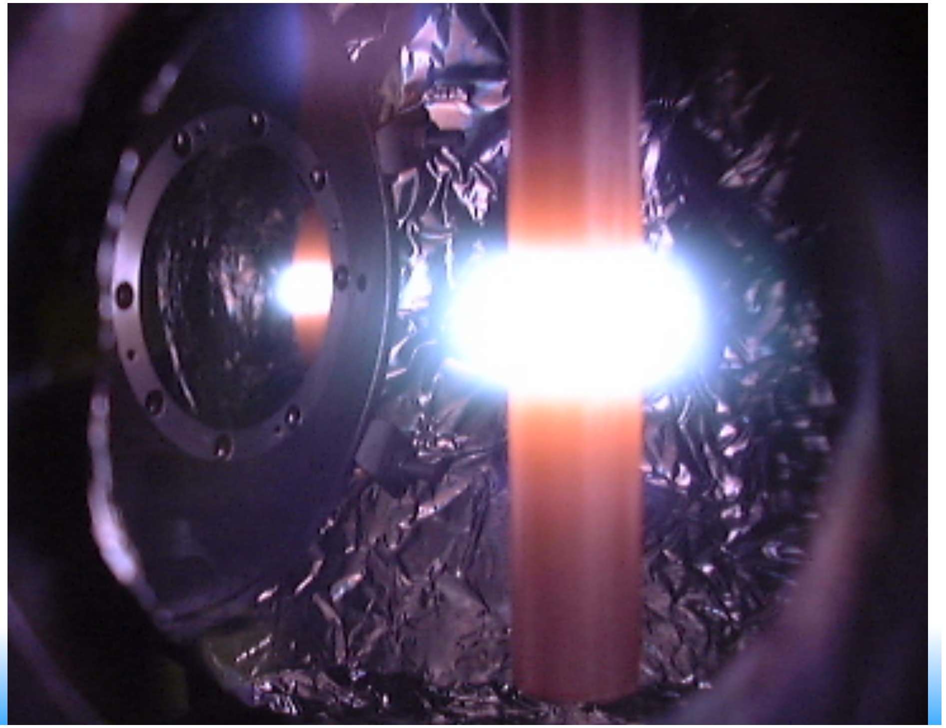


- Optimization of the cavity quality
  - a long (painful !!) process

Mirror technology



- Copper substrates
  diamond machining
    ~shape accuracy 300 nm ptv
    ~rugosity 10 nm
  Toroidal surface ➔ single mode



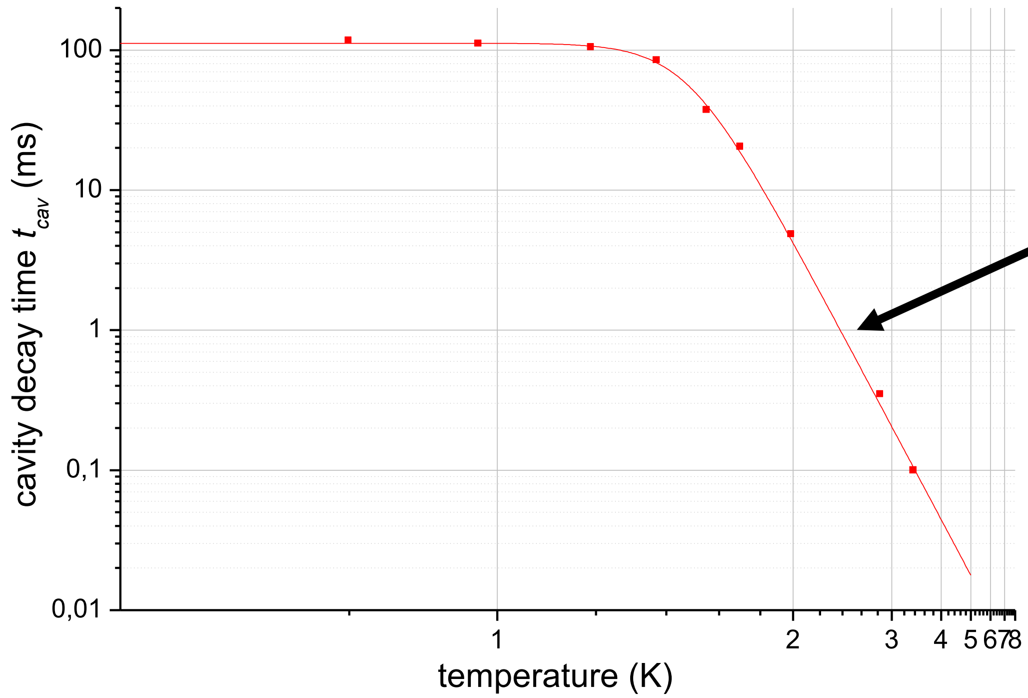
- 12 µm Niobium layer
  Cathode plasma sputtering
  CEA, Saclay

  [E. Jacques, B. Visentin, P. Bosland]

S. Kuhr et al, APL, **90**, 164101

An unprecedented quality factor

- Best cavity damping time T_c=130 ms !!



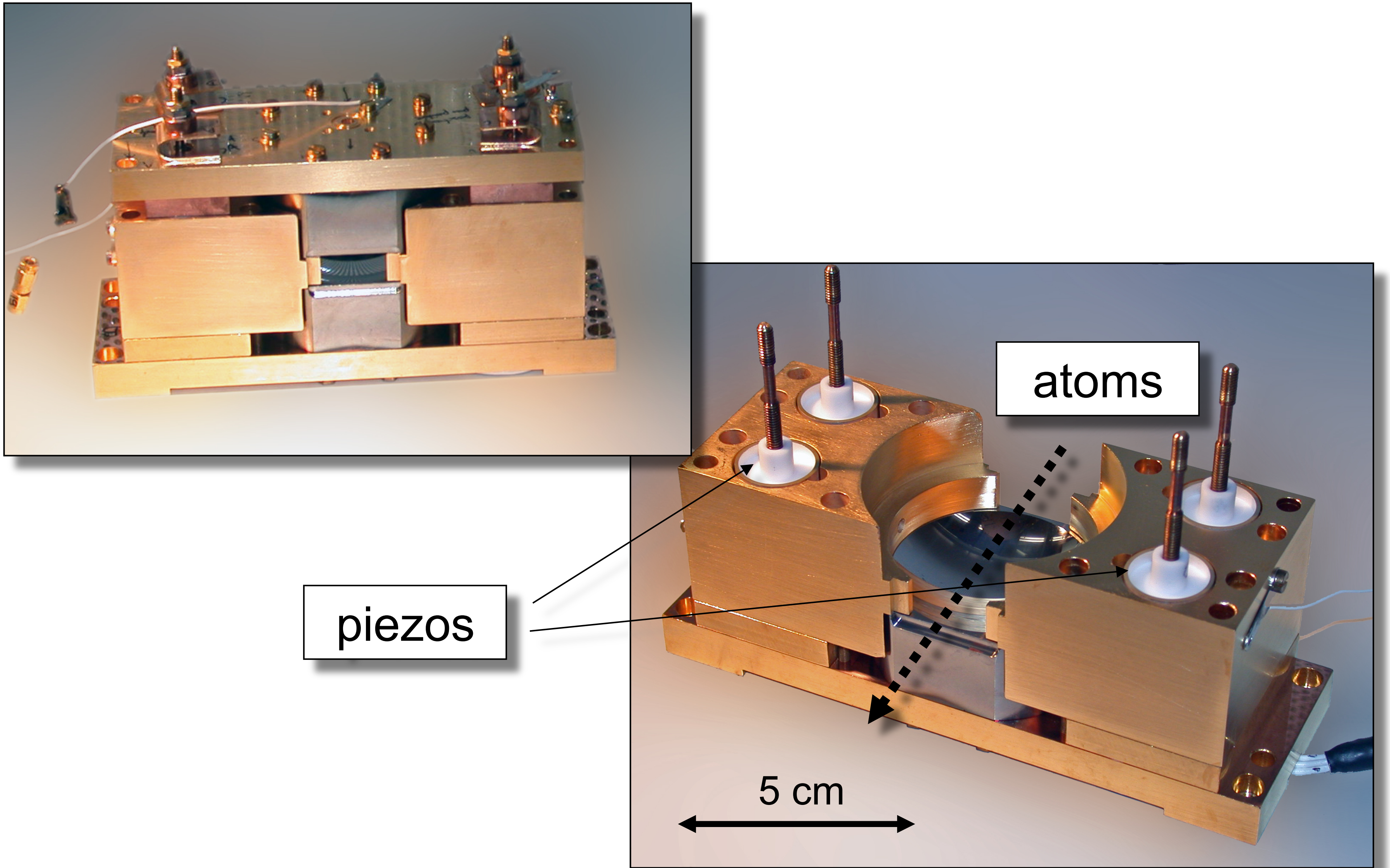
Q ~ Exp(Δ/kT)
BCS limit

- At 0.8 K
- $Q = \omega\, T_c = 4.2 \cdot 10^{10}$
- $F = 4.6 \cdot 10^{9}$

  – best mirrors so far
  – 1.1 billion bounces on the mirrors
  – 40000km travel between mirrors
    - plenty of time for atom-field interaction

S. Kuhr et al, APL **90**, 164101 (2007)

Cavity assembly



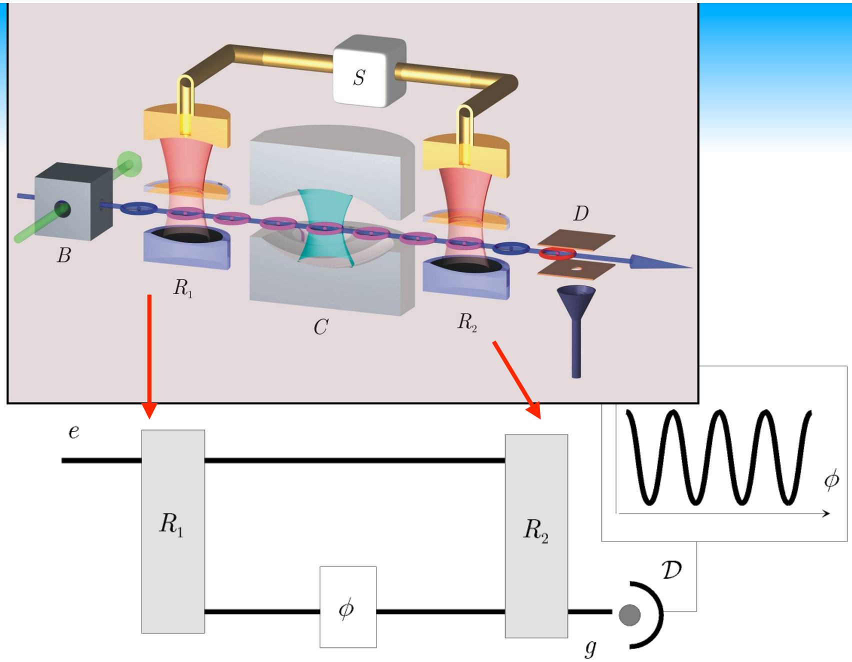
Preparation of a coherent state and Q measurement

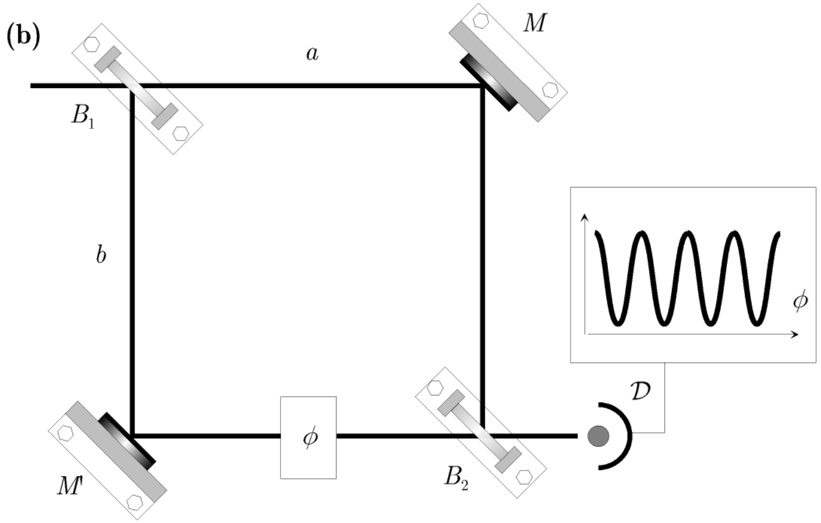
- Coherent state injection
  - Intense pulse generated by a classical microwave source
  - Sent in the spacing between cavity mirrors
  - Couples into the cavity mode through the diffraction loss channels
  - Injected photon numbers adjustable between much less than one and millions
  - Phase coherent injection

- Cavity lifetime measurement
  - Inject a large field (N photons) and probe it with atoms in the n=52 multiplicity in a large dc electric field after an adjustable delay
    - Atoms absorb field on a wide frequency range
  - Measure atomic response as a function of delay. S-curve
  - Repeat experiment with N/e^2 photons. New S-curve
  - The two atomic responses are translated in time by $2T_c$

Ramsey interferometer

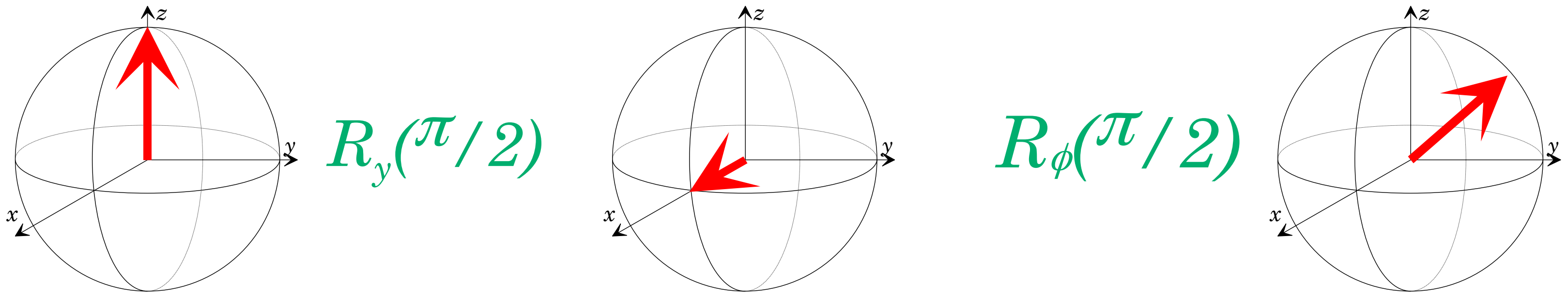
- An atomic version of the Mach-Zehnder interferometer
  - Two π/2 classical resonant pulses before and after cavity interaction
  - First pulse creates an e/g coherence
  - Second pulse probes this coherence
  - Transfer probability sinusoidal function of the relative phase of the second pulse and atomic coherence
  - An extremely sensitive probe of atomic state change during interaction with cavity



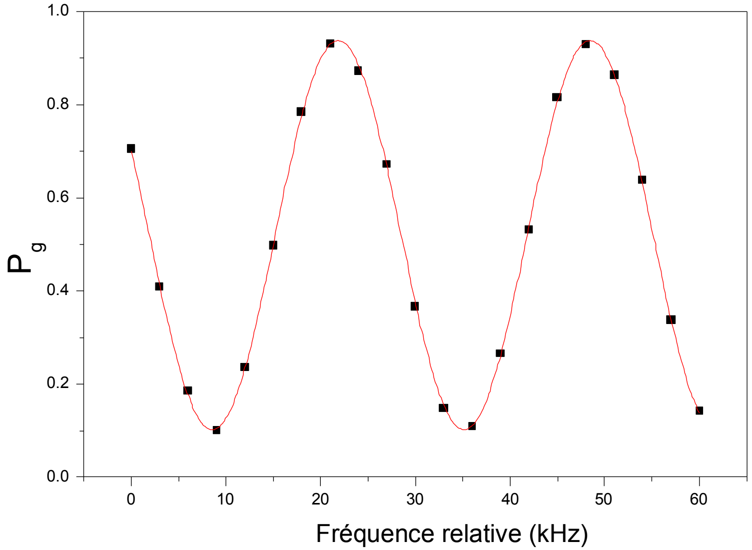


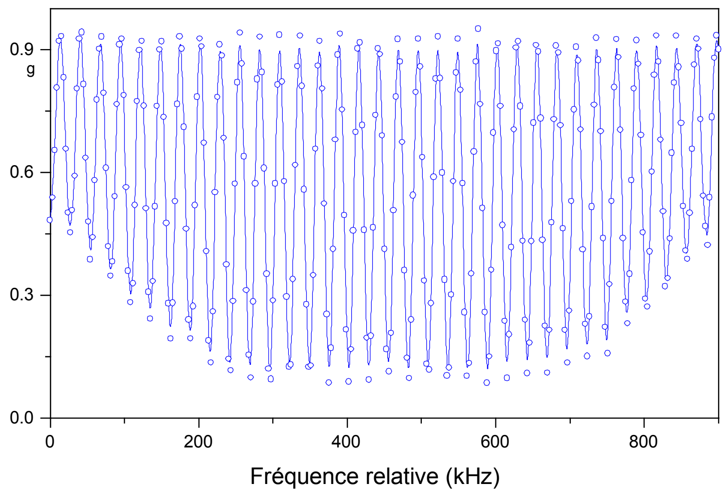
Ramsey as a spin interferometer

Two rotations of the spin ½ representing the atomic transition around different axes

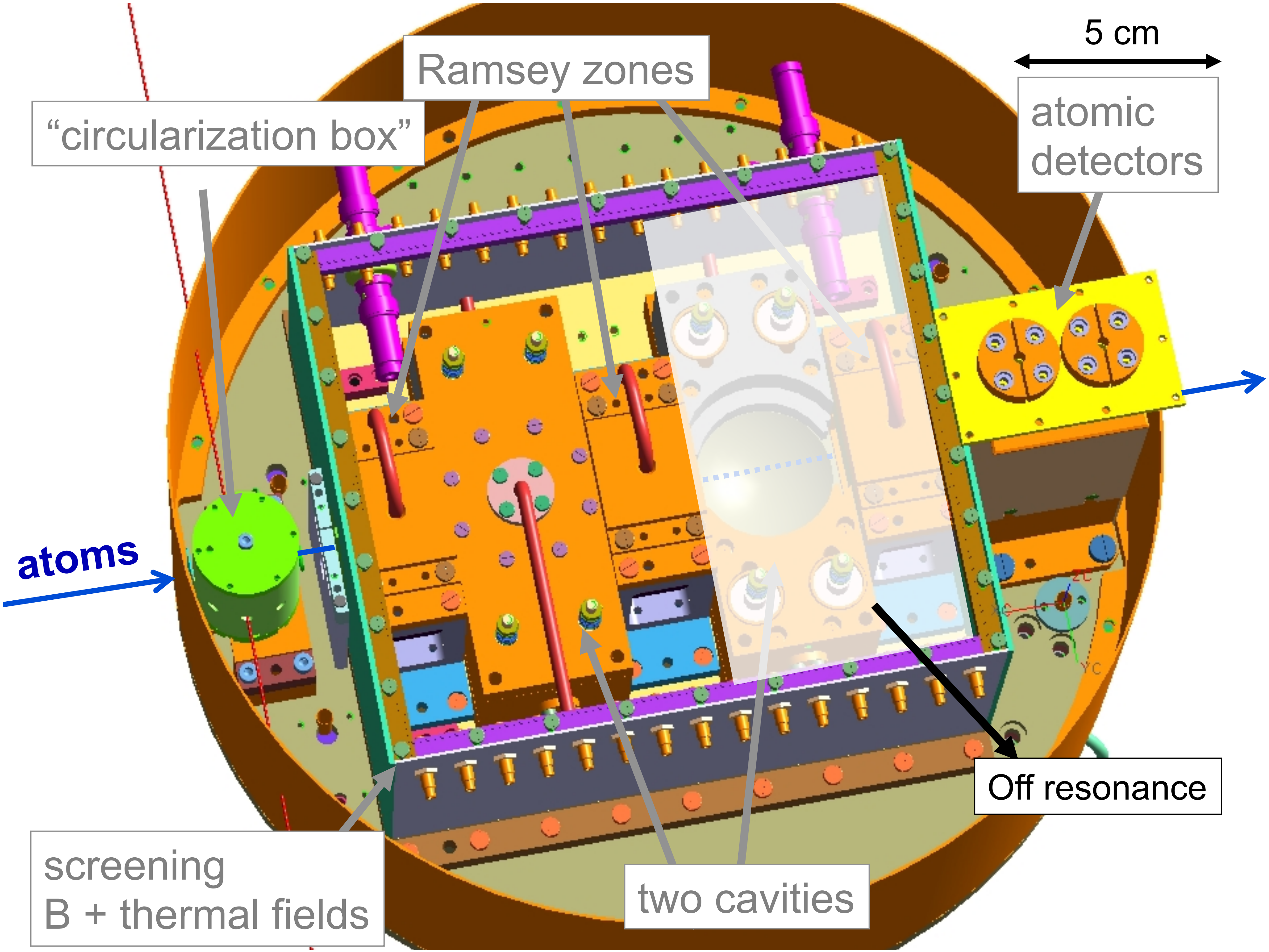


- Two Stern and Gerlach devices
- Polarizer and analyzer

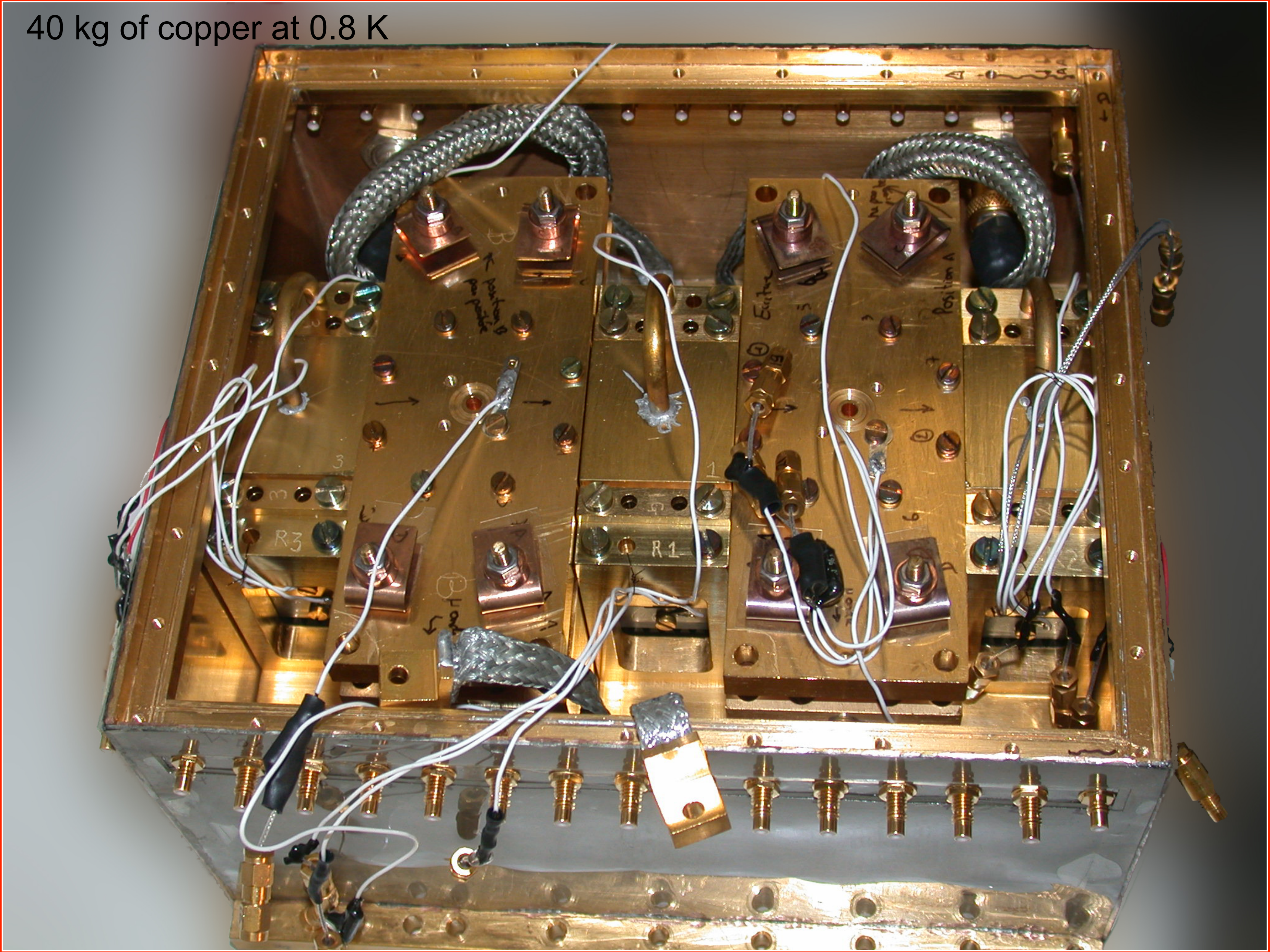




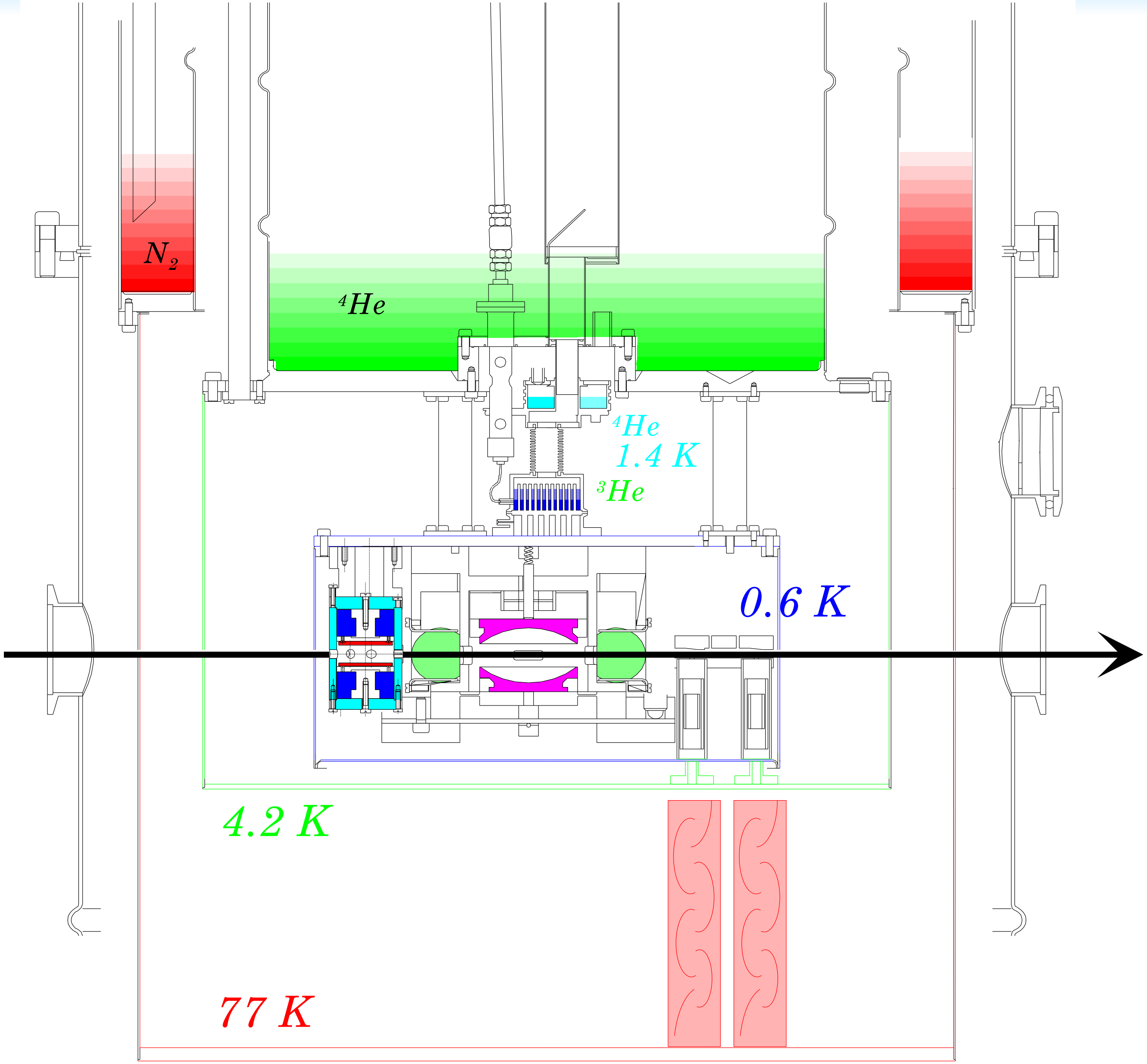
High (80-90%) contrast fringes


5 cm
Ramsey zones
"circularization box"
atomic detectors
atoms
Off resonance
screening
B + thermal fields
two cavities

40 kg of copper at 0.8 K



The ^3He-^4He refrigerator



These lectures

I) Introduction

II) Experimental tools for microwave CQED

III) Theoretical tools for microwave CQED

IV) Resonant microwave CQED

V) Dispersive microwave CQED

VI) Conclusion and perspectives

A cavity field mode

- Quadratic hamiltonian as in the case of mechanical oscillator (field is a collection of harmonic oscillators)
- Photon annihilation and creation operators

$$H_c = \hbar\omega_c(N + 1/2) \qquad H_c' = \hbar\omega_c N$$

- Electric field operator

$$\mathbf{E}_c = i\mathcal{E}_0\left(\mathbf{f}(\mathbf{r})a - \mathbf{f}^*(\mathbf{r})a^\dagger\right) \qquad \mathbf{f}(\mathbf{r}) = \boldsymbol{\epsilon}_c f(\mathbf{r})$$

  - $\mathcal{E}_0$ normalization factor (dimension of a field)
  - $\boldsymbol{\epsilon}_c$ local polarization
  - $\mathbf{f}(\mathbf{r})$ relative field mode amplitude and polarization (1 at field maximum) a solution of Helmoltz equation with cavity limiting conditions

A normalization issue

- Normalization of the mode function
  - Up to now in these lectures, integral of $|f|^2$ is the volume of the fictitious quantization box.
  - In CQED we have an actual quantization box
    - Define instead the normalization of f so that it is 1 at the field maximum
    - Defines the cavity mode volume as the integral of $|f|$^2
  - Only a matter of coefficients definition. No impact on physical results

Field normalization

- Energy of Fock states

$$\langle n|\int \varepsilon_0 |\mathbf{E}_c|^2\, d^3\mathbf{r}|n\rangle = \hbar\omega_c(n+\frac{1}{2})$$

$$\langle n|\varepsilon_0 \mathcal{E}_0^2 \mathcal{V}(2N+1)|n\rangle = \hbar\omega_c(n+\frac{1}{2})$$

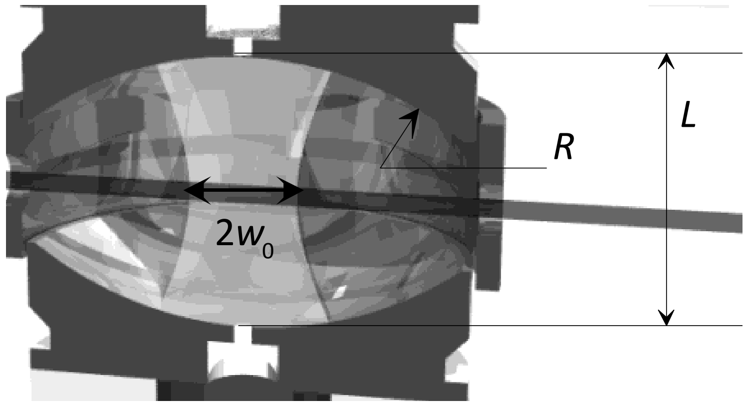
- Cavity mode volume

$$\mathcal{V} = \int |\mathbf{f}(\mathbf{r})|^2\, d^3\mathbf{r}$$

$$\mathcal{E}_0 = \sqrt{\frac{\hbar\omega_c}{2\varepsilon_0\mathcal{V}}}$$

- For the Gaussian mode of a FP cavity

$$\mathcal{V} = \frac{\pi}{4} w_0^2 L$$

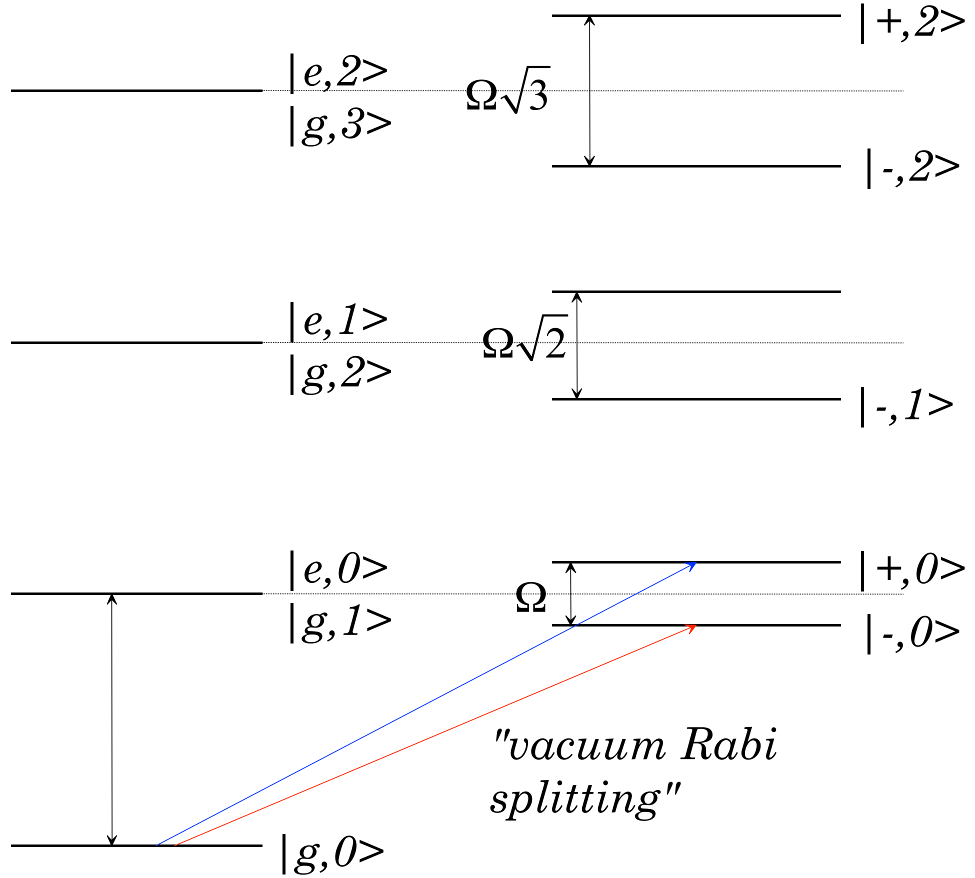


- For cavities used in mm-wave CQED
  - $V = 0.7\,\mathrm{cm}^3$, $E_0 = 1.5$ mV/m

The resonant case

- Atom-cavity at exact resonance

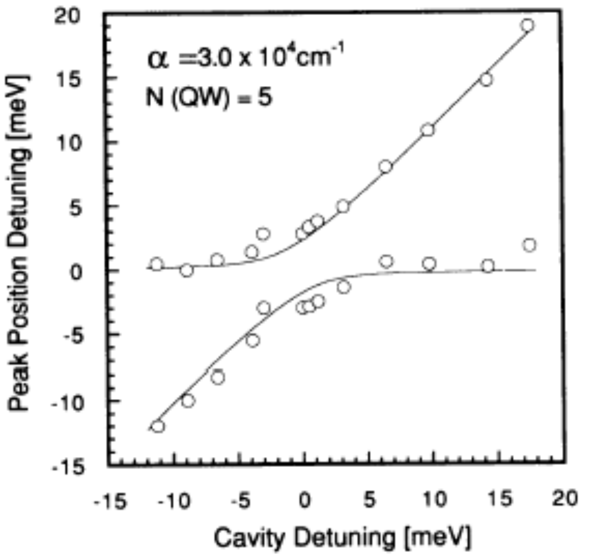
$$|\pm, n\rangle = \frac{1}{\sqrt{2}}\left[|e,n\rangle \pm i|g,n+1\rangle\right]$$



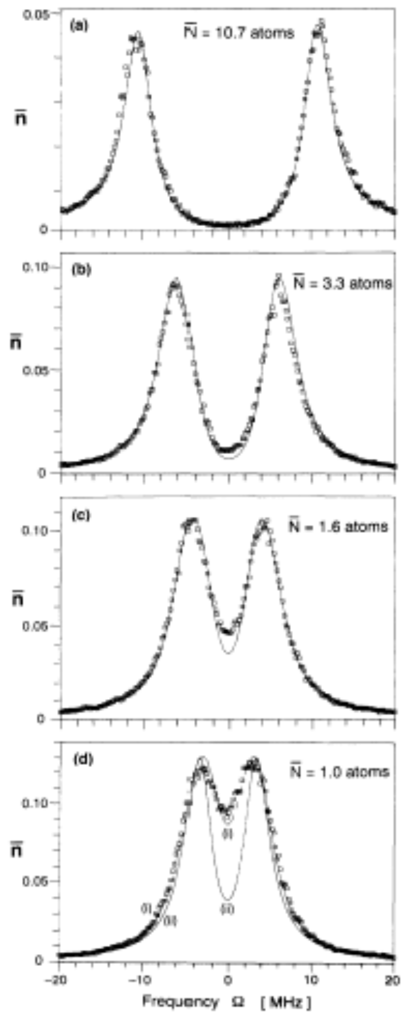
- A doublet for the weak excitation from the ground state:
  - the vacuum Rabi splitting
- *e* and *g* are no longer eigenstates
  - a quantum Rabi oscillation between these levels

The vacuum Rabi splitting

- Equivalent to the normal mode splitting for two coupled oscillators.
  – Observed for an atom in an optical cavity and for excitons in a semiconducting cavity



Weisbuch et al 92



Kimble et al 1992

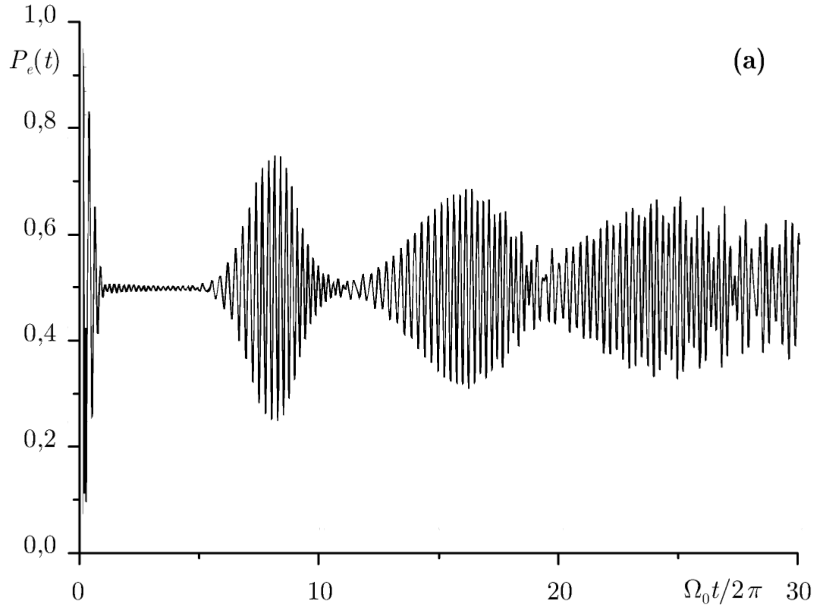
Rabi oscillation in a coherent field

- Intermediate regime of a few tens of photons.
  - A simple theoretical problem $|\alpha\rangle = \sum_n c_n |n\rangle$;

$$|\Psi(t)\rangle = \sum_n c_n \cos\frac{\Omega_0\sqrt{n+1}t}{2}|e,n\rangle + c_n \sin\frac{\Omega_0\sqrt{n+1}t}{2}|g,n+1\rangle \ ,$$

$$P_e(t) = \sum_n p_c(n)\frac{1+\cos\Omega_0\sqrt{n+1}t}{2} \ .$$

  - A surprisingly complex behavior



Collapse and revival

- Collapse:
  - dispersion of field amplitudes due to dispersion of photon number

$$t_c \approx \pi/\Omega_0 \ .$$

- Revival:
  - rephasing of amplitudes at a finite time such that oscillations corresponding to n and n+1 come back in phase

$$t_r \approx \frac{4\pi}{\Omega_0}\sqrt{\overline{n}} \ .$$

  - Revival is a genuinely quantum effect

Action of an atom on a coherent field in the dispersive regime

- Define an effective hamiltonian for shifts

$$H_{eff} = \hbar s_0 \left[\sigma_Z (a^\dagger a + \frac{1}{2}) + \frac{1}{2}\mathbb{1} \right]$$

$$U_{eff}(t) = e^{-i\Phi/2} e^{-i\Phi\sigma_z a^\dagger a} e^{-i\Phi\sigma_z/2} \qquad \Phi = s_0 t.$$

- Apply to $|e, \alpha\rangle$

$$e^{-i\Phi a^\dagger a}|\alpha\rangle = \qquad e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\Phi a^\dagger a}|n\rangle$$

$$= e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-in\Phi}|n\rangle = |\alpha e^{-i\Phi}\rangle$$

$$|e, \alpha\rangle \longrightarrow e^{-i\Phi}|e, \alpha e^{-i\Phi}\rangle$$

$$|g, \alpha\rangle \longrightarrow |g, \alpha e^{i\Phi}\rangle \ .$$

  - The atom (quantum system) controls the classical phase of the field
  - At the heart of Schrödinger cat states generation

Taking into account atomic motion

- Real atoms cross gaussian mode

$$f(vt) = e^{-v^2t^2/w_0^2}$$

- Simple expressions only in resonant and dispersive cases
  - Resonant case

$$\widetilde{H}(t) = f(t)\widetilde{H}(0).$$

$$U^r_\infty = \exp((i/\hbar)\int H(t)\,dt) = \exp((i/\hbar)H(0)t^r_i)$$

$$t^r_i = \int f(vt)\,dt = \sqrt{\pi}\frac{w_0}{v}\ .$$

- All expressions obtained at r=0 remain valid when replacing real time by the effective interaction time taking account mode geometry

Taking into account atomic motion

– Dispersive case
  - Use effective hamiltonian, proportional to f^2

$$U_\infty^d = \exp((i/\hbar) H_e(0) t_i^d)$$

$$t_i^d = \int f^2(vt)\, dt = \sqrt{\frac{\pi}{2}} \frac{w_0}{v}$$

  - The r=0 results also valid when using the effective interaction time
  - Note that resonant and non-resonant effective interaction times are not equal

These lectures

Resonant microwave CQED

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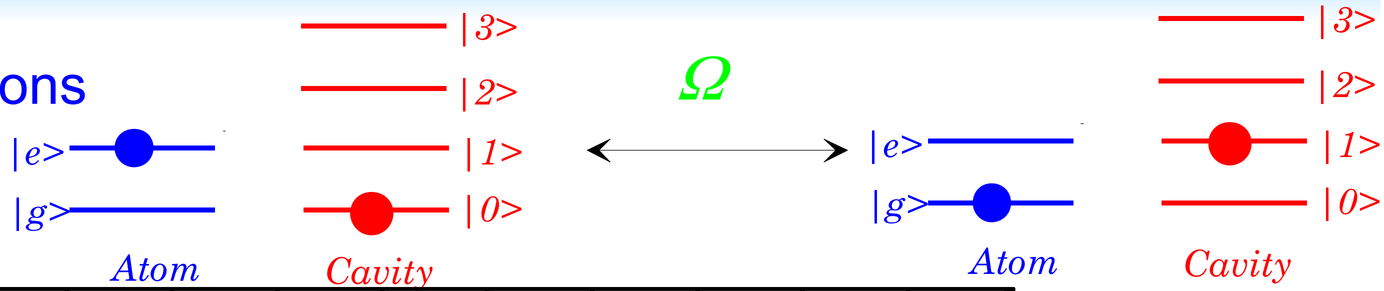
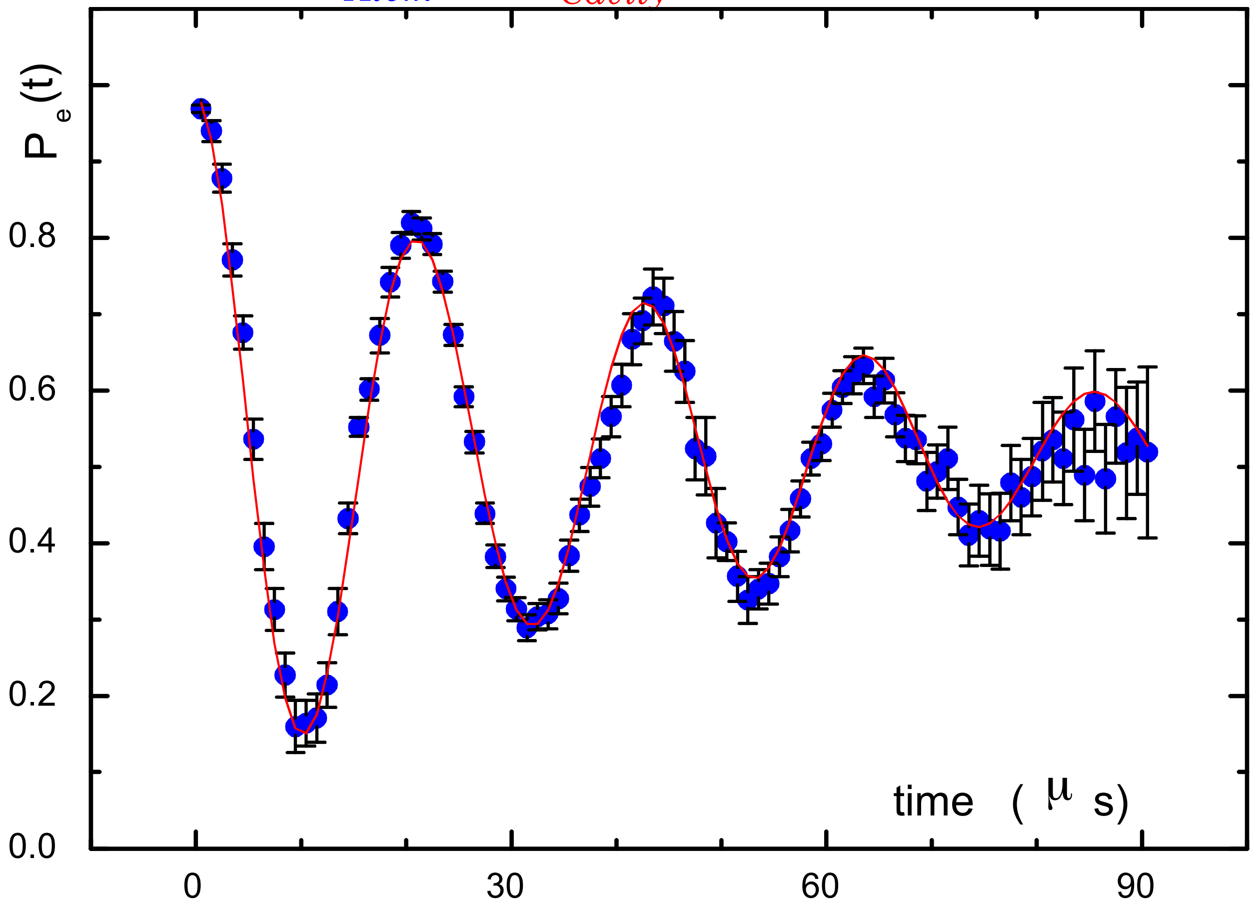
Resonant atom-cavity interaction

Quantum Rabi oscillations

Initial state |e,0>



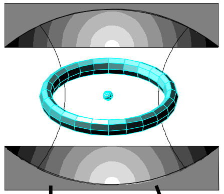


Brune et al, PRL **76,** 1800 (96)

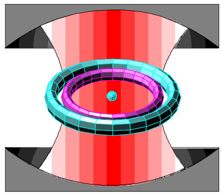
Oscillatory Spontaneous emission and strong coupling regime

Quantum Rabi oscillations: state transformations

Initial state

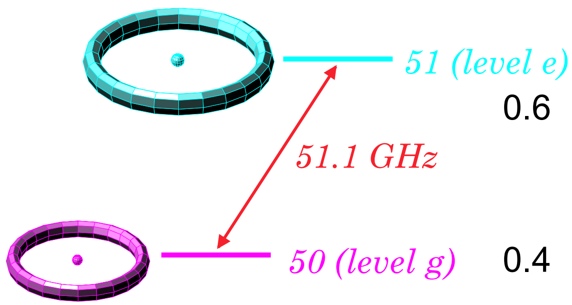


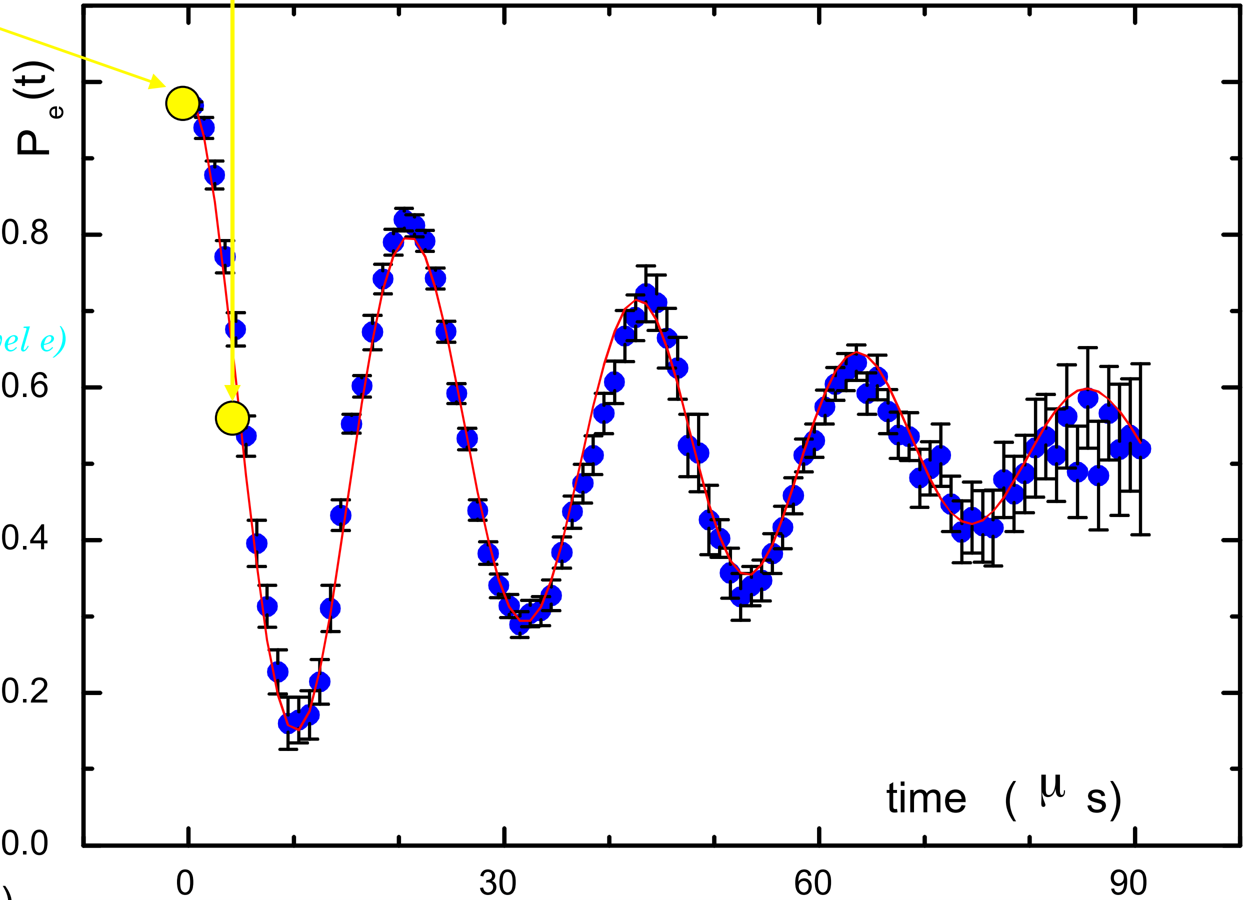
$$|e,0\rangle$$



$$|e,0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|e,0\rangle + |g,1\rangle\right)$$

π/2 spontaneous emission pulse
Entanglement creation
Atom-cavity EPR pair

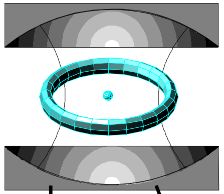




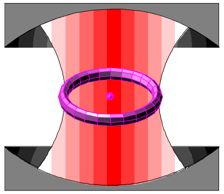
Brune et al, PRL **76,** 1800 (96)

Quantum Rabi oscillations: state transformations

Initial state



$|e,0\rangle$

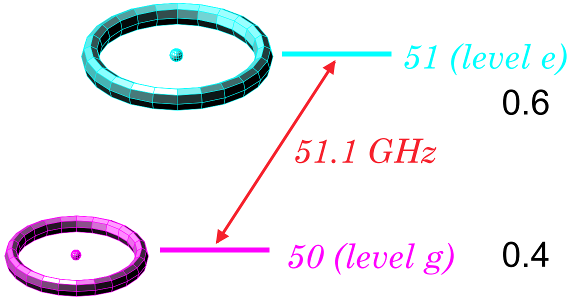


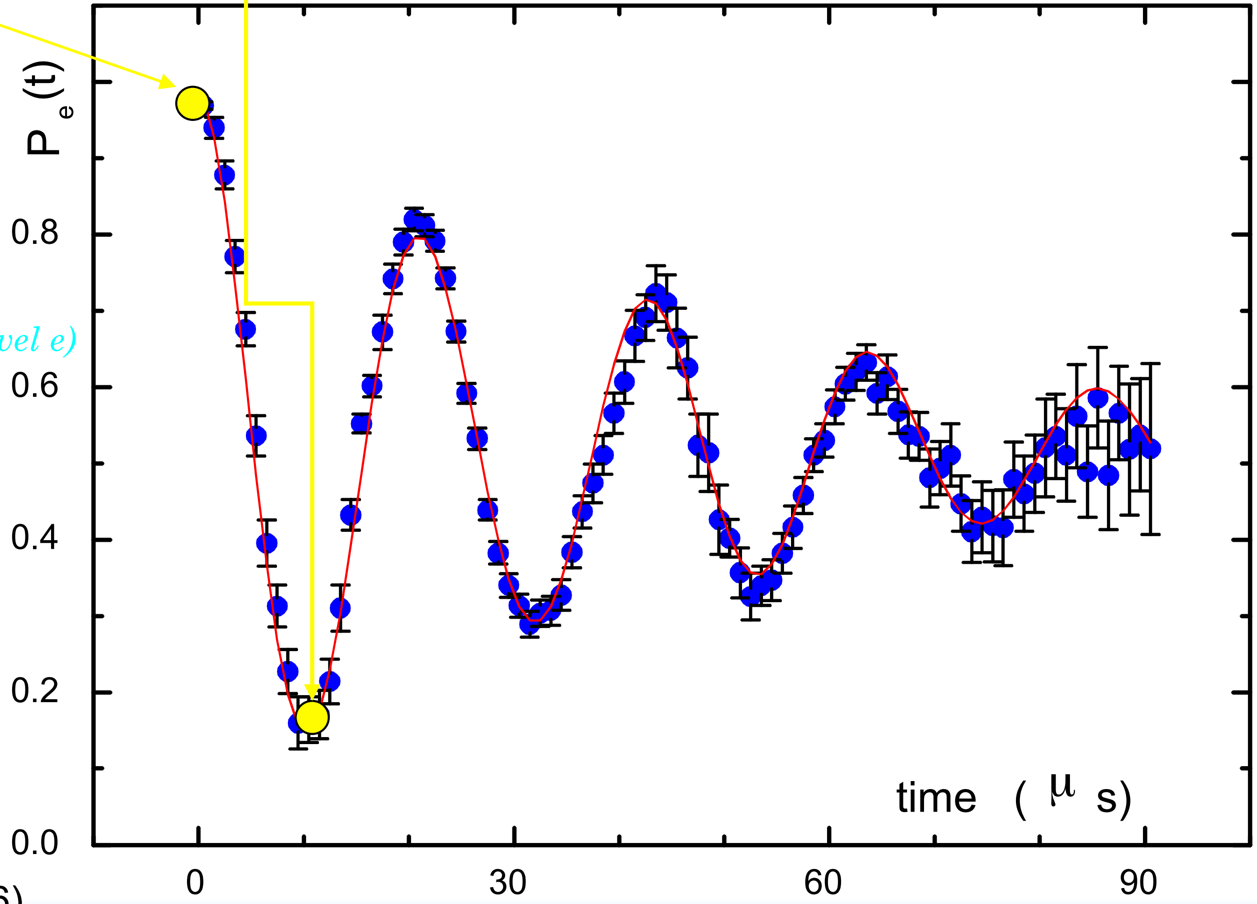
$$|e,0\rangle \longrightarrow |g,1\rangle$$

$$\left(c_e|e\rangle + c_g|g\rangle\right)|0\rangle \longrightarrow |g\rangle\left(c_e|1\rangle + c_g|0\rangle\right)$$

π spontaneous emission pulse

Atom/cavity state copy





Brune et al, PRL **76,** 1800 (96)

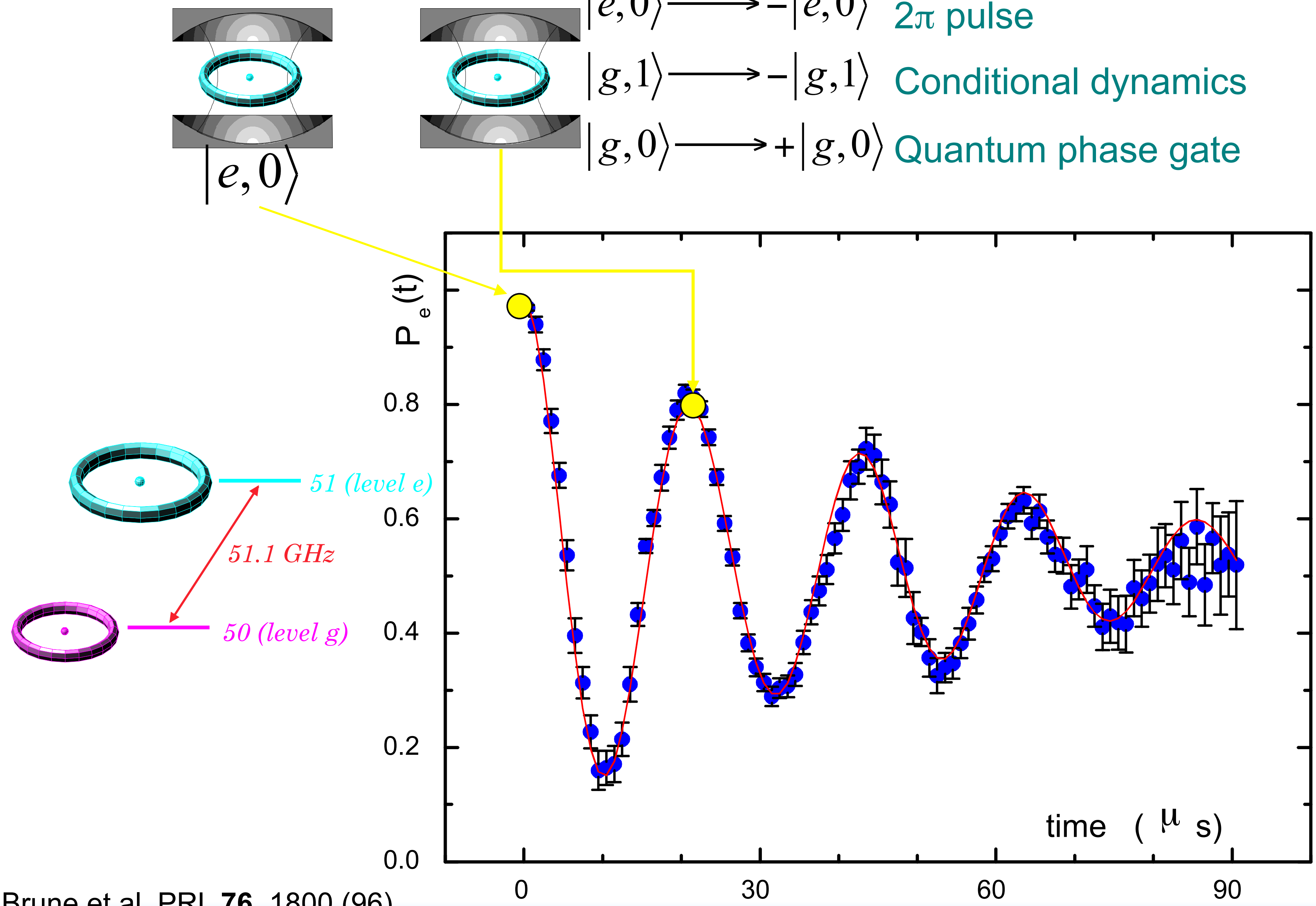
Quantum Rabi oscillations: state transformations

Initial state

$$|e,0\rangle \longrightarrow -|e,0\rangle \quad 2\pi \text{ pulse}$$

$$|g,1\rangle \longrightarrow -|g,1\rangle \quad \text{Conditional dynamics}$$

$$|g,0\rangle \longrightarrow +|g,0\rangle \quad \text{Quantum phase gate}$$



Brune et al, PRL **76,** 1800 (96)

Three "stitches" to "knit" quantum entanglement

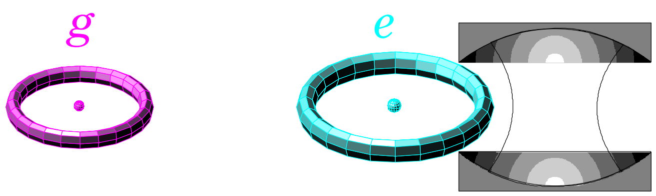
Combine elementary transformations to create complex entangled states

- State copy with a p pulse
  - Quantum memory : PRL **79**, 769 (97)
- Creation of entanglement with a p/2 pulse
  - EPR atomic pairs : PRL **79**, 1 (97)
- Quantum phase gate based on a 2p pulse
  - Quantum gate : PRL **83**, 5166 (99)
  - Absorption-free detection of a single photon: Nature **400**, 239 (99)
- Entanglement of three systems (six operations on four qubits)
  - GHZ Triplets : Science **288**, 2024 (00)
- Entanglement of two radiation field modes
  - Phys. Rev. A **64**, 050301 (2001)
- Direct entanglement of two atoms in a cavity-assisted collision
  - Phys. Rev. Lett. **87**, 037902 (2001)

Creation of an EPR atom pair

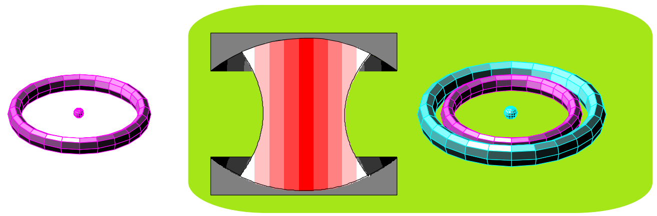
A simple entanglement manipulation experiment

- Initial state



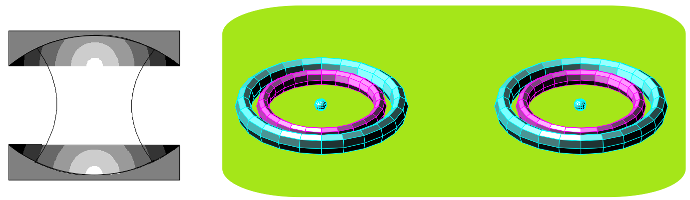
$$|e,g,0\rangle$$

- $\pi/2$ pulse:
- Entanglement creation



$$\frac{1}{\sqrt{2}}\left(|e,0\rangle+|g,1\rangle\right)|g\rangle$$

- State copy



- Final state $\frac{1}{\sqrt{2}}\left(|e,g\rangle-|g,e\rangle\right)$ in spin terms: $\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle-|\downarrow,\uparrow\rangle\right)$

$$=\frac{1}{\sqrt{2}}\left(|\rightarrow,\leftarrow\rangle-|\leftarrow,\rightarrow\rangle\right)$$

Estimated fidelity 48%

Hagley et al, PRL **79**, 1 (97)

Resonant microwave CQED

1. Quantum Rabi oscillation and entanglement knitting
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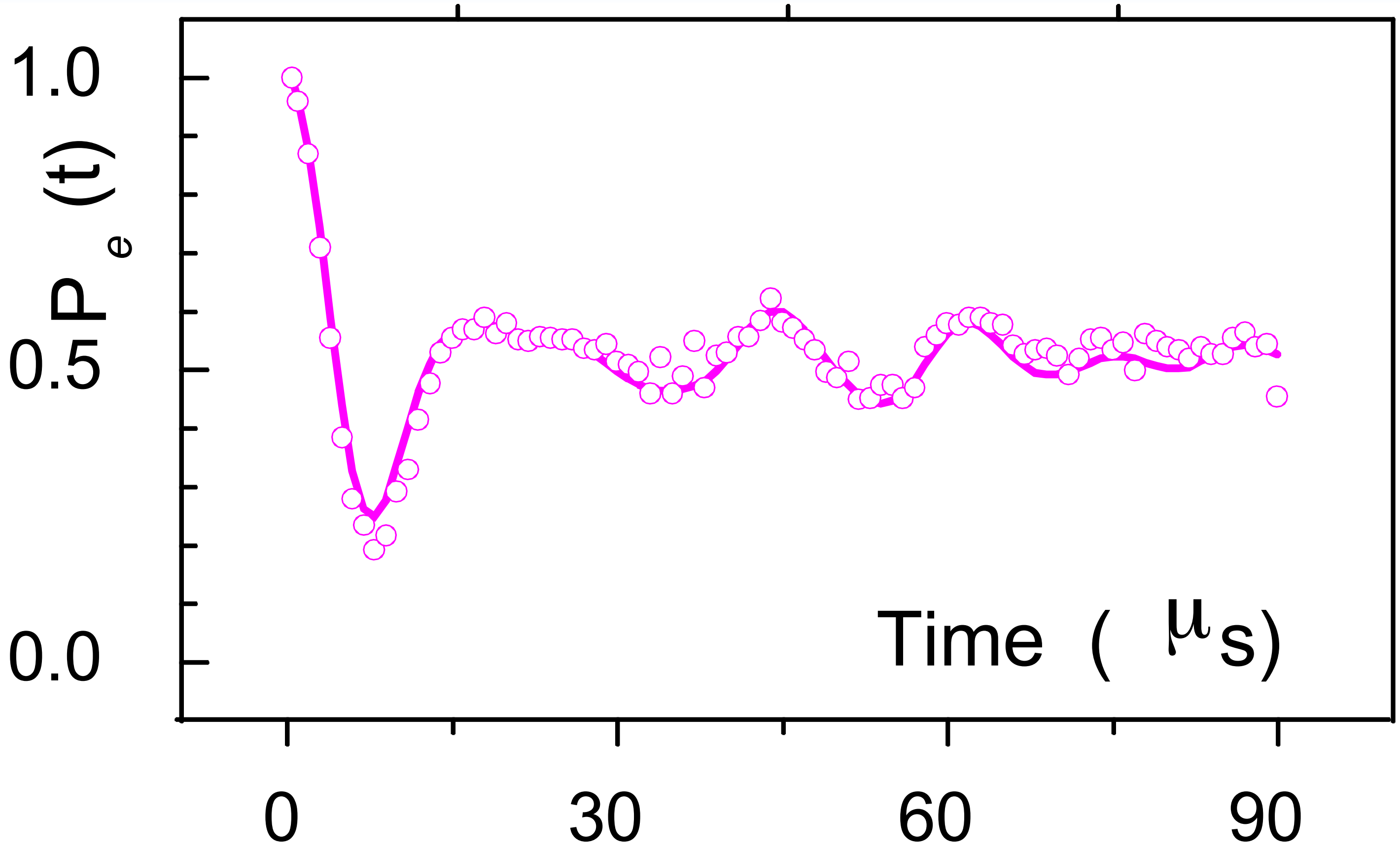
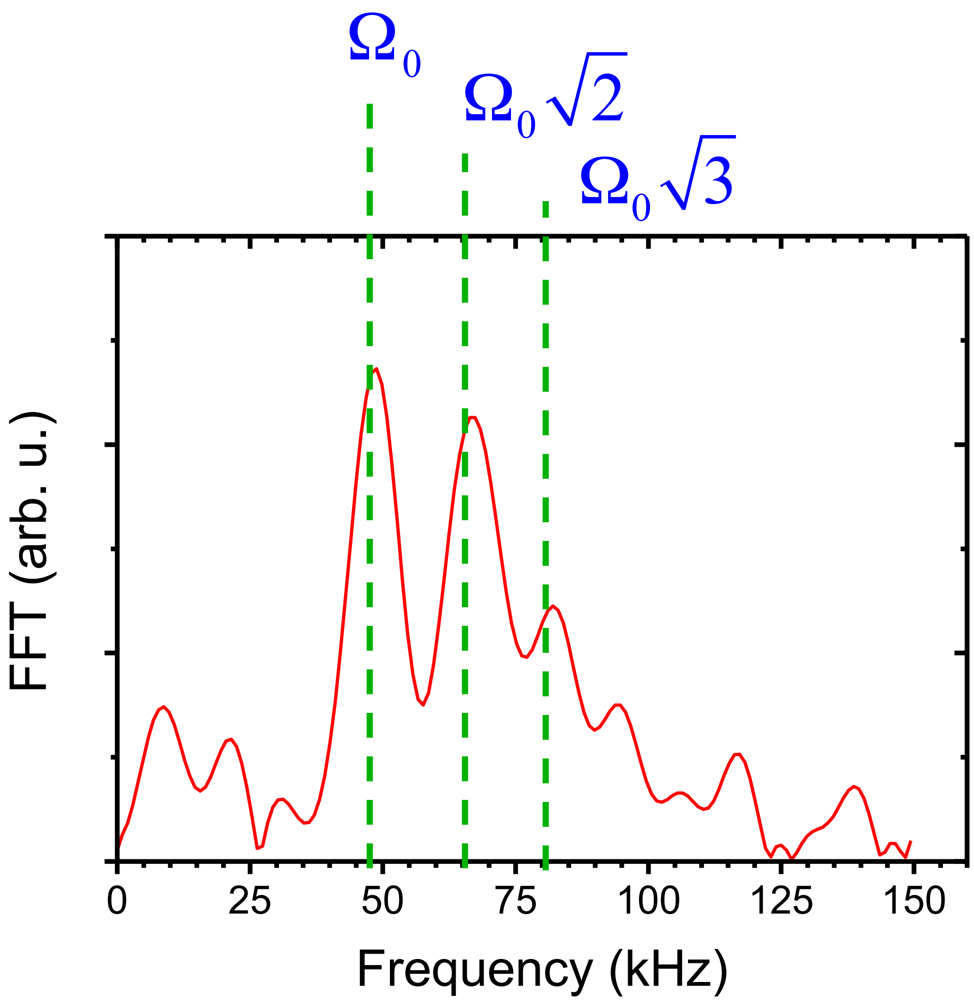
Rabi oscillation in a small coherent field



- A more complex signal
- $\pi/2$ pulse possible for any cavity field by proper tuning of interaction time

Rabi oscillation in a small coherent field: observing discrete Rabi frequencies

Fourier transform of the Rabi oscillation signal



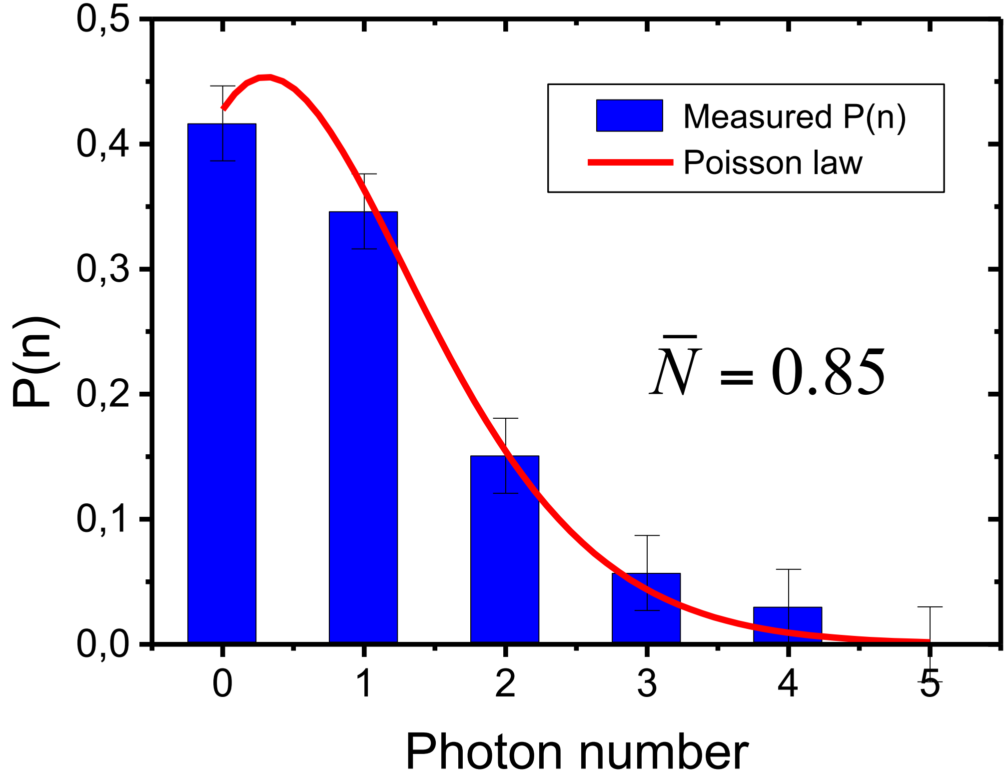
Discrete peaks corresponding to discrete photon numbers

→ Direct observation of field quantization in a "box"

Rabi oscillation in a small coherent field: Measuring the photon number distribution

$$P_g(t) = \sum_N P(N)\frac{1}{2}\left(1-\cos\left(\Omega_0 t\sqrt{N+1}\right).e^{-t/\tau}\right)$$

→ Fit of P(n) on the Rabi oscillation signal:



→ accurate field statistics measurement

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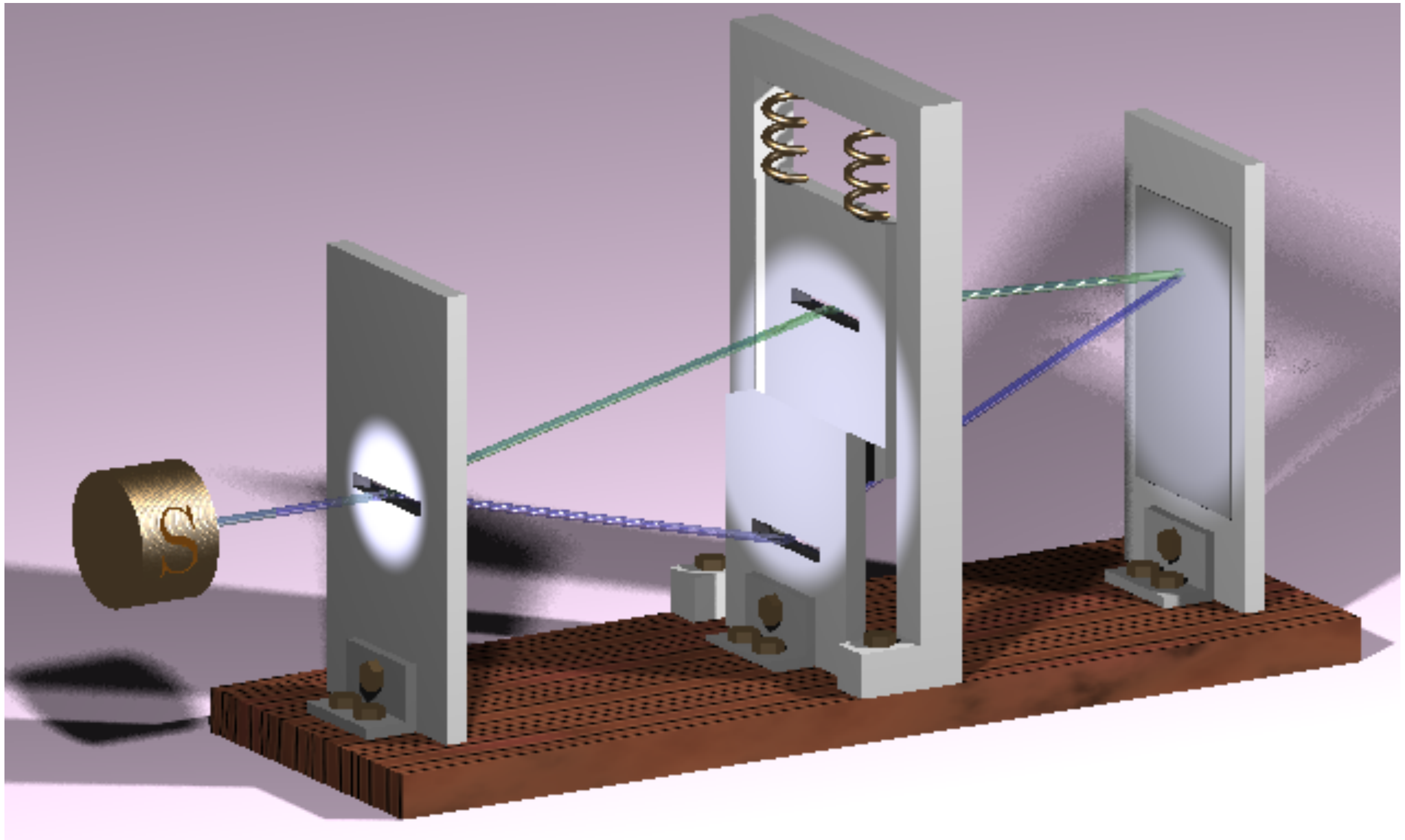
Bohr's thought experiment on complemetarity

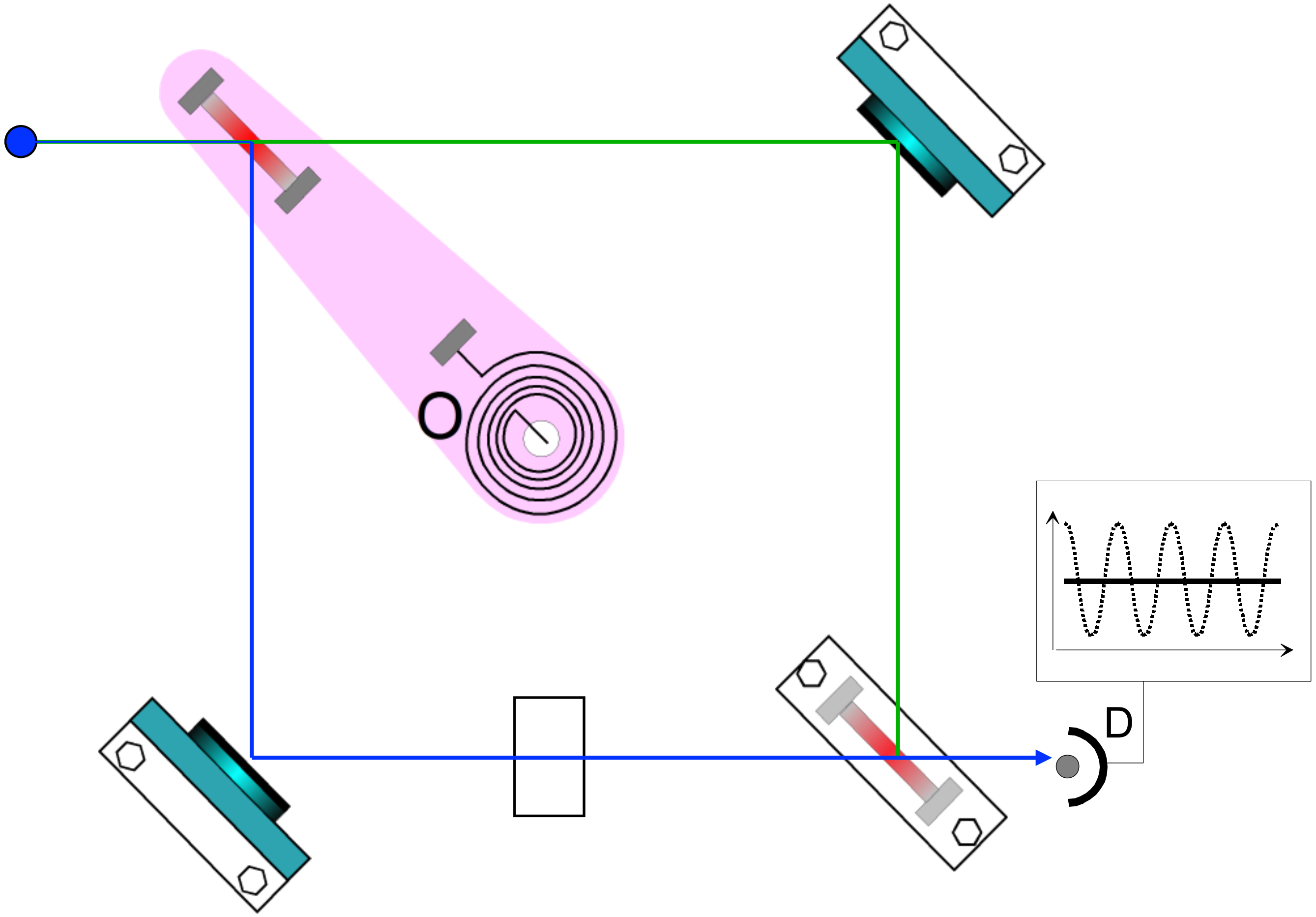
- Complementarity (From Einstein-Bohr at the 1927 Solvay congress)



  – Microscopic slit: set in motion when deflecting particle.
    - Which path information and no fringes
  – Macroscopic slit: impervious to interfering particle.
    - No which path information and fringes
  – Wave and particle are complementary aspects of the quantum object.

A “modern” version of Bohr’s proposal

- Mach-Zehnder interferometer with a moving slit



- Massive slit: negligible motion, no which- path information, fringes
- Microscopic slit: which path information and no fringes

Complementarity and uncertainty relations

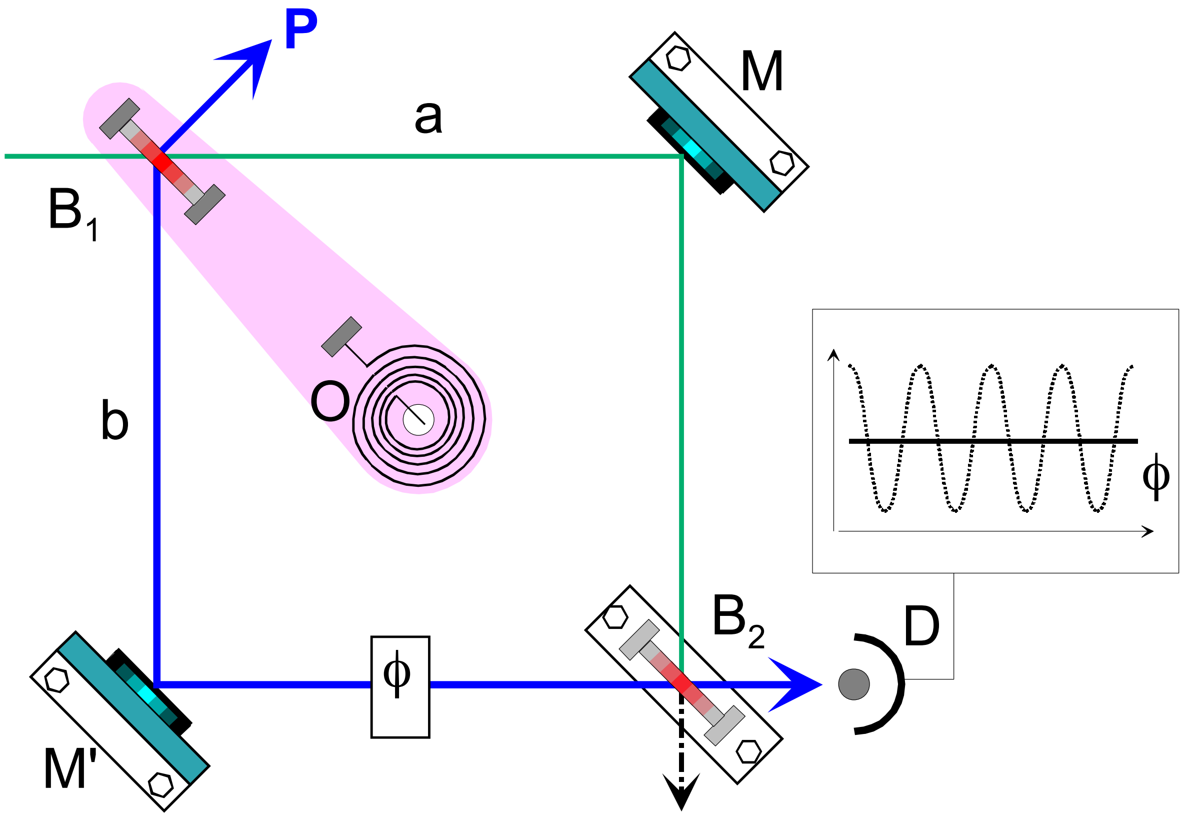
Get a which path information?

$P > \Delta p$

(Δp quantum fluctuations of beam splitter's momentum)

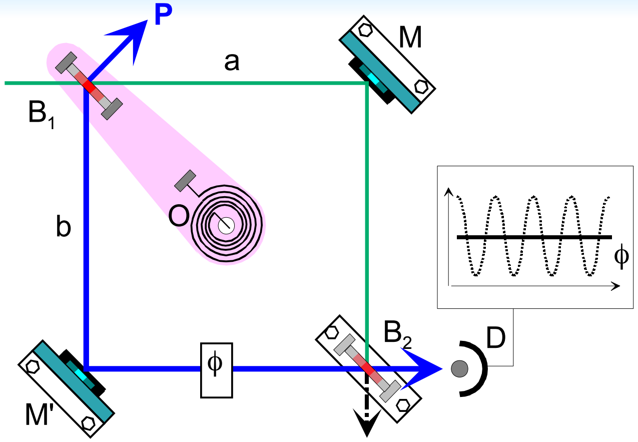
Hence

$\Delta x > h/\Delta p > h/P = \lambda$



Beam splitter's quantum position fluctuations larger than wavelength: no fringes

Complementarity and entanglement



- A more general analysis of Bohr's experiment
  - Initial beam-splitter state $|0\rangle$
  - Final state for path b $|\alpha\rangle$
  - Particle/beam-splitter state $|\Psi\rangle = |\Psi_a\rangle|0\rangle + |\Psi_b\rangle|\alpha\rangle$
    - Particle/beam-splitter entanglement
    - (an EPR pair if states orthogonal)
  - Final fringes signal $\left|\langle\Psi_a|\Psi_b\rangle\langle 0|\alpha\rangle\right|$
    - Small mass, large kick $|\langle 0|\alpha\rangle| = 0$

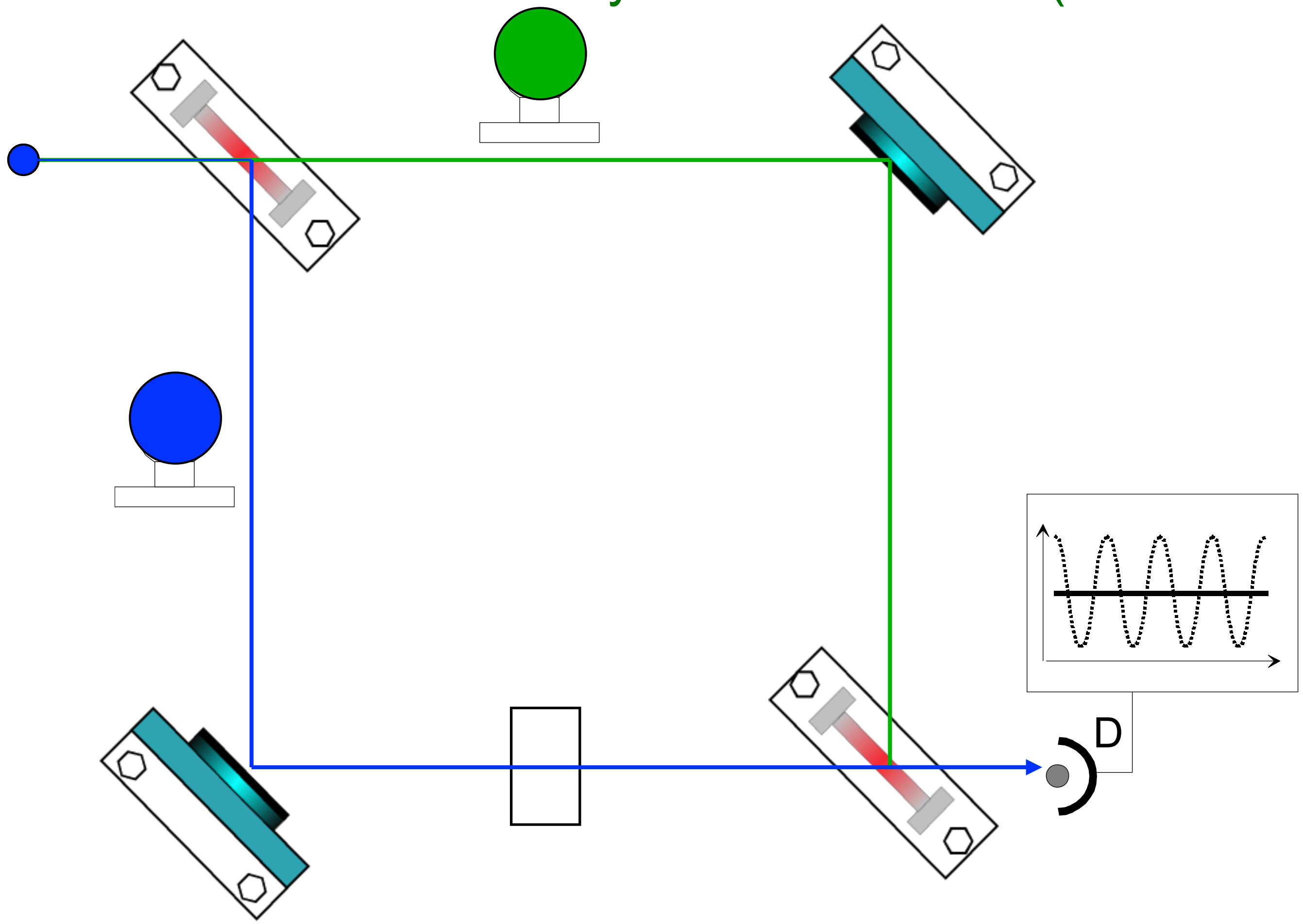
      NO FRINGES
    - Large mass, small kick $|\langle 0|\alpha\rangle| = 1$

      FRINGES

Entanglement and complementarity

Entanglement with another system destroys interference

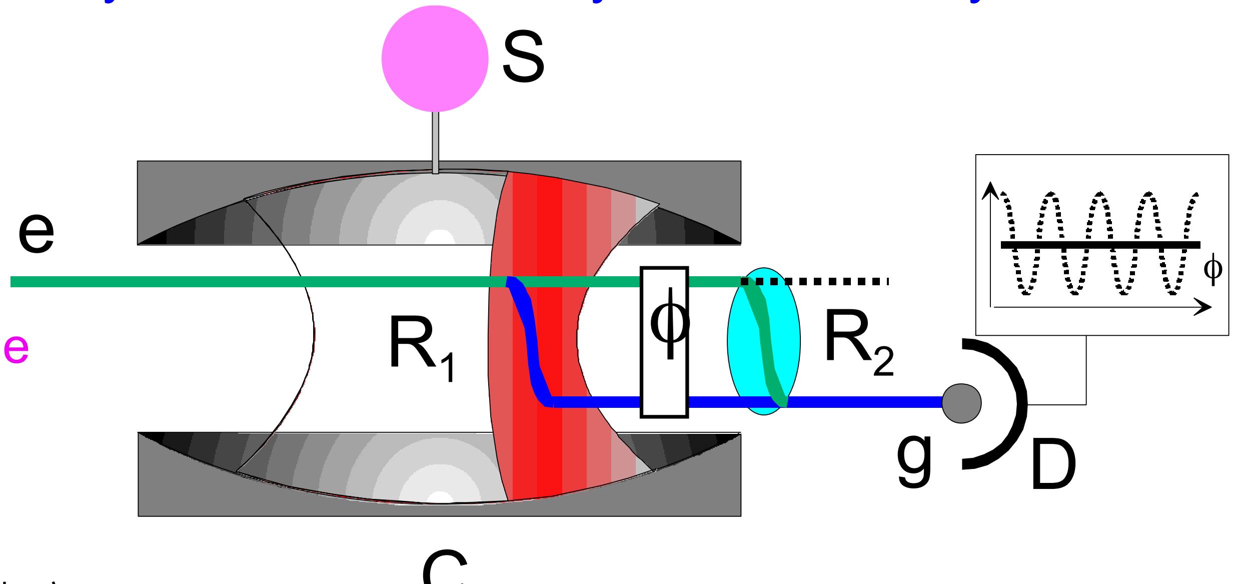
- explicit detector (beam-splitter/ external)
- uncontrolled measurement by the environment (decoherence)



Complementarity, decoherence and entanglement intimately linked

Bohr's experiment with a Ramsey interferometer

- Illustrating complementarity: Store one Ramsey field in a cavity

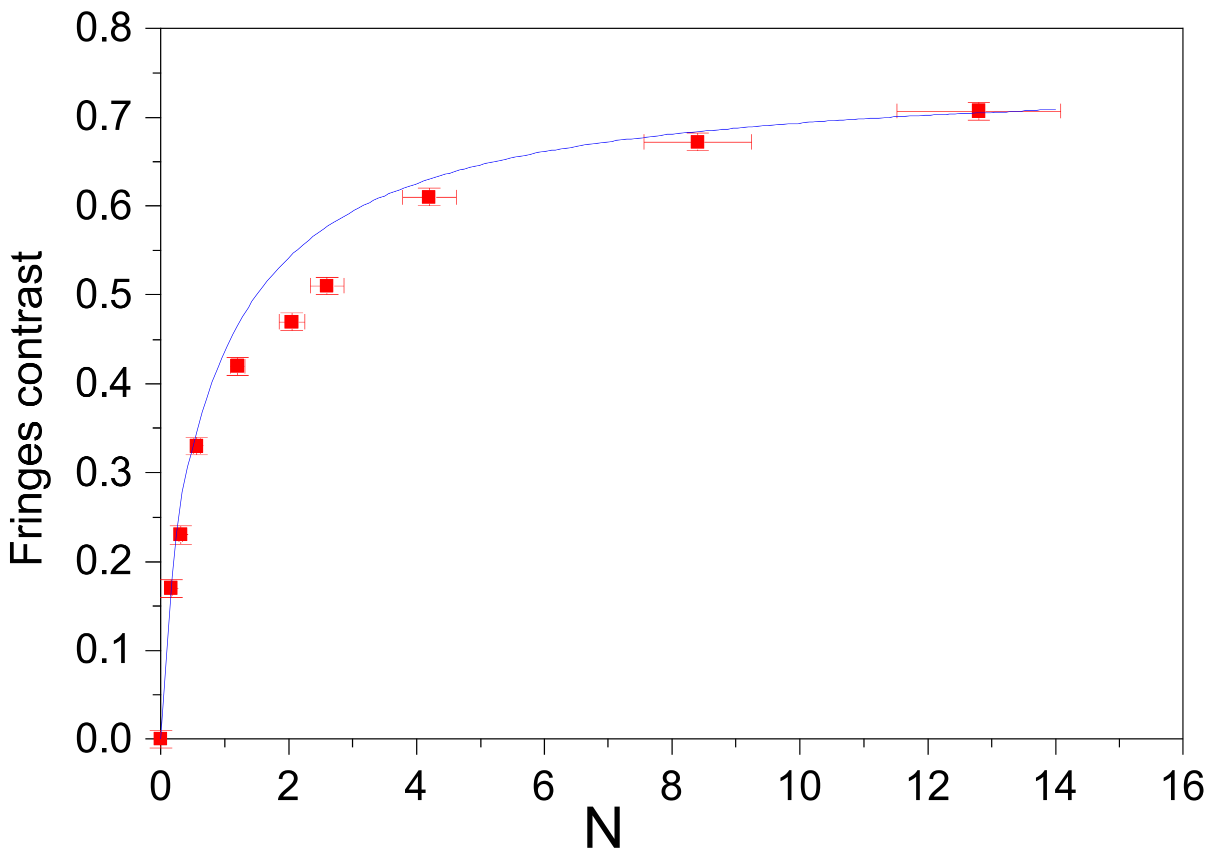


Atom-cavity interaction time
Tuned for π/2 pulse
Possible even if C empty

- Initial cavity state $|\alpha\rangle$
- Intermediate atom-cavity state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|e,\alpha_e\rangle + |g,\alpha_g\rangle\right)$
  - Ramsey fringes contrast $\left|\langle\alpha_e|\alpha_g\rangle\right|$
- Large field
  - $|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle$ FRINGES
- Small field
  - $|\alpha_e\rangle = |0\rangle, |\alpha_g\rangle = |1\rangle$ NO FRINGE

Quantum/classical limit for an interferometer

Fringes contrast versus photon number N



Nature, **411**, 166 (2001)

Fringes vanish for quantum field

photon number plays the role of the beam-splitter's "mass"

An illustration of the DNDF uncertainty relation :

- Ramsey fringes reveal field pulses phase correlations.
- Small quantum field: large phase uncertainty and low fringe contrast

Not a trivial blurring of the fringes by a classical noise: atom/cavity entanglement can be erased

Resonant microwave CQED

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Rabi oscillation in a classical field

Oscillation in a large coherent field

$$H_I = \hbar \frac{\Omega_r}{2} \sigma_Y \quad \Omega_r = \Omega_0 \sqrt{n} \propto E$$

$$|\Psi(t)>= \frac{1}{2}\left[e^{-i\Omega_{cl}t/2}(|e> + i|g>) + e^{i\Omega_{cl}t/2}(|e> - i|g>)\right] \otimes |\alpha\rangle$$

Atomic eigenstates

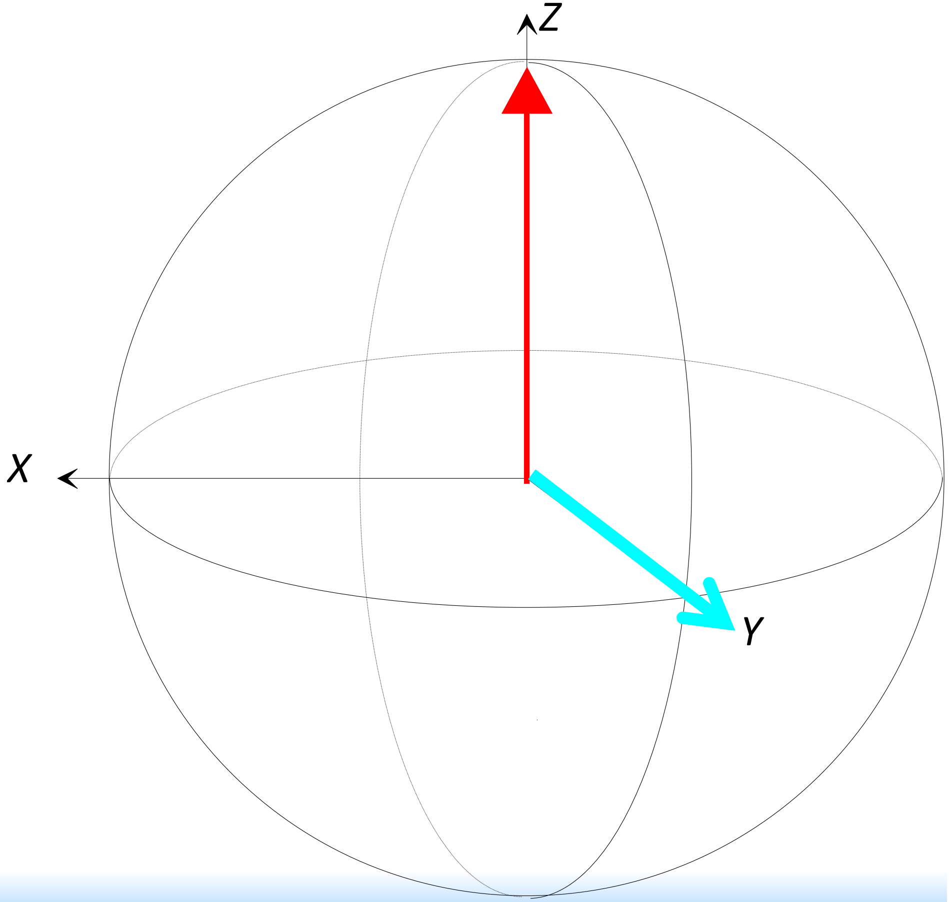
$$|\pm_Y\rangle = \frac{1}{\sqrt{2}}\left[|e\rangle \pm i|g\rangle\right]$$

In-phase and π-out-of-phase with respect to field

Quantum beat between eigenstates:

- Sinusoidal Rabi oscillation between *e* and *g*

In terms of Bloch sphere



An insightful quasi-exact solution

$$\left|\Psi(t)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\Psi_a^+(t)\right\rangle\left|\Psi_c^+(t)\right\rangle + \left|\Psi_a^-(t)\right\rangle\left|\Psi_c^-(t)\right\rangle\right]$$

$$\left|\Psi_a^\pm\right\rangle = \frac{1}{\sqrt{2}} e^{\pm i\Omega_0\sqrt{\bar{n}}t/2}\left[e^{\pm i\Phi}\left|e\right\rangle \mp i\left|g\right\rangle\right] \qquad \Phi = \frac{\Omega_0 t}{4\sqrt{\bar{n}}}$$

– Atomic states slowly ($\bar{n}$ times slower than Rabi oscillation) rotating in the equatorial plane of the Bloch sphere

$$\left|\Psi_c^\pm\right\rangle = e^{\mp i\Omega_0\sqrt{\bar{n}}t/4}\left|\alpha e^{\pm i\Phi}\right\rangle$$

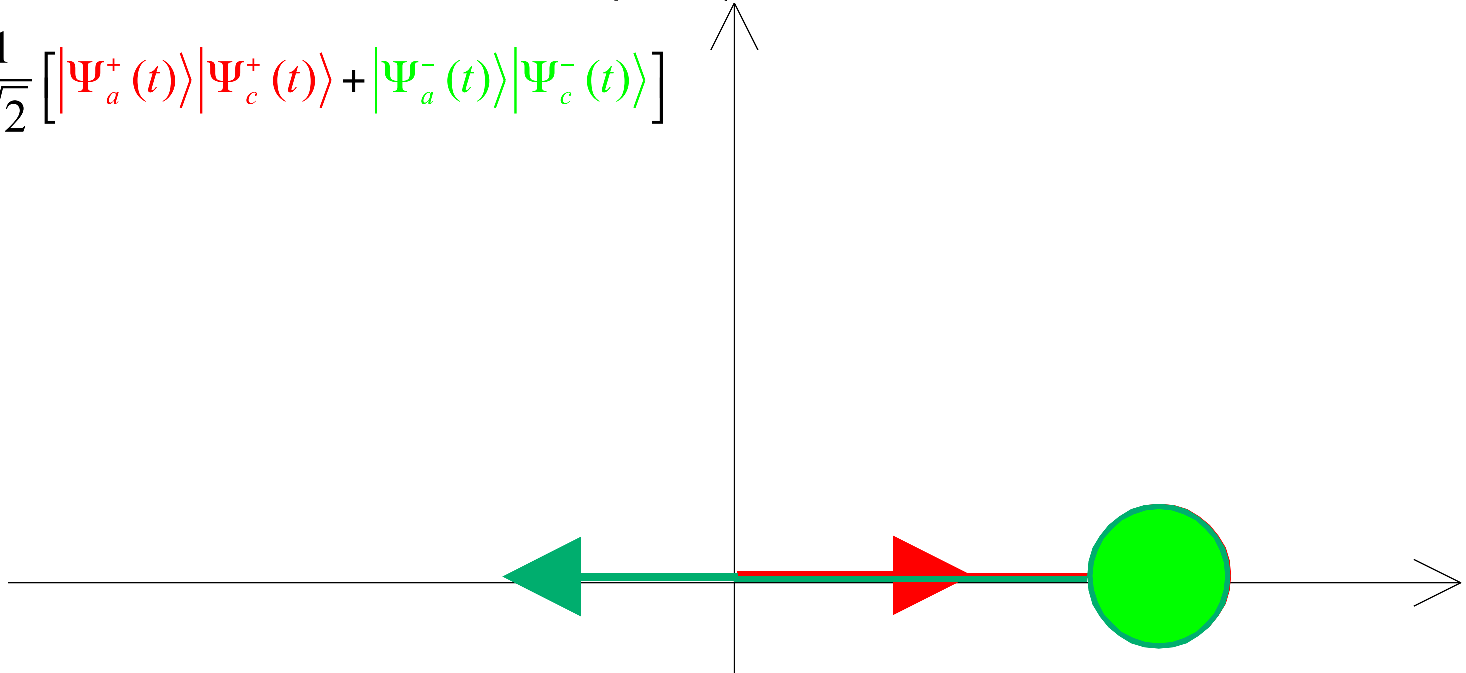
– A slowly rotating field state in the Fresnel plane

- Graphical representation of the joint atom-field evolution in a plane
- t=0:
  – both field states coincide with original coherent state
  – Atomic states are the classical eigenstates

Atom-field states evolution

$|\Psi_c^+\rangle$ $|\Psi_c^-\rangle$ $|\Psi_a^+\rangle$ $|\Psi_a^-\rangle$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\Psi_a^+(t)\rangle|\Psi_c^+(t)\rangle + |\Psi_a^-(t)\rangle|\Psi_c^-(t)\rangle\right]$$

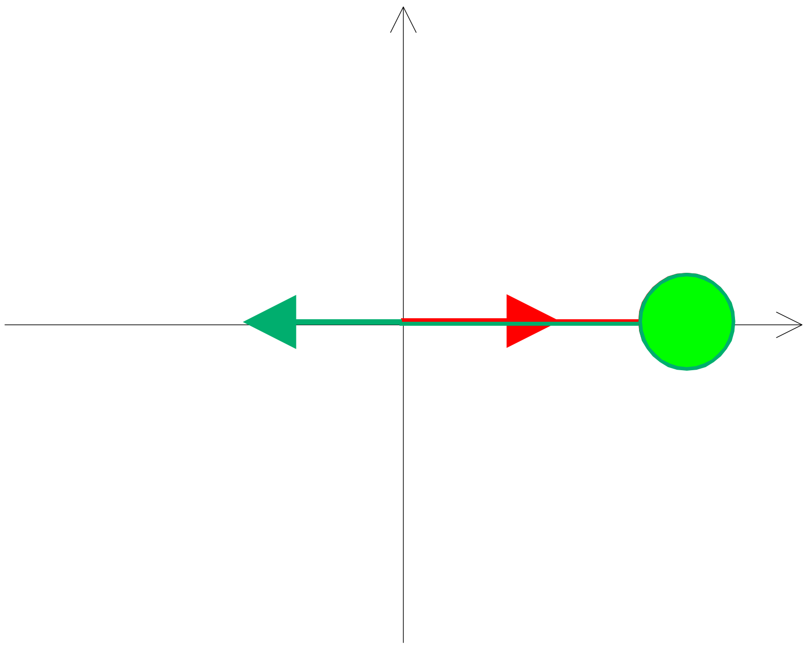


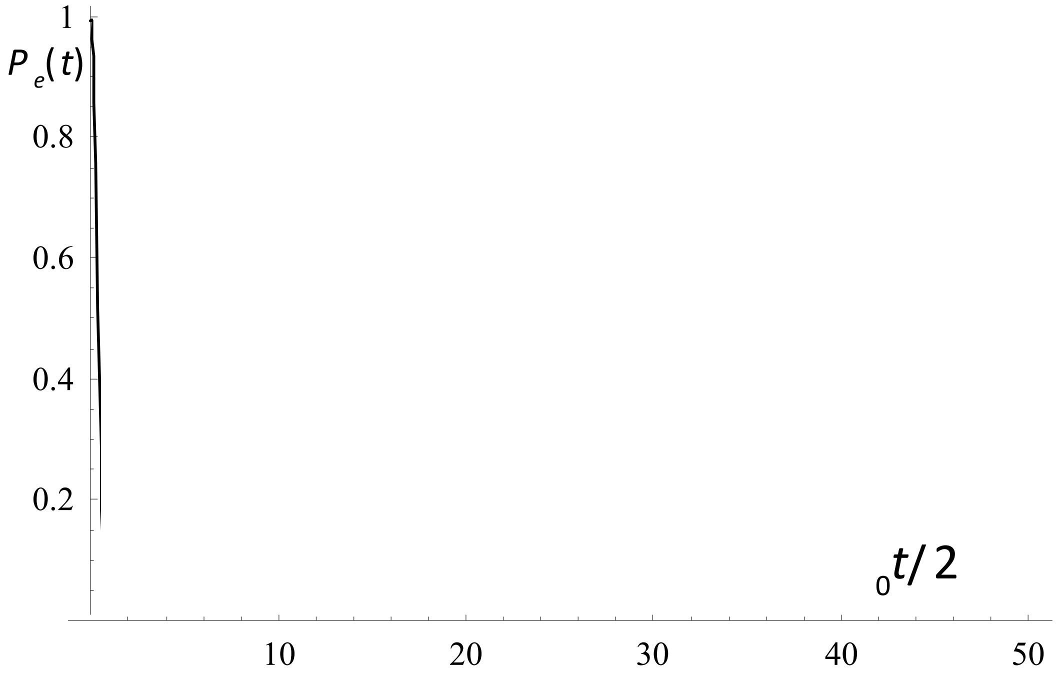
- At most times: $\langle\Psi_c^+|\Psi_c^-\rangle = 0$ an atom-field entangled state
- In spite of large photon number: considerable reaction of the atom on the field

$$\left|\Psi(t)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\Psi_a^+(t)\right\rangle\left|\Psi_c^+(t)\right\rangle + \left|\Psi_a^-(t)\right\rangle\left|\Psi_c^-(t)\right\rangle\right]$$

Rabi oscillation: quantum interference between $\left|\Psi_a^+\right\rangle$ and $\left|\Psi_a^-\right\rangle$

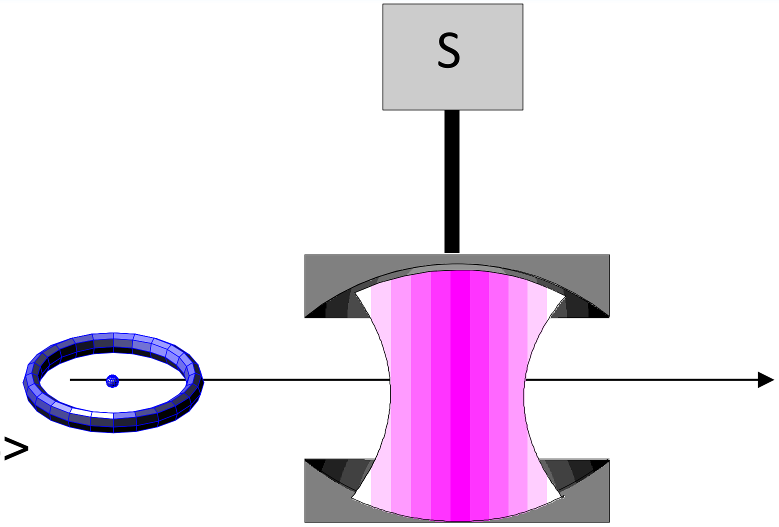
- Contrast vanishes when $\left\langle\Psi_c^+\middle|\Psi_c^-\right\rangle = 0$:
  - A direct link between Rabi collapse and complementarity



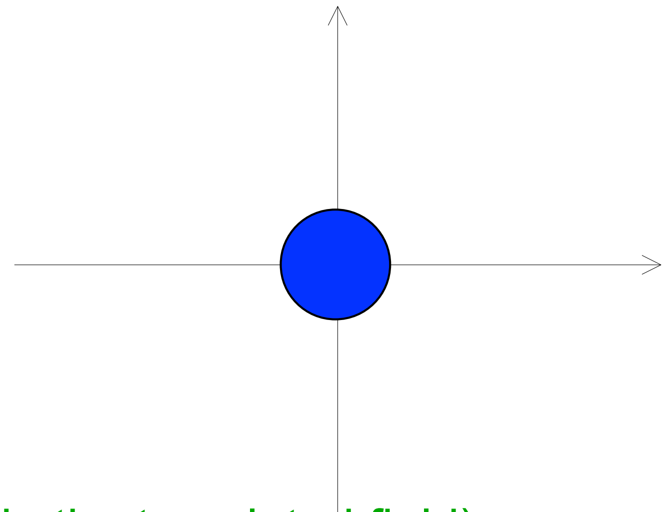


Field phase distribution measurement

Homodyning a coherent field



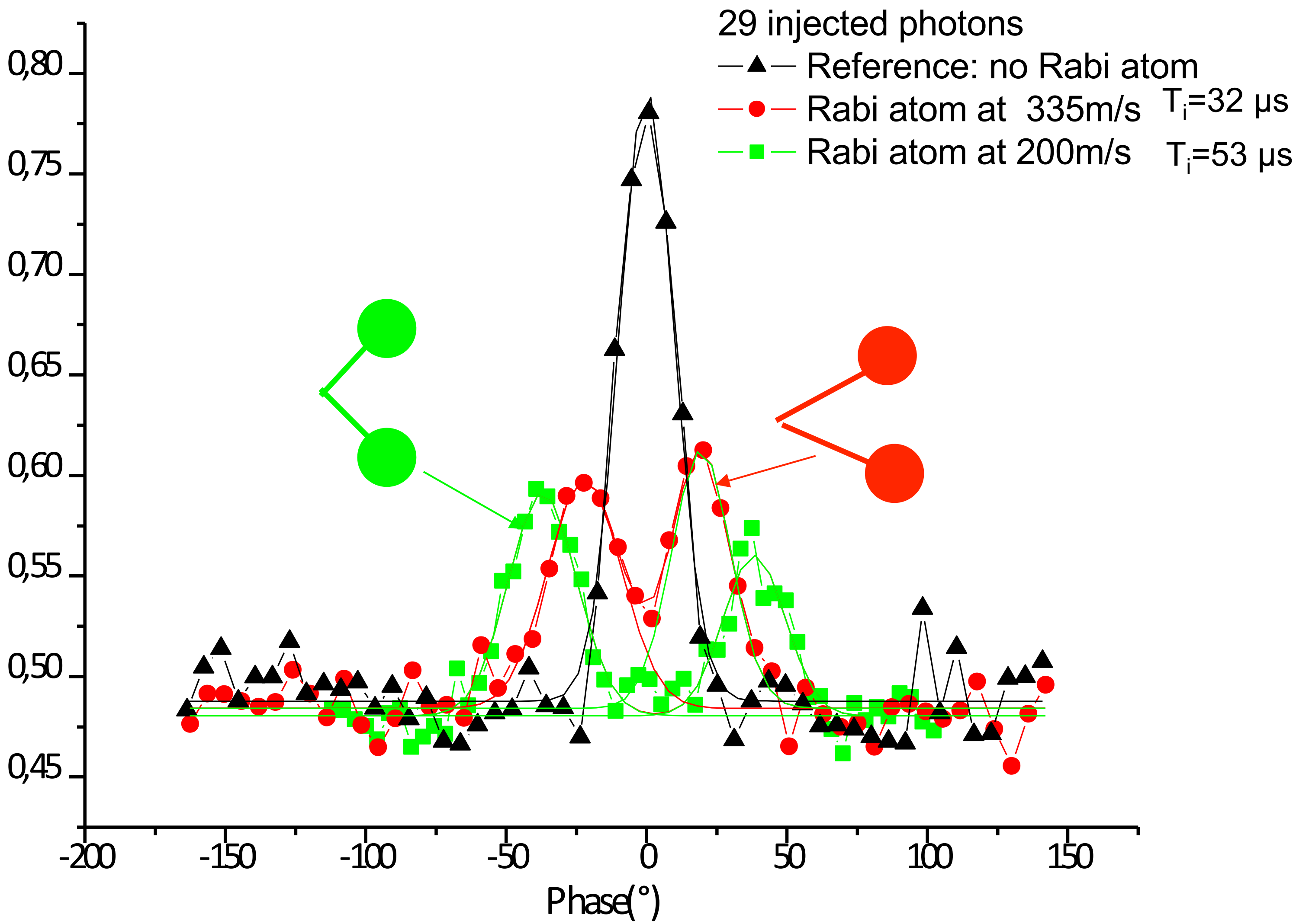
- Inject a coherent field $|\alpha\rangle$
- Add a coherent amplitude $-\alpha e^{i\phi}$
  - Resulting field (within global phase) $|\alpha(1-e^{i\phi})\rangle$
  - Zero final amplitude for $\phi=0$
- Probe final field amplitude with atom in *g*
  - $P_g=1$ for a zero amplitude
  - $P_g=1/2$ for a large amplitude



- More generally: $P_g(\phi)$ reveals field phase distribution
  - $P_g(\phi) \sim Q$ distribution (probability for zero photons in the translated field)

Phase splitting in quantum Rabi oscillation

Experimental phase distributions



These lectures

I) Introduction

II) Experimental tools for microwave CQED

III) Theoretical tools for microwave CQED

IV) Resonant microwave CQED

V) Dispersive microwave CQED

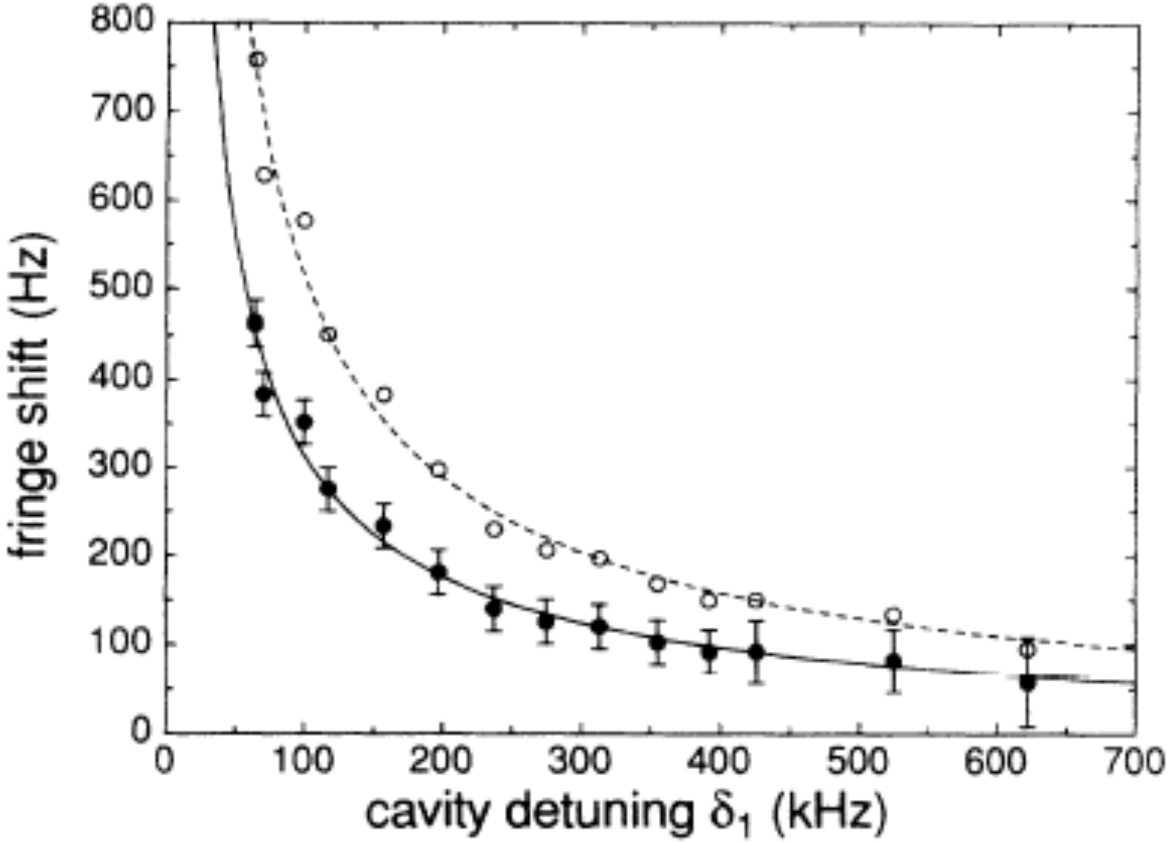
VI) Conclusion and perspectives

Dispersive microwave CQED

1. Lamb, light and phase shifts
2. A QND measurement of the photon number
3. Zeno effect
4. Fock states reconstruction and decoherence
5. Return on Bohr's complementarity, cats and decoherence
6. Quantum feedback

Lamb shifts

- Interaction with the ‘vacuum’



PRL 72, 3339 (94)

Solid line corrected for residual thermal field (0.32 photons)

A remarkable single mode Lamb shift effect

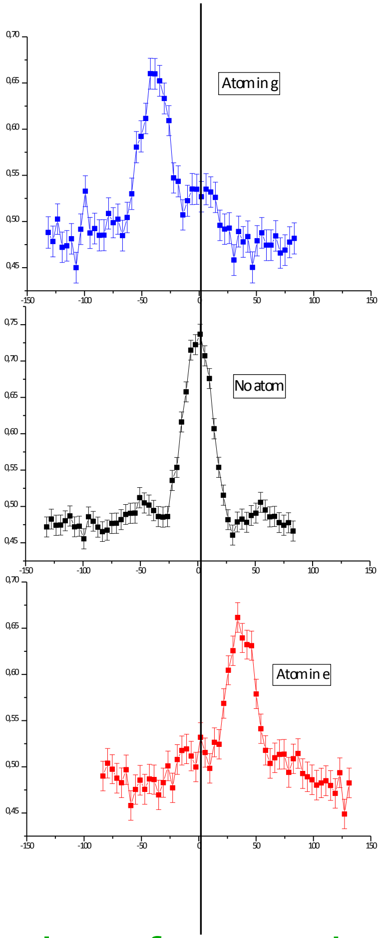
Phase shift with dispersive atom-field interaction

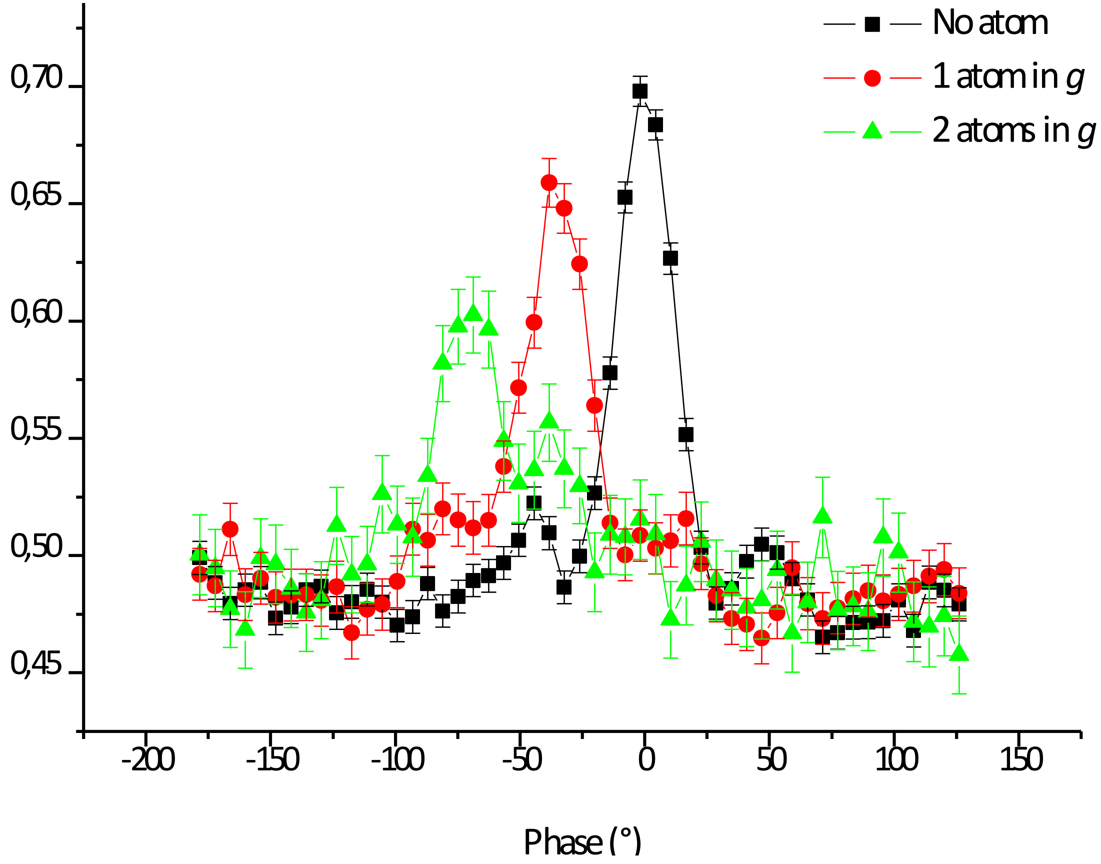
- Non resonant atom: no energy exchange but cavity mode frequency shift (atomic index of refraction effect).
  - Phase shift of the cavity field (slower than in the resonant case)

Atom in *g*

No atom

Atom in *e*





Opposite values for *e* and *g*

Proportional to atom number

PRL 94, 113601 (05)

Dispersive microwave CQED

1. Lamb, light and phase shifts
2. A QND measurement of the photon number
3. Zeno effect
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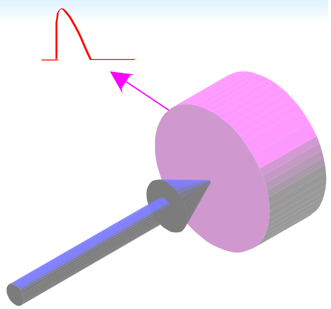
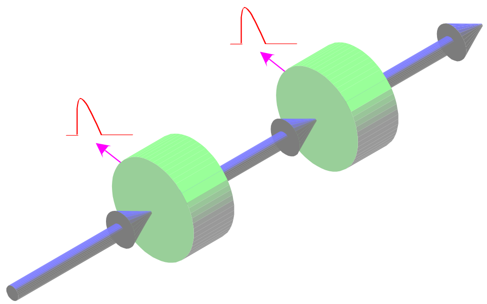
5. Return on Bohr's complementarity, cats and decoherence
6. Quantum feedback

Ideal photon number counting

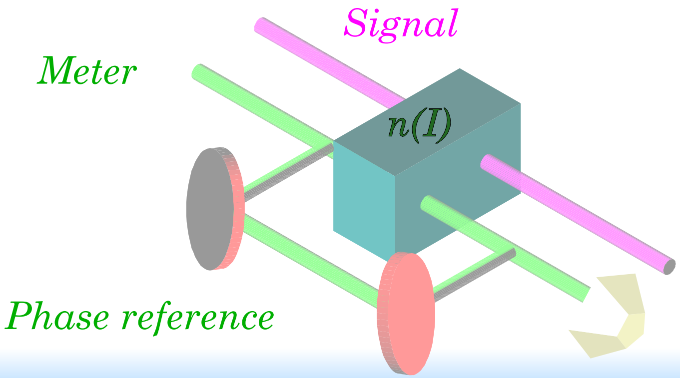
- Most quantum measurements are far from being projective
  - Light detection:
    - Photons are destroyed when detected



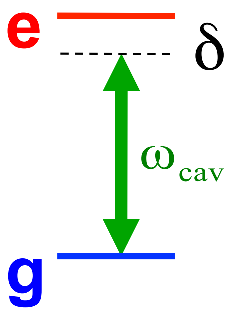
- Quantum non-demolition measurements (Braginsky, 70s)
  - A transparent photocounter
    - 'see' the same photon twice
    - Should allow observation of the quantum jumps of light

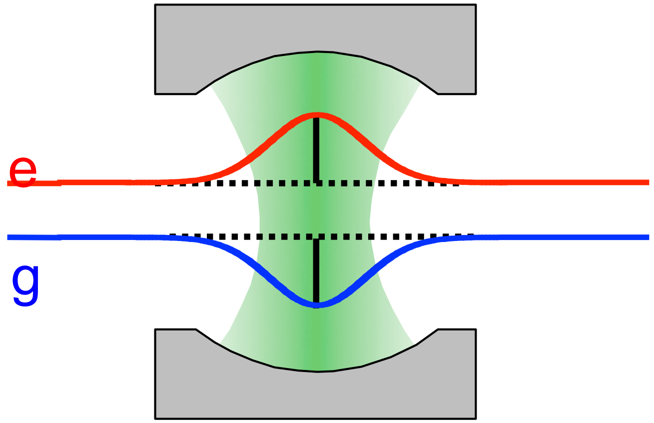


- Realized in the optical domain (Grangier et al, Nature, 396, 537)
  - no single photon resolution
    - weak non-linearity
  - propagating fields:
    - repetition difficult



Dispersive atom-field interaction

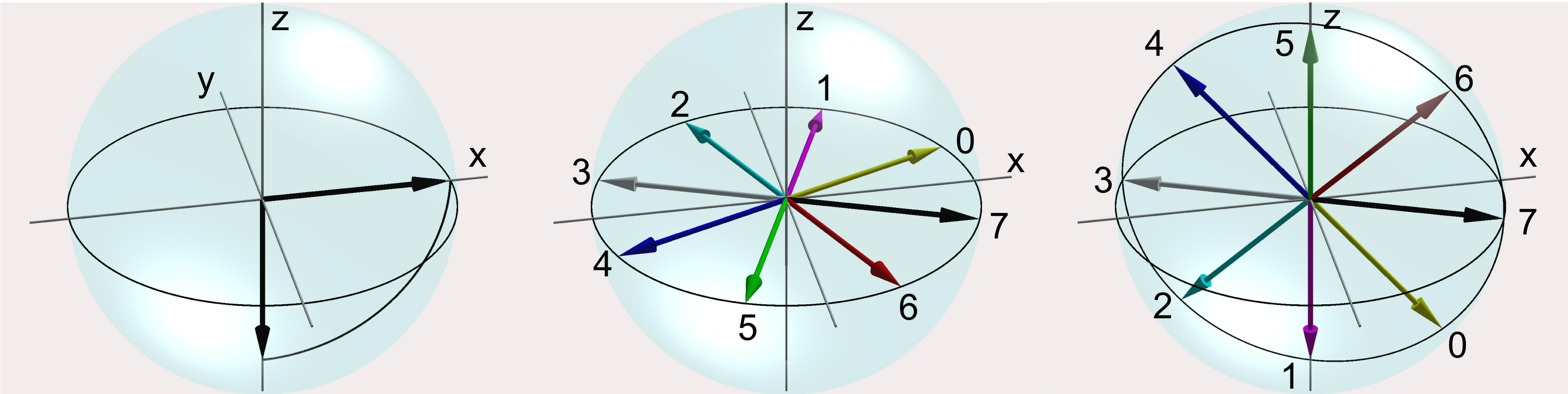




- Atomic frequency shift inside the cavity
  - Light and Lamb shifts:
    - An atomic clock ticking rate modification
    - A phase shift of the atomic coherence $\phi_0\,(n + 1/2)$
- Adiabatic coupling in and out of the atom-cavity interaction
  - negligible spurious absorption rate ($<10^{-4}$ for $\delta \sim \Omega$)

A pictorial representation of the interaction

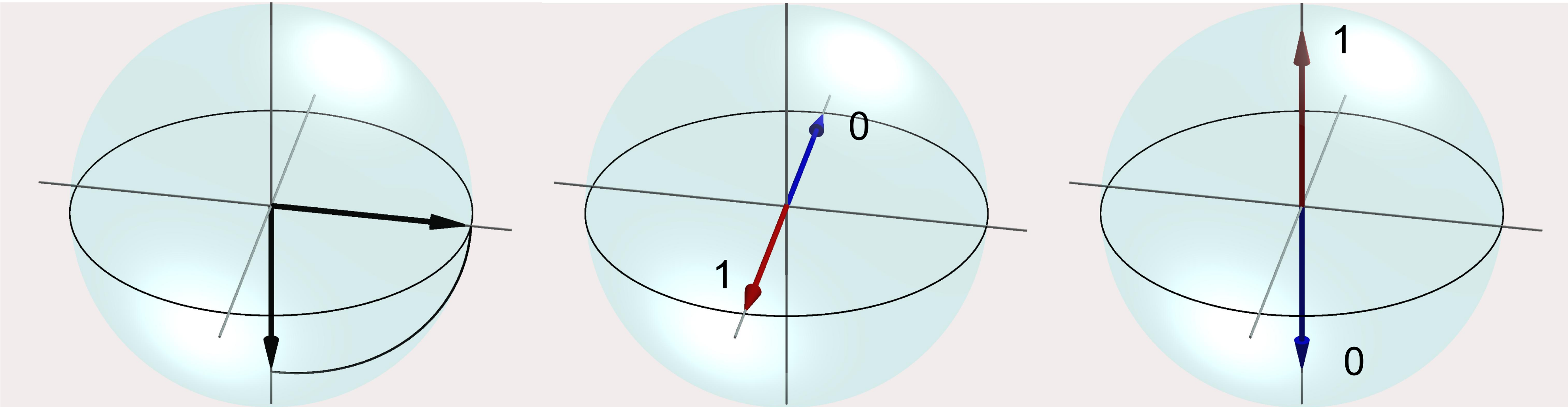
- Evolution of the atomic state on the Bloch sphere.
  - $\pi/4$ phase shift per photon



- The Bloch vector direction is the clock's hand
- In general non-orthogonal final atomic states correspond to different photon numbers: A single atom does not tell all the story
- A simple case: π phase shift per photon and 0/1 photon (a 'qubit' situation)

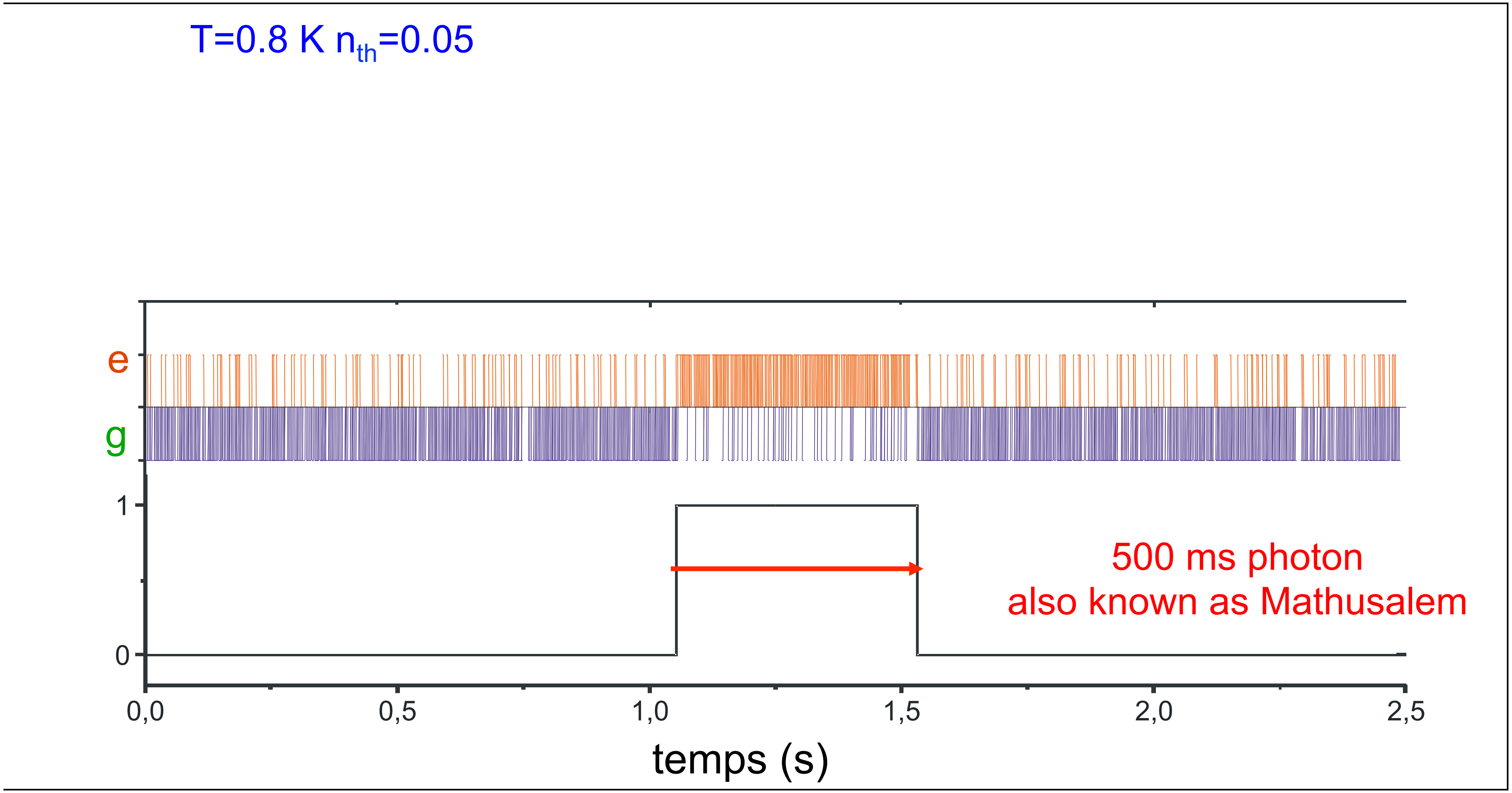
The single photon case

- A zero or one-photon field
  - π phase shift per photon



- Two orthogonal final atomic states
  - in principle, a single atomic detection unambiguously tells the photon number.

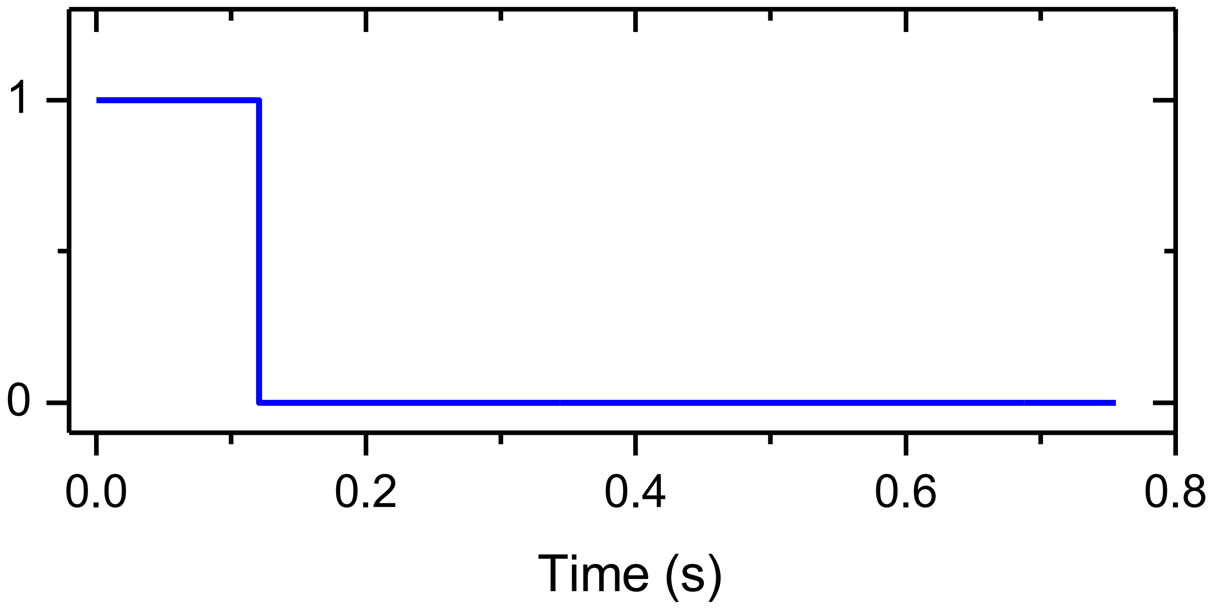
Birth, life and death of a photon



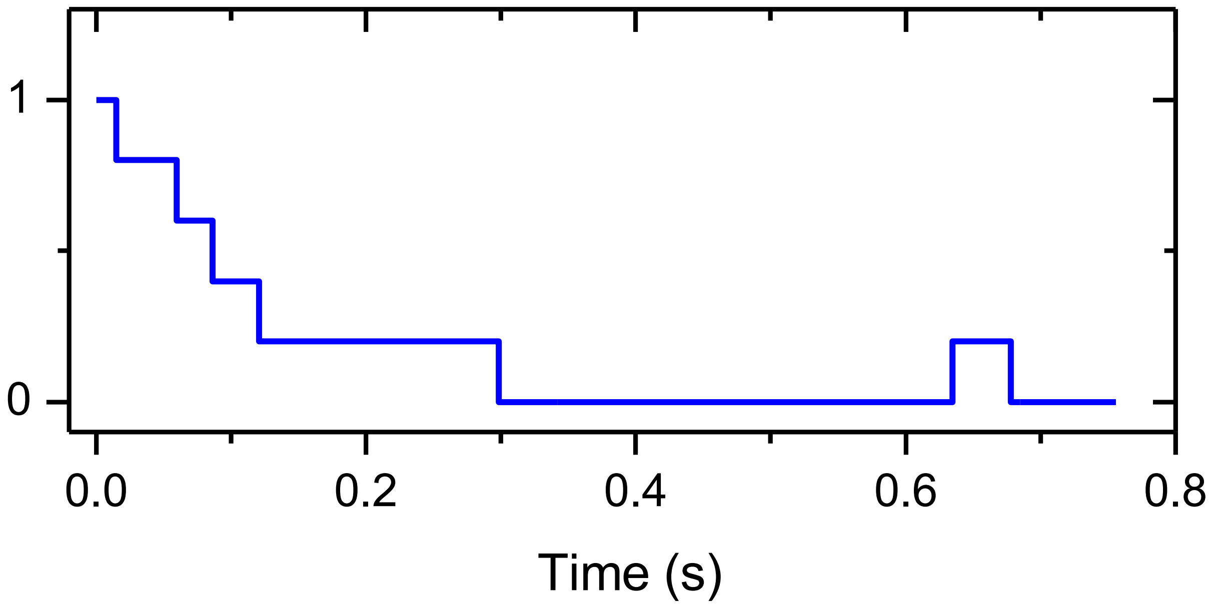
Gleyzes et al, Nature, **446**, 297 (2007)

|n=1> Lifetime

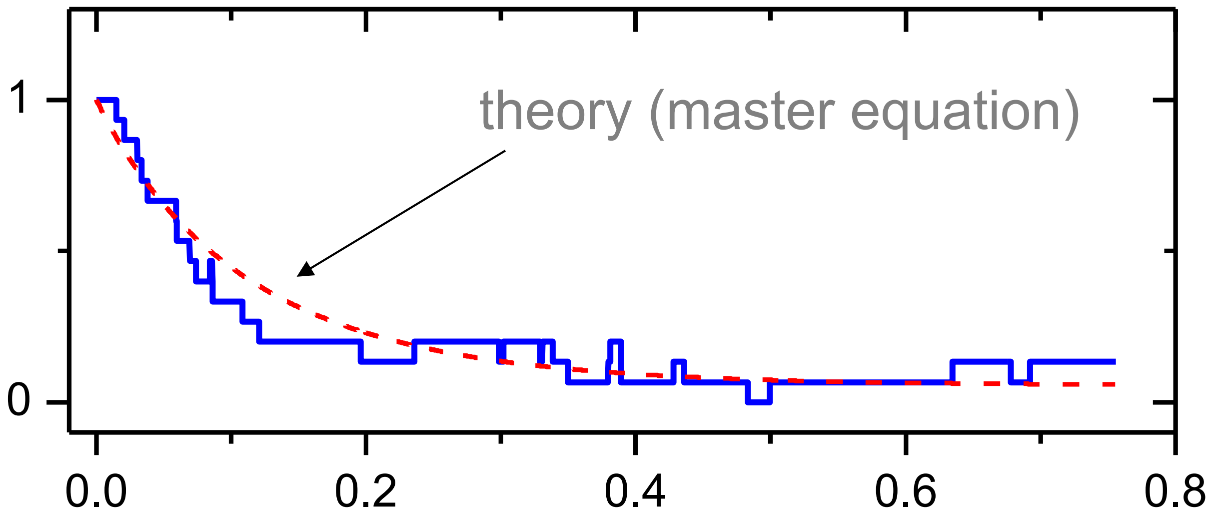
1 sequence :



5 sequences :

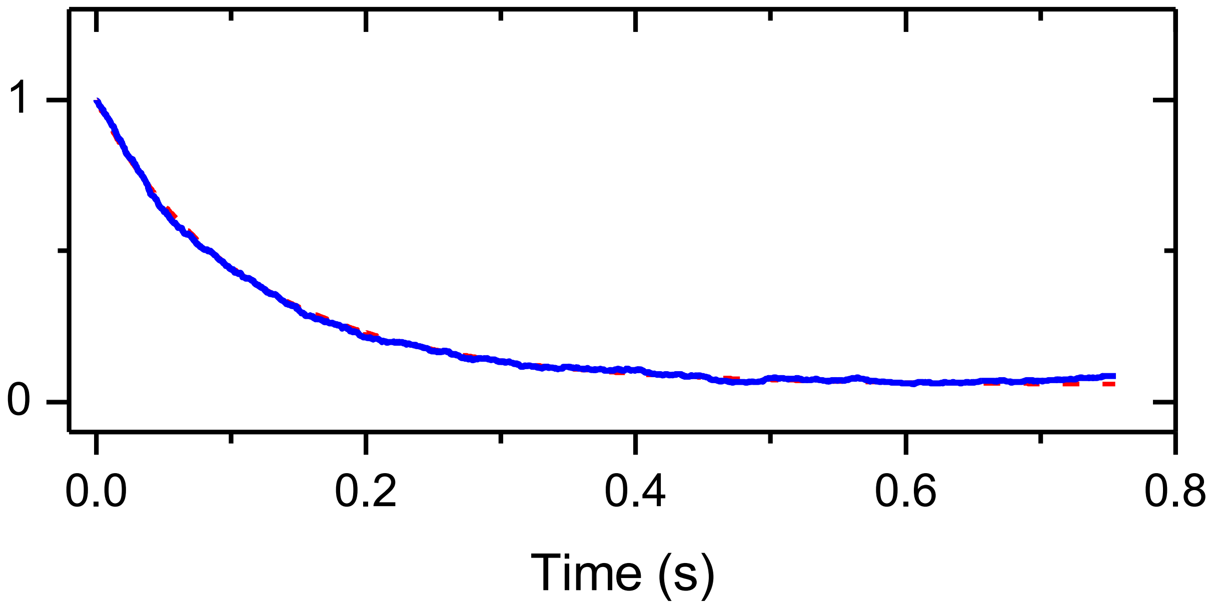


15 sequences :



|n=1> Lifetime

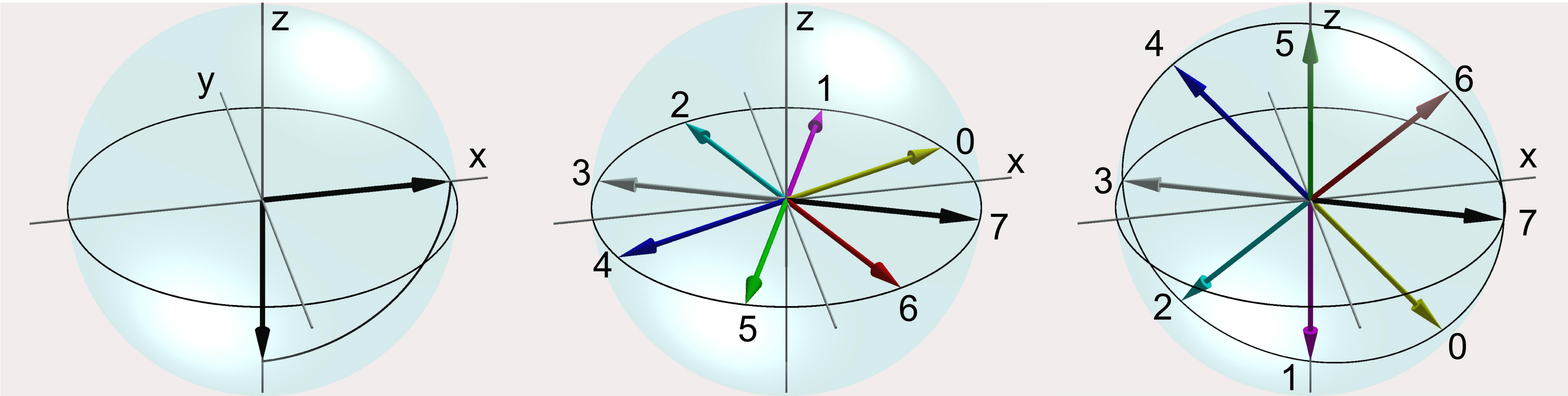
904 sequences :



Excellent agreement with the quantum predictions (no adjustable parameter)

Counting from 0 to 7

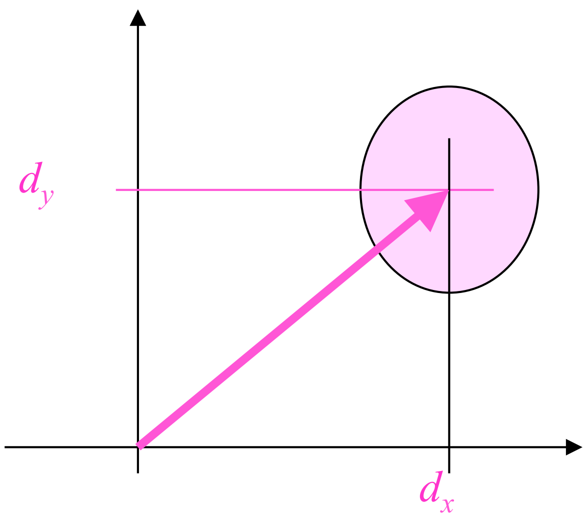
- $\pi/4$ phase shift per photon
  - Evolution of the atomic state on the Bloch sphere.



  - In general non-orthogonal final atomic states correspond to different photon numbers: A single atom does not tell all the story

Photon counting by information accumulation

- One atom exits cavity with a spin direction correlated to *n*
- QND interaction: N atoms exit cavity with the same spin direction correlated to *n*
  - Entanglement of the photon number with a mesoscopic atomic sample
- Split atomic sample in two parts
  - On *N*/2 atoms, measure S_x
  - On *N/2* atoms, measure S_y
  - Estimate spin direction with $1/\sqrt{N}$ uncertainty



S. Gleyzes et al. Nature **446**, 297, C. Guerlin et al. Nature **448**, 889

“Forward” estimation of the photon number at time *t*

- Density operator ρ including all available information from 0 to t
  - Updated according to each atomic detection result

$$\rho_{p-1}^{f} \longrightarrow \rho_{p}^{f} = \frac{M_j \rho_{p-1}^{f} M_j^\dagger}{\pi_j(\phi_r|\rho_{p-1}^{f})}$$

    - Measurement operators

$$M_g = \sin\left[\frac{\phi_r + \phi_0(N+1/2)}{2}\right]$$
$$M_e = \cos\left[\frac{\phi_r + \phi_0(N+1/2)}{2}\right]$$

$$\pi_j(\phi_r|\rho) = \mathrm{Tr}\left(M_j \rho M_j^\dagger\right)$$

  - Updated according to cavity relaxation between detections
    - Liouvillian evolution

A Bayesian inference process

- Photon number distribution
  - Relaxation and measurement operators diagonal in the Fock states basis
    - Updated according to atomic detection results

$$P_p^f(n) = \frac{\pi_j(\phi_r|n)}{\pi_j(\phi_r|\rho)} P_{p-1}^f(n)$$

      - Detection probabilities:

$$\pi_j(\phi_r|\rho) = \sum P(n)\pi_j(\phi_r|n)$$

$$\pi_e(\phi_r|n) = 1 - \pi_g(\phi_r|n) = \frac{1}{2}\left(1 + \cos\left[\phi_r + \phi_0(n + 1/2)\right]\right)$$

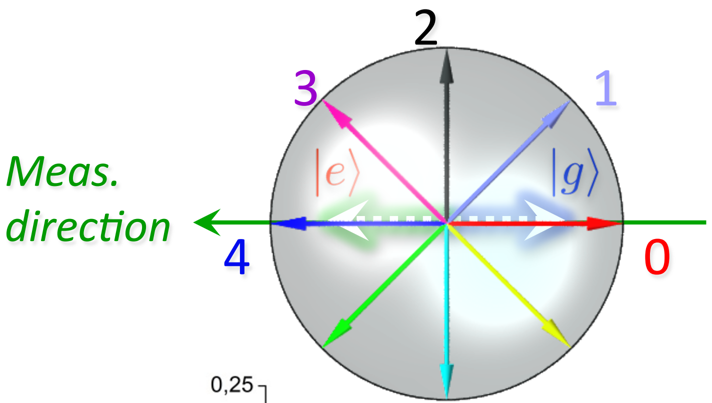
    - Updated according to cavity relaxation

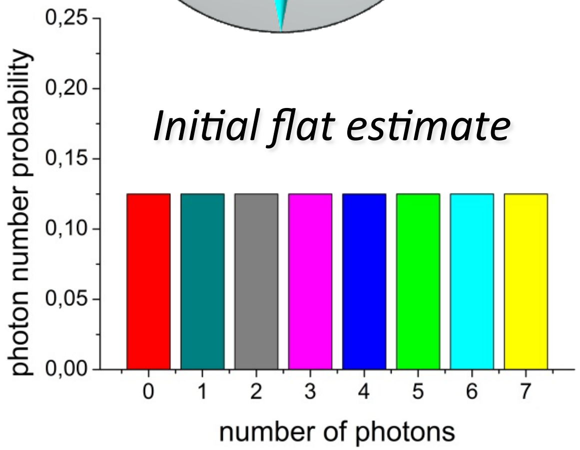
$$\frac{dP^f(n,t)}{dt} = \sum_m K_{n,m} P^f(m,t)$$

- A Bayesian inference of P(n) by photon decimation, proceeding forward in time
  - About 8^2 atoms required to count from 0 to 7

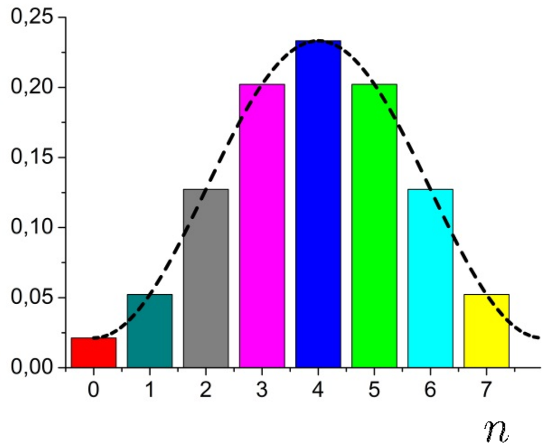
Single atom detection

Each detection brings partial information on the photon number

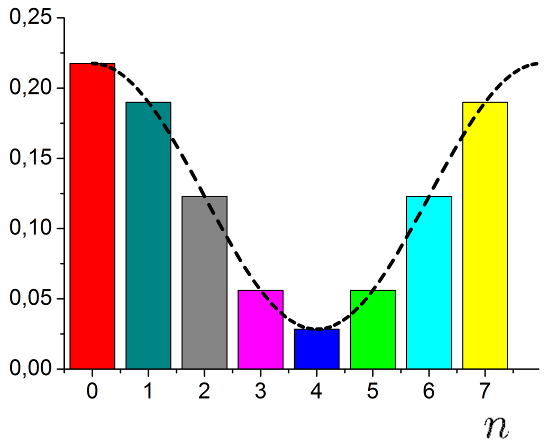




detection $|e\rangle$

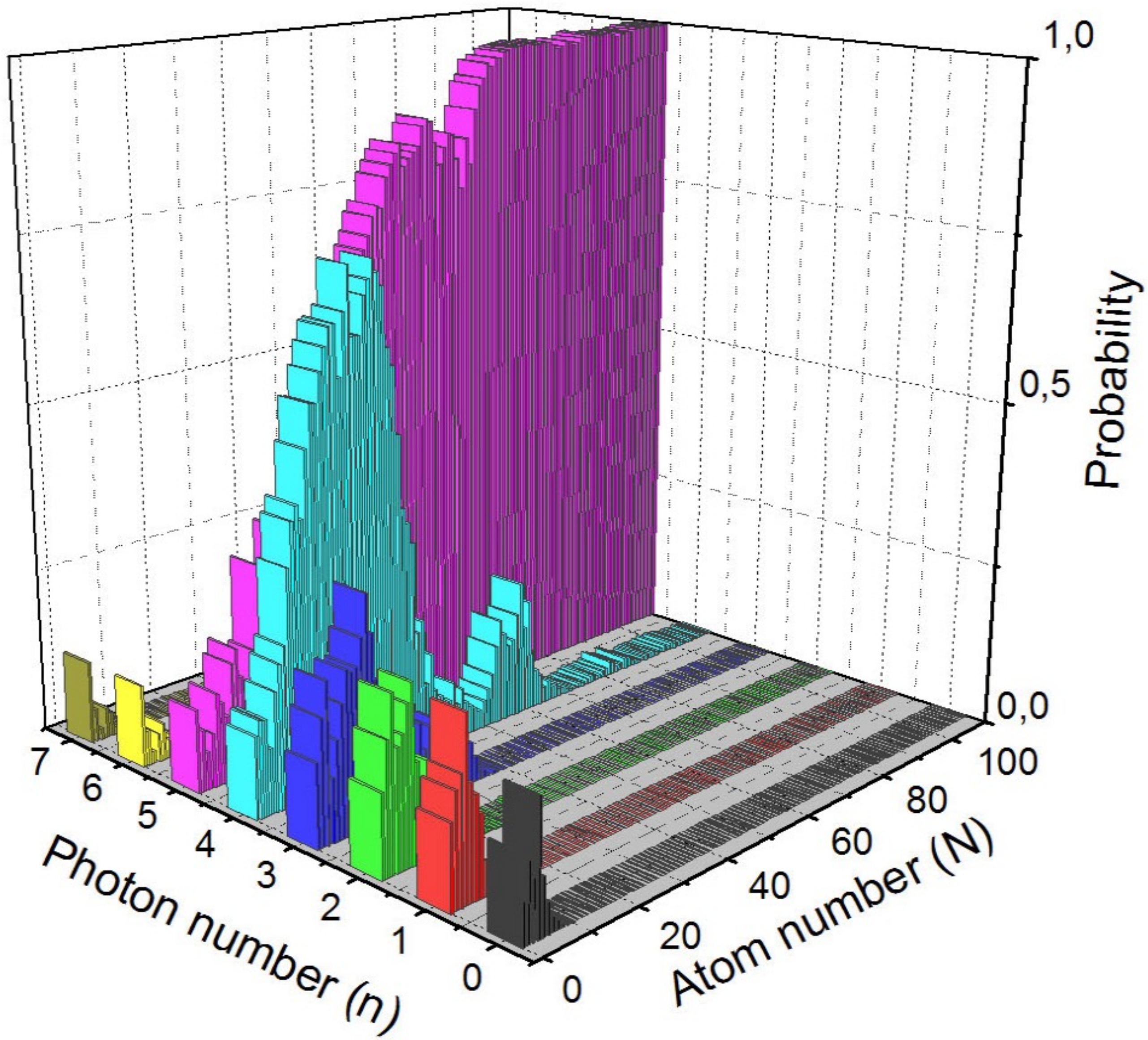


detection $|g\rangle$



To speed-up convergence, the measurement phase ϕ_r is randomly chosen for each atom among the four values corresponding to atomic states

Wave-function collapse in real time



- Evolution of P(n) while detecting 110 atoms in a single sequence

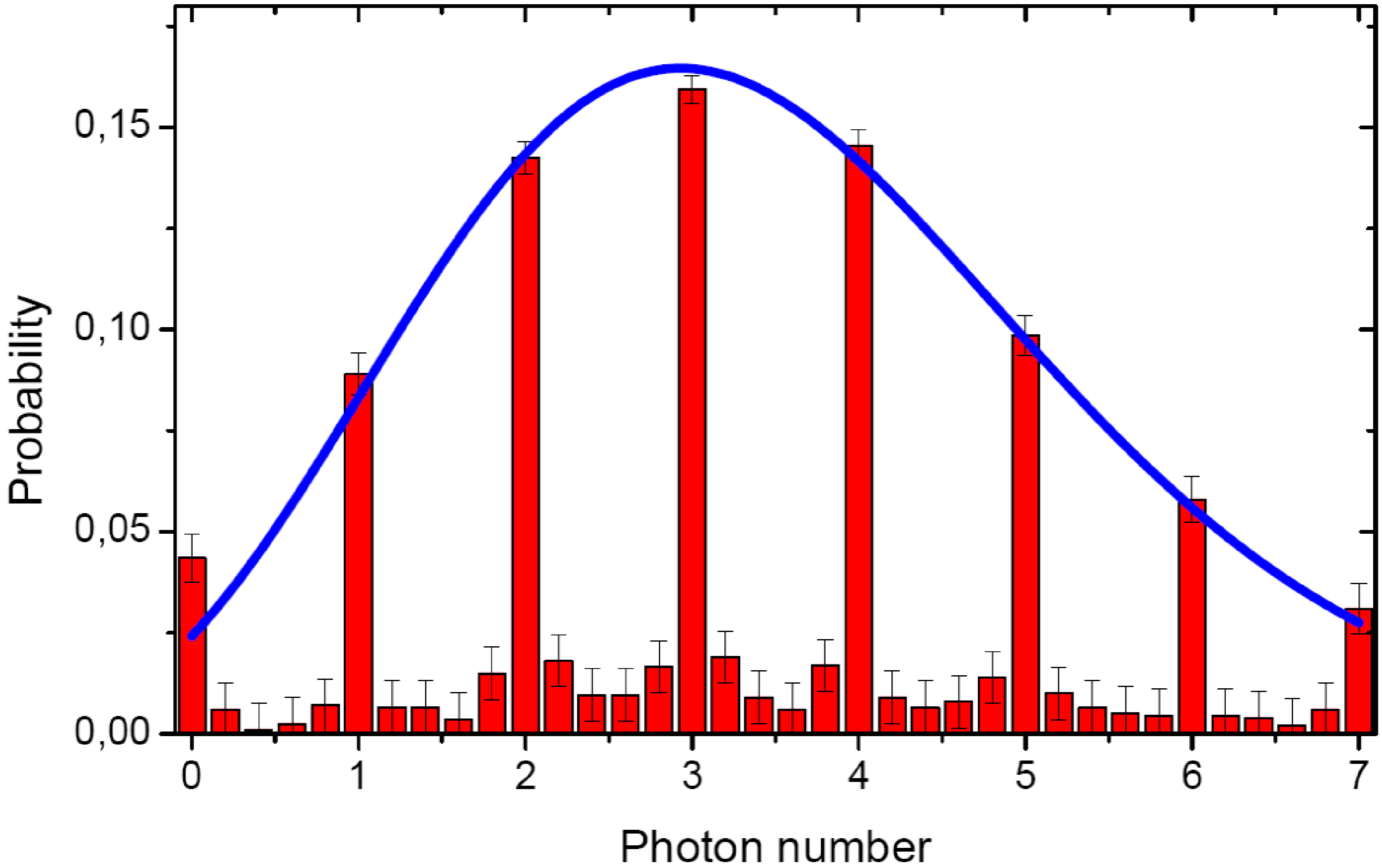
- Initial coherent field with 3.7photons

- Initial inferred distribution flat (no information) but final result independent of initial choice

- Progressive collapse of the field state vector during information acquisition

C. Guerlin et al, Nature, **448**, 889

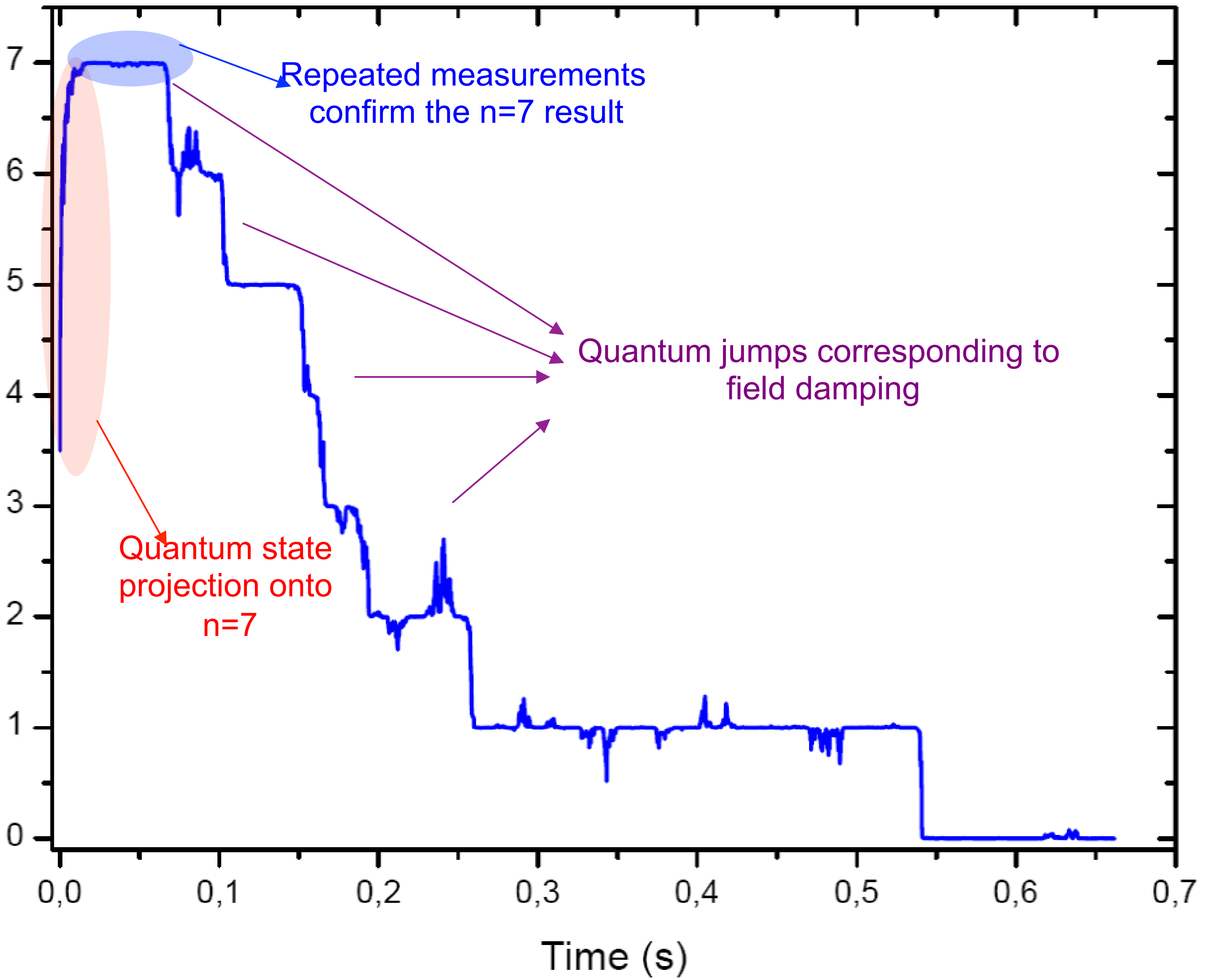
Photon number statistics



Excellent agreement with the expected Poisson distribution

A vivid illustration of quantum measurement postulates

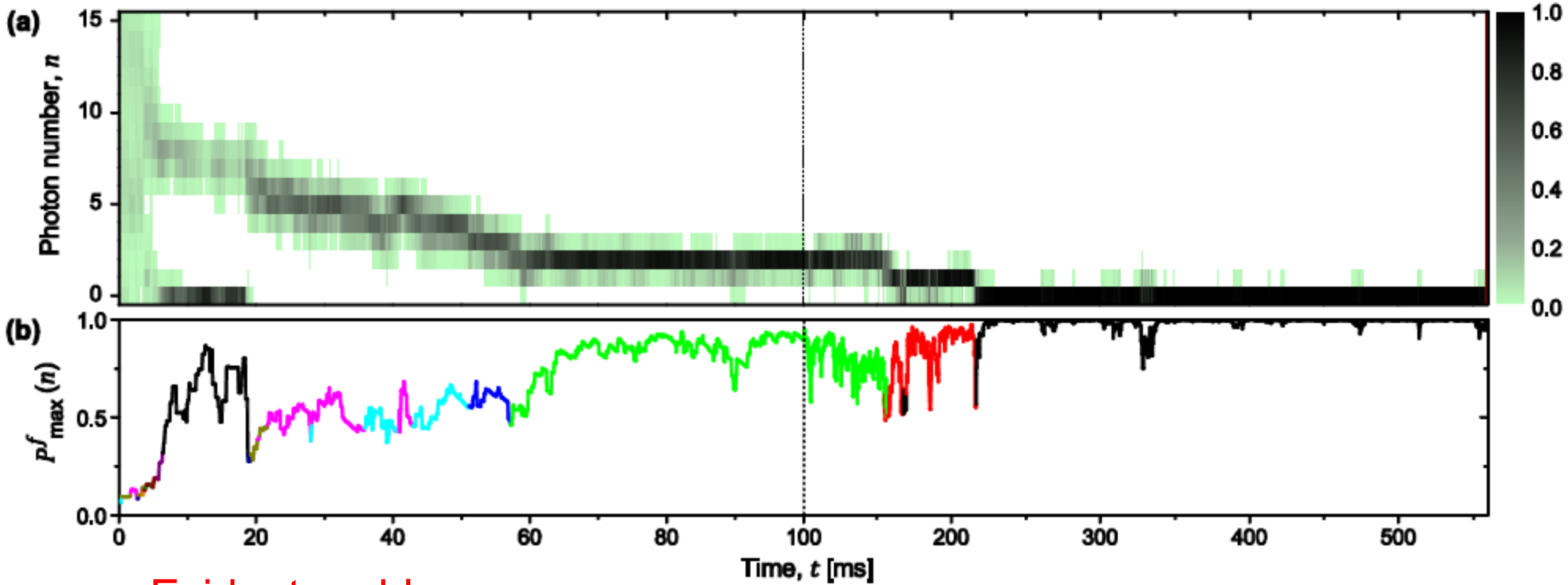
Cascade down the Fock states ladder



C. Guerlin et al, Nature, **448**, 889

A single quantum trajectory with a large initial field

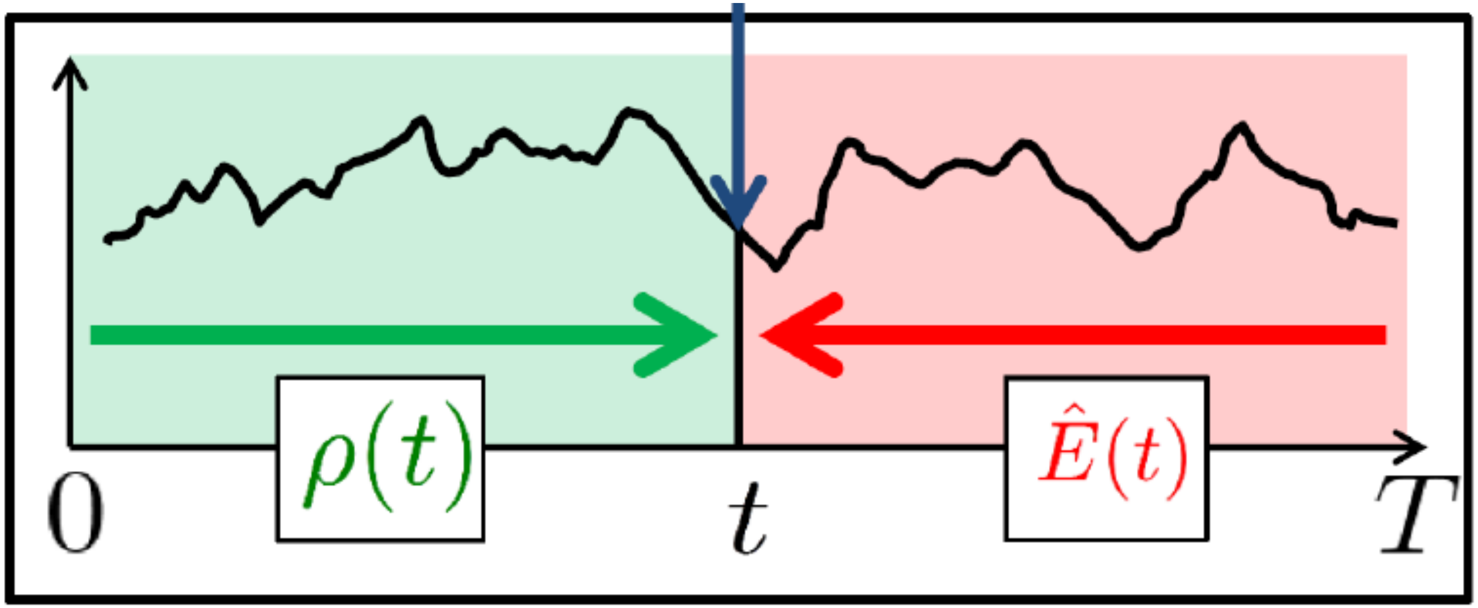
- Forward estimation at time *t*



  - Evident problems
    - Initial ambiguity in the photon number due to the periodicity of the measurement operators
    - Absurd photon number jumps (from 0 to 7)
    - Noise due to statistical fluctuations of atomic detections
  - Improvement by taking into account measurements to come after *t*

The Past Quantum State approach

- A posteriori estimation of the photon number at *t* based on all available information, gathered from 0 to *t* AND from *t* to *T*
  - From the journalist's to the historian's perspective
- A quantum formalism (S. Gammelmak et al. PRL 111, 160401)
  - The Past quantum state



  - Best estimate for the results of a quantum measurement at *t* based on the density matrix ρ computed forward in time AND on an effect matrix *E* computed backwards in time

Forward-backward estimation

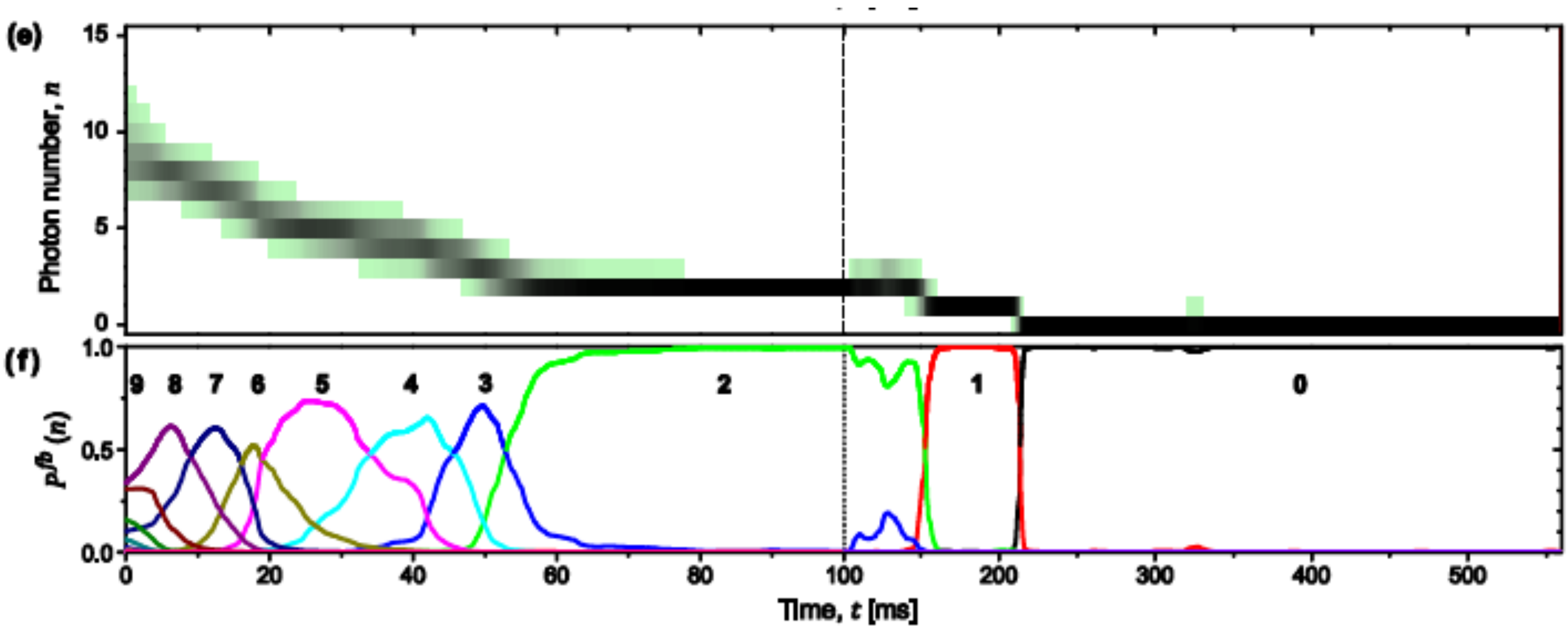
- For diagonal measurement/relaxation operators

$$P^{fb}(n,t) = \frac{P^f(n,t)P^b(n,t)}{\sum_m P^f(m,t)P^b(m,t)}$$

  - PQS reduces to the forward/backward smoothing algorithm, which can be safely used in this quantum context
  - P(n) is the product of two photon number distributions computed forward and backward in time.

- Backwards estimation
  - Flat distribution at *T*
  - Same measurement operators
  - ‘inverse’ relaxation (annihilation and creation operators exchanged)
    - Exponential growth of the photon number

PQS estimation



- Ambiguities lifted
  - Measurement of photon number beyond the intrisic periodicity of atomic signal
- Considerable noise reduction
  - All estimations take into account ALL available information

T. Rybarczyk et al., PRA 91 062116

Dispersive microwave CQED

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Quantum Zeno effect

- A watched kettle never boils
  - coherent evolution of a system and frequently repeated quantum measurements
    - a quantum jumps evolution between eigenstates of the measured quantity
    - an evolution much slower than without measurements
    - no evolution at all in the limit of zero delay between measurements
  - No Zeno effect for incoherent relaxation processes

Quantum Zeno effect

- A simple description of the Zeno effect
  - A quantum system initially in |0> evolves under the action of the hamiltonian *V* during time *t*.
  - During this time, *n* measurements of an observable *O* with the non-degenerate eigenstate |0> are performed, at times *t/n*, *2t/n*…
  - At *t/n* probability for finding |0> is

$$\Pi_0\left(\frac{t}{n}\right) = 1 - \frac{\Delta^2 V}{\hbar^2}\frac{t^2}{n^2} + \cdots \quad ; \quad \Delta^2 V = \sum_{i\neq 0}\left|\langle 0|V|i\rangle\right|^2$$

    - A quadratic function of the time interval *t/n*
  - Final probability for finding |0>:

$$\Pi_0^{(n)}(t) = \left[1 - \frac{\Delta^2 V}{\hbar^2}\frac{t^2}{n^2} + \cdots\right]^n = 1 - \frac{\Delta^2 V}{\hbar^2}\frac{t^2}{n} + \cdots \xrightarrow[n\to\infty]{} 1$$

    - 1 if the time interval between measurements is close to zero.
    - Efficient inhibition of coherent evolution

Quantum Zeno effect

- Coherent evolution
  - Probability for leaving |0> quadratic in *t/n*
    - Efficient inhibition of coherent evolution

- Incoherent evolution (relaxation)
  - Probability for leaving |0> in the first step $\Gamma t/n$ (exponential decay)
  - Final probability for staying in |0> (assume $\Gamma t \ll 1$):

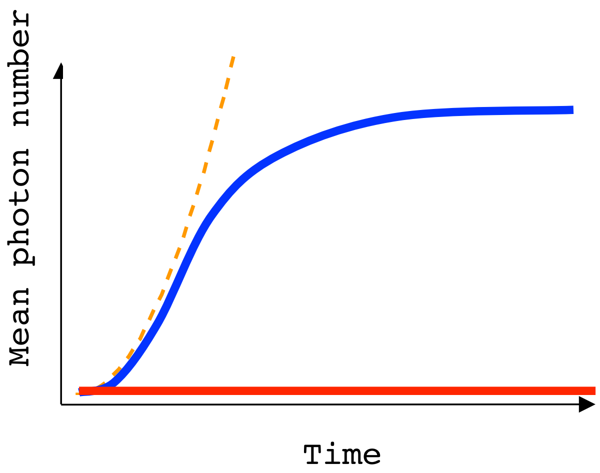
$$\left(1-\Gamma\frac{t}{n}\right)^{n}\approx 1-\Gamma t$$

    - Same decay without measurements
  - Zeno effect does not affect relaxation processes
    - Unless measurements frequently repeated on the scale of the environment's correlation time

Quantum Zeno effect

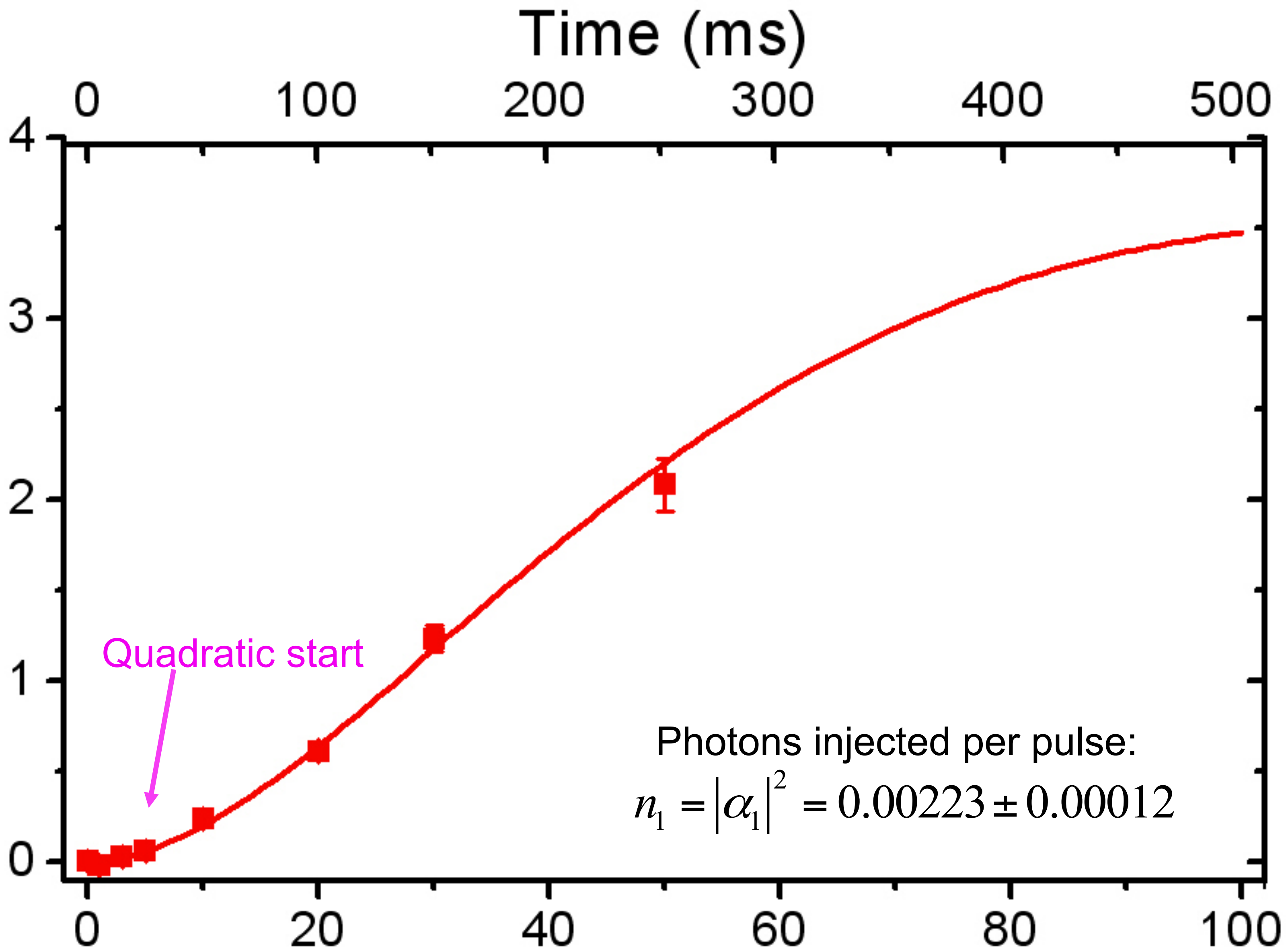
- Coherent evolution: injection of a coherent field by a classical source

–Repeated injection of phase coherent pulses: an amplitude varying linearly with the number of injections (photon number varies quadratically).

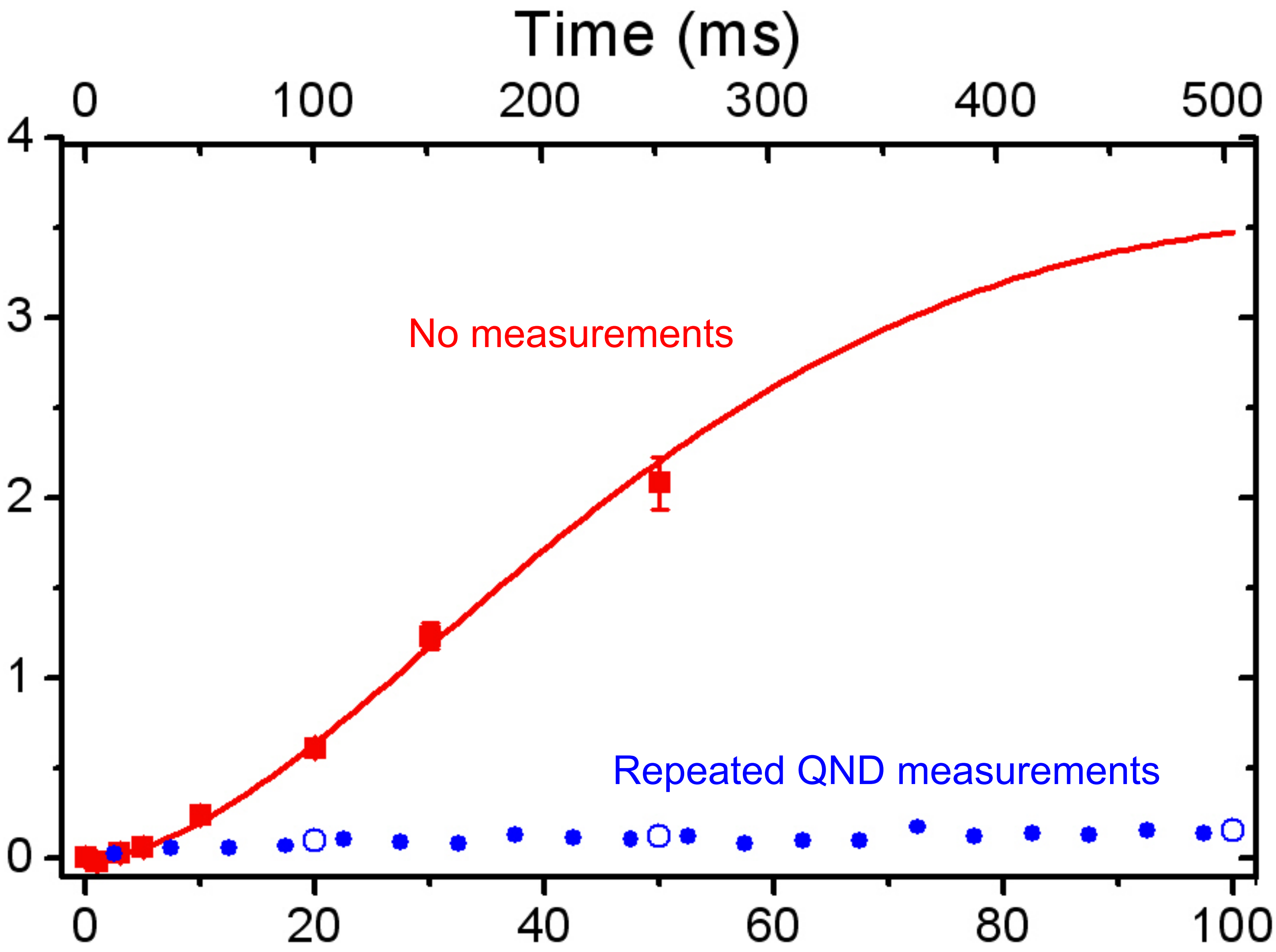


Principle of the experiment: perform QND measurements of photon number between two pulses

Growth of a coherent field

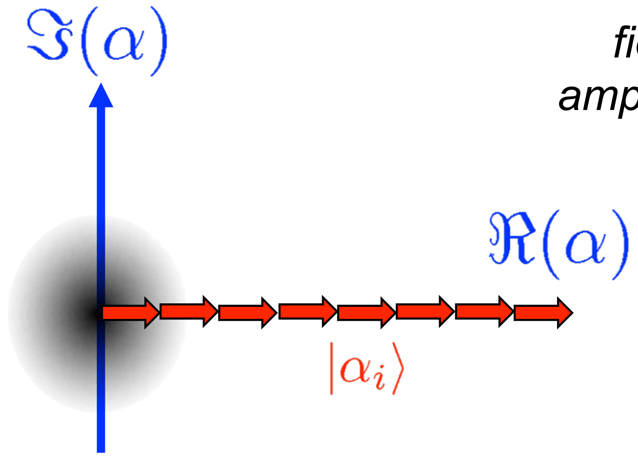


Inhibited growth



Random walk in phase-space

No measurement



number of pulses

field amplitude

amplitude injected in each pulse

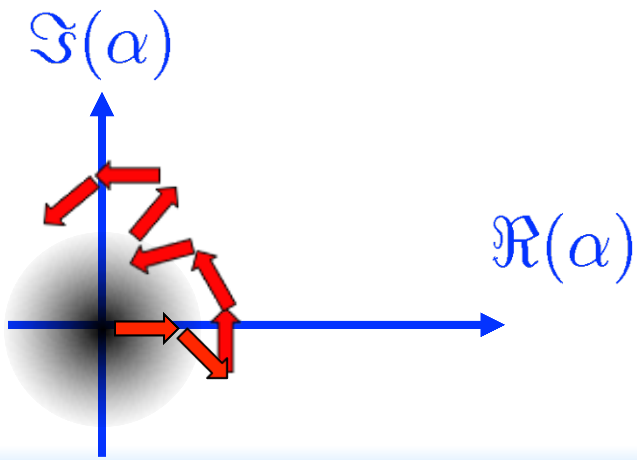
$$A = N \cdot \alpha_i$$

$$n = A^2 = N^2 |\alpha_i|^2$$

quadratic growth

Back-action: measurement of the photon number destroys the phase

With measurements



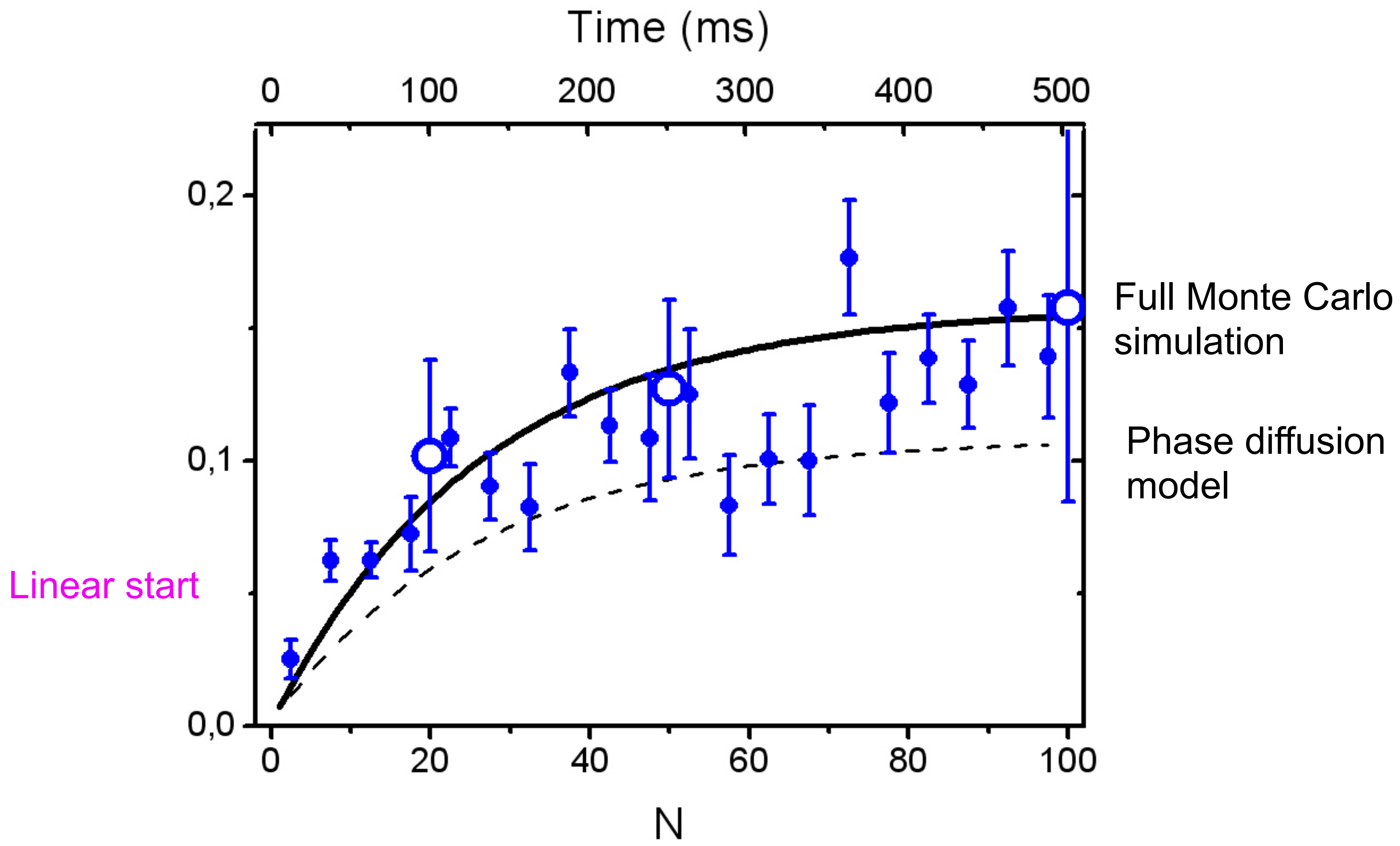
random walk

$$A = \sqrt{N}\, \alpha_i$$

$$n = N |\alpha_i|^2$$

linear growth

Residual field growth



J. Bernu et al, PRL, **101**, 180402 (2008)

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Fock state preparation

- Ideal projective measurement
  - After measurement the field is in a photon number state
  - Prepare all Fock states from 0 to 7
    - Not an easy task in quantum optics

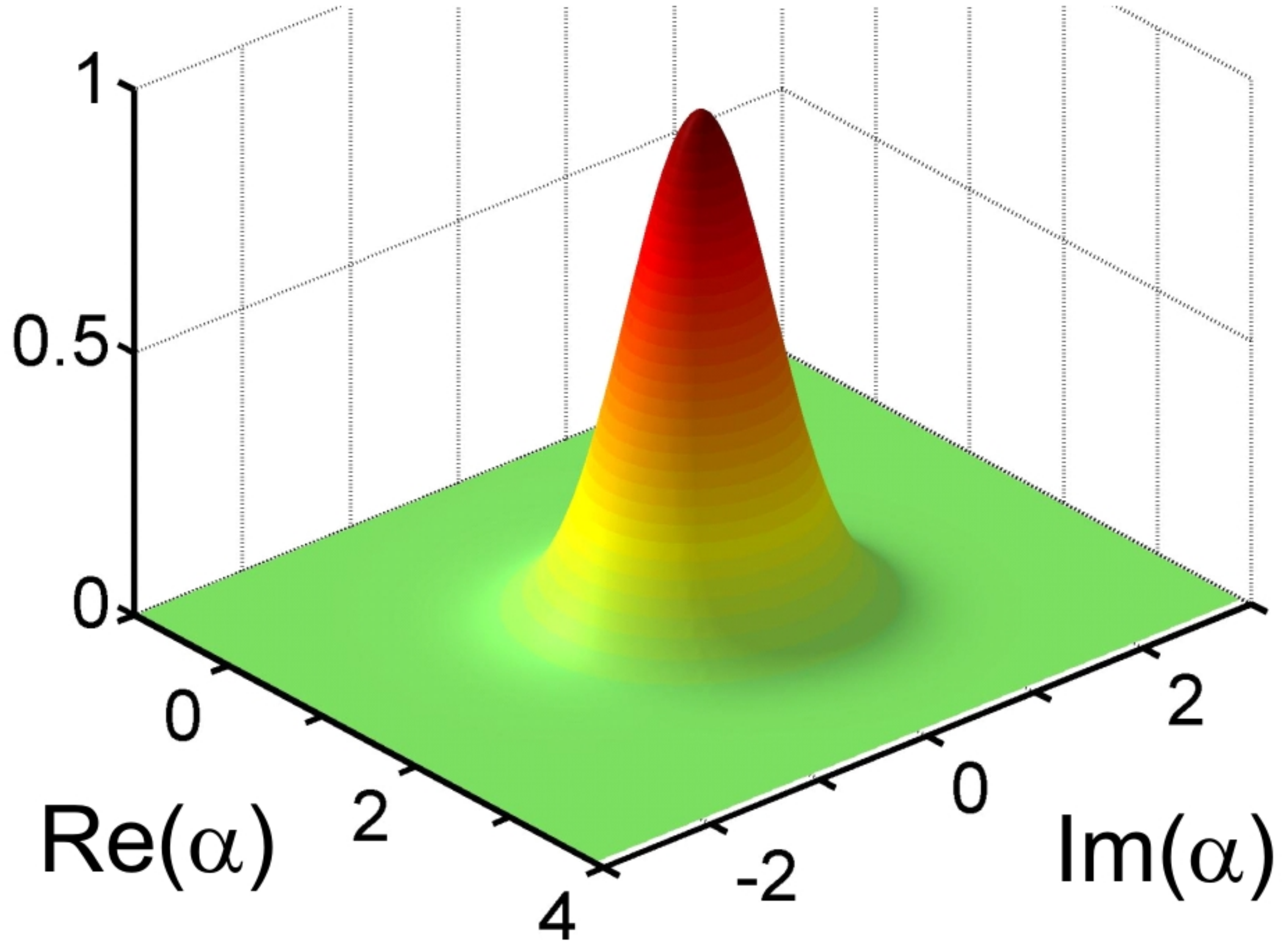
- Check the produced state ?
  - Full measurement of the cavity quantum state
    - Also based on the QND interaction
    - Get all the density operator describing the field state
  - Present it in terms of the field's Wigner function:
    - A 'wavefunction' in the phase plane (Fresnel plane)
    - A quasiprobability distribution for the complex field amplitude.

S. Deléglise et al, Nature, **455**, 510 (2008)

Coherent states

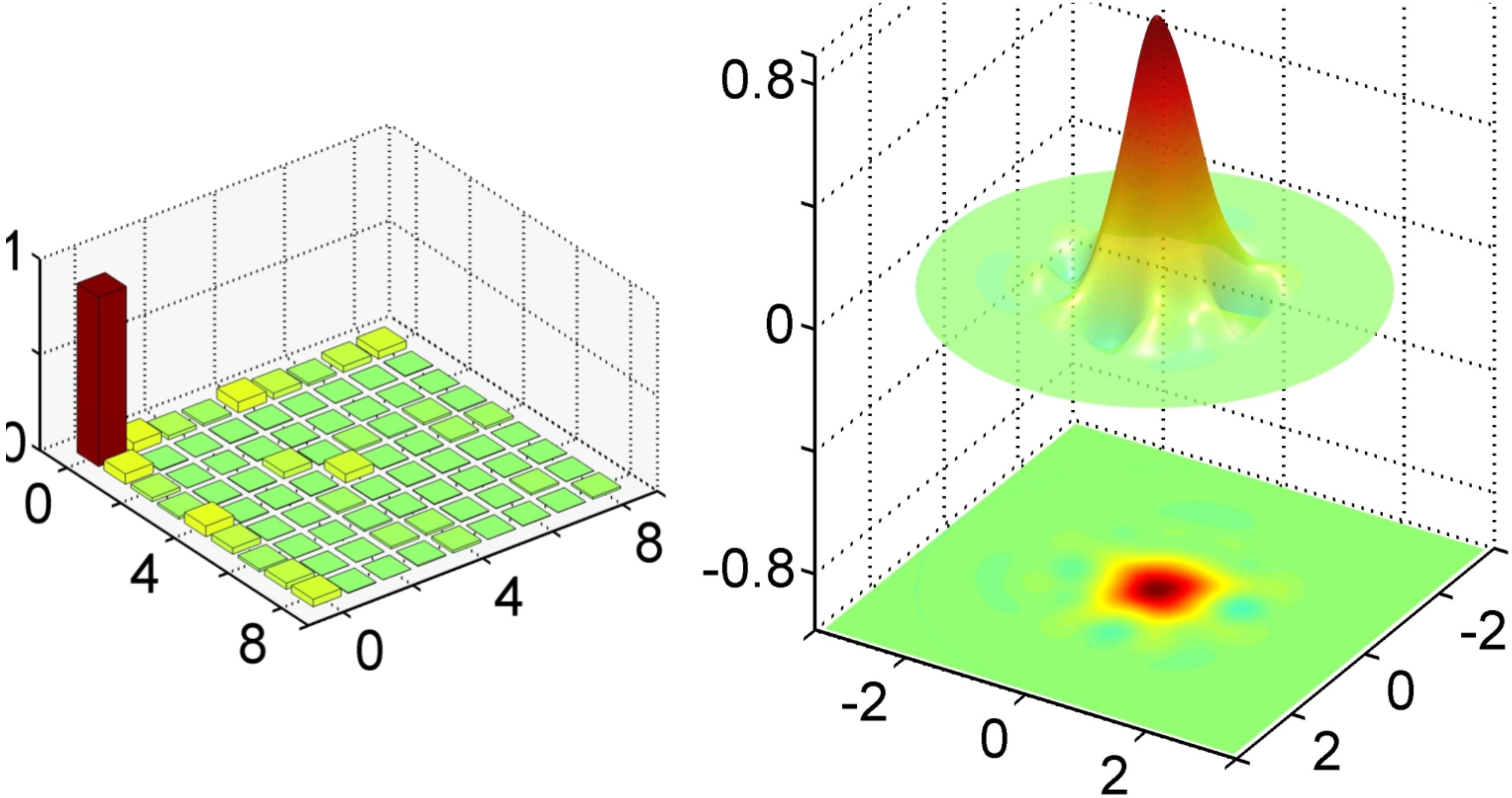
- A coherent state with 2.5 photons

F=0.98



Fock states

- n=0

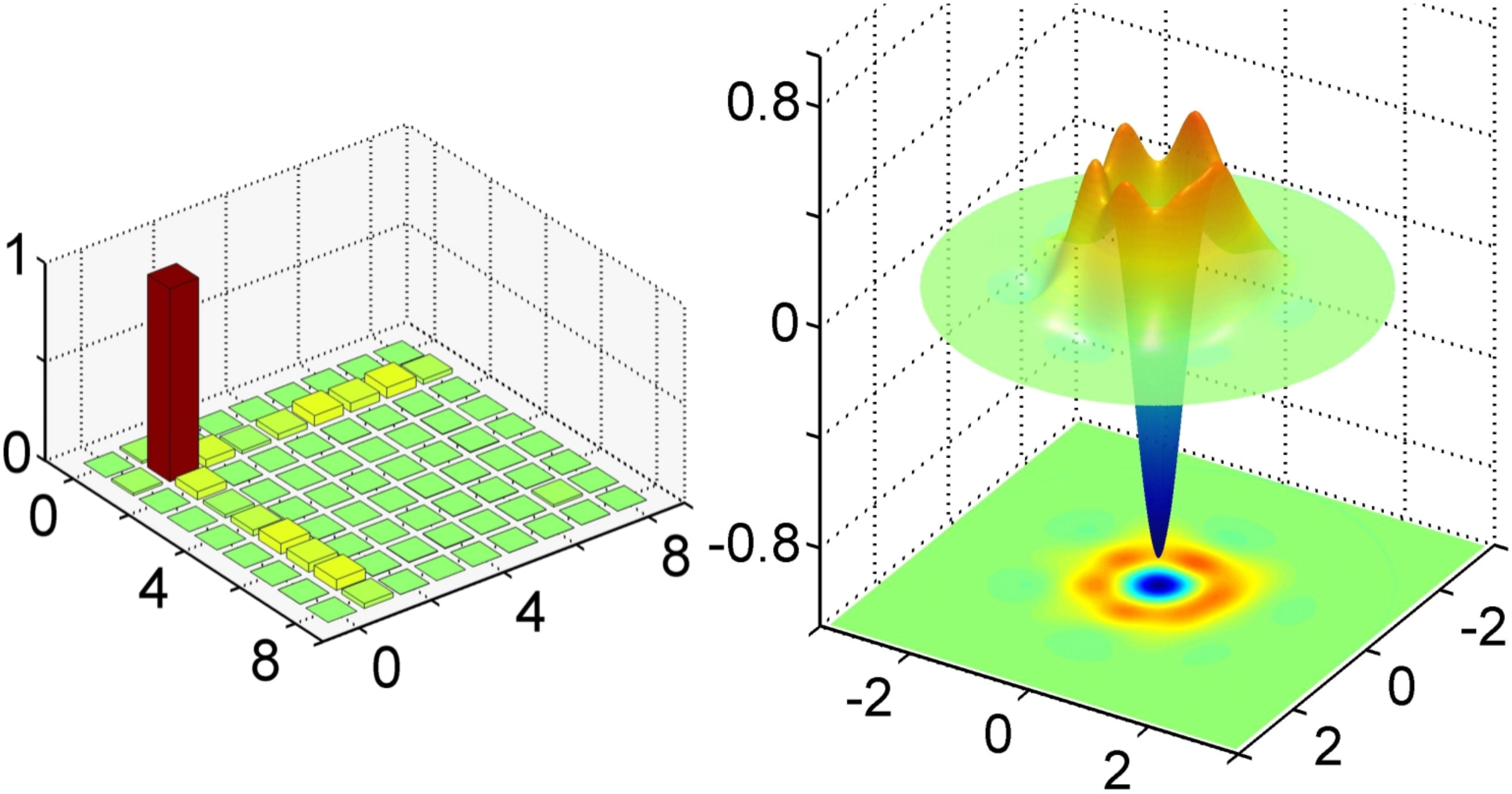


S. Deléglise et al, Nature, **455**, 510 (2008)

F=0.89

Fock states

- n=1

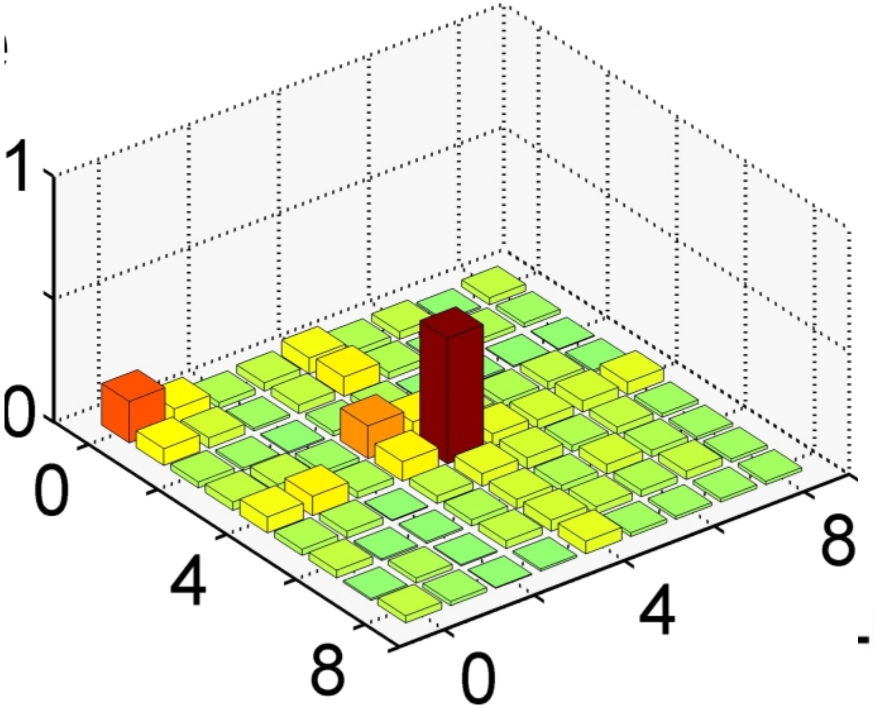


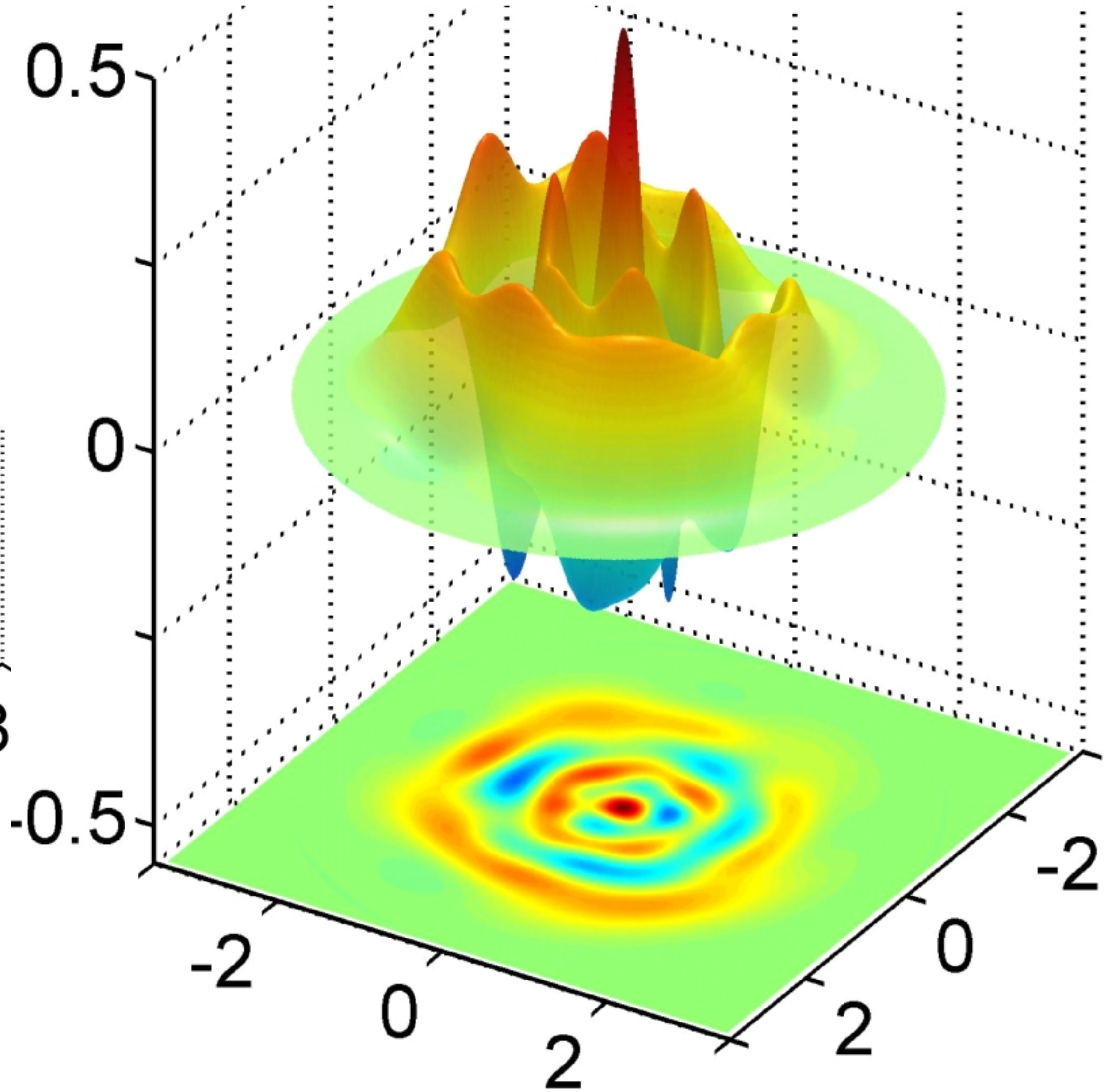
S. Deléglise et al, Nature, **455**, 510 (2008)

F=0.98

Fock states

- n=4





S. Deléglise et al, Nature, **455**, 510 (2008)

F=0.51

Decoherence of Fock states

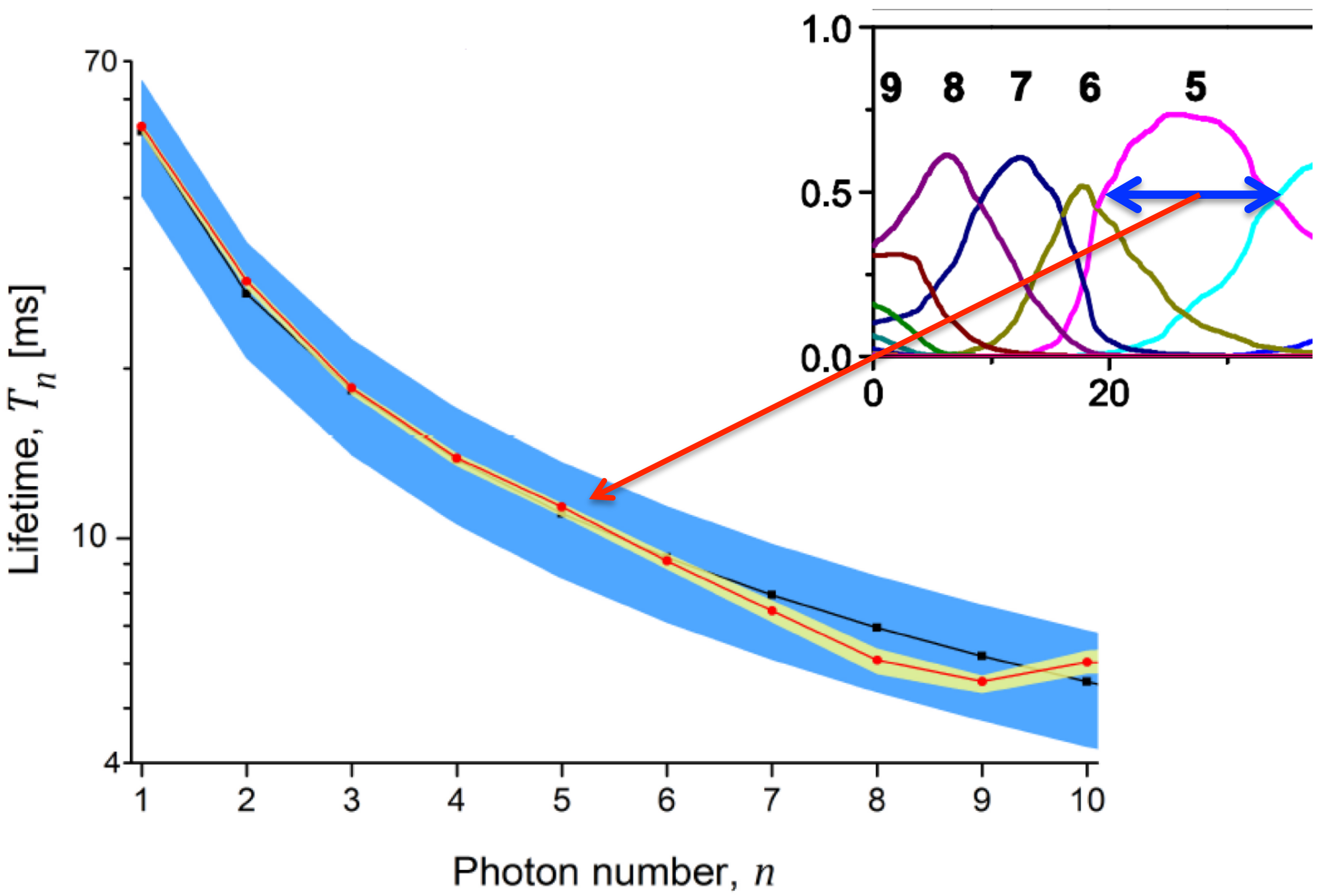
- Non-classical states are short-lived
  - Rapidly transformed into more classical ones by unavoidable relaxation processes
    - Here: cavity damping T_c=0.13 s

- Single photon lifetime (at zero temperature)
    - $\kappa^{-1}=T_c$ the classical field energy damping time
    - Also applies to coherent states
      - Fock states superpositions produced by classical sources
      - Pointer states of the cavity-environment interaction

- |n> lifetime : T_c/n
  - Relaxation time much shorter than the energy lifetime
  - Relaxation time decreases with the size of the state
    - A typical decoherence effect
    - A Fock state is quite similar to the Schrödinger cat!

Lifetime of the *n* photon Fock state using PQS

- Analyze average time between jumps
  - Fock states lifetime T_c/n



  - An impossible feat with forward estimation only due to spurious noise-induced jumps (Brune et al. PRL 101 240402)

T. Rybarczyk et al., PRA 91 062116

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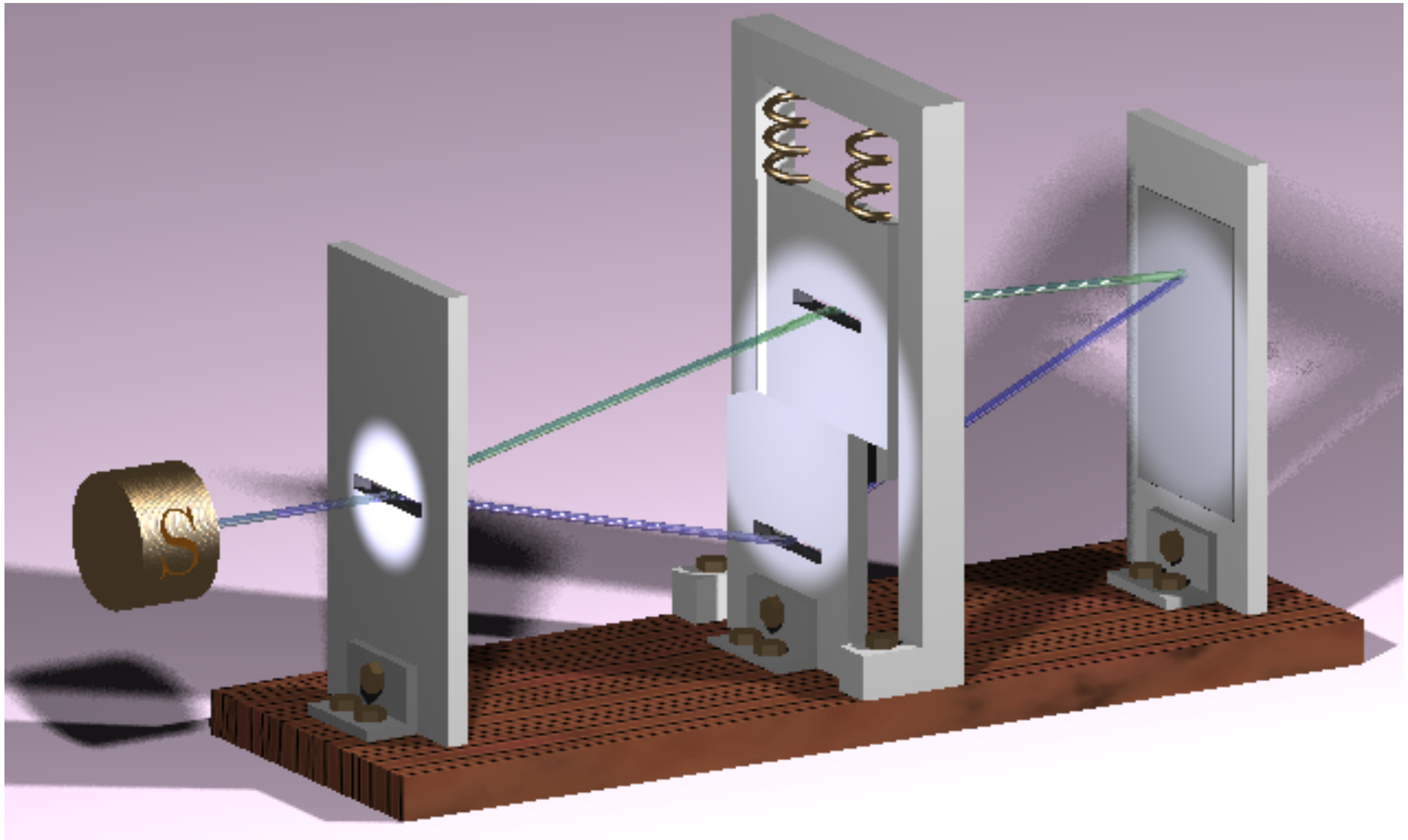
Bohr's thought experiment on complemetarity

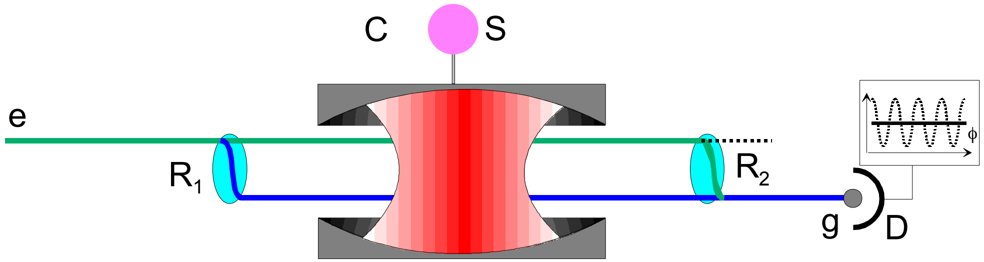
- Complementarity (From Einstein-Bohr at the 1927 Solvay congress)



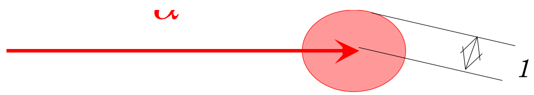
  - Moving slit records the trajectory of the particle in the interferometer
    - Which path information but no fringes
    - Or no which path but fringes
  - Wave and particle are complementary aspects of the quantum object.

Cavity field as a which path detector

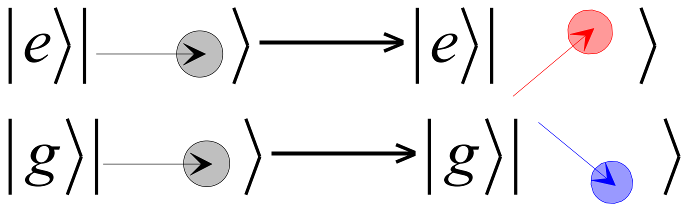
- Insert non-resonant cavity inside the interferometer



  – Cavity contains initially a mesoscopic coherent field



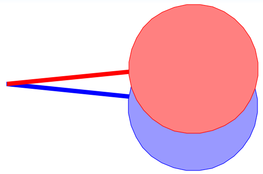
  – The two atomic levels produce opposite phase shifts of the cavity field

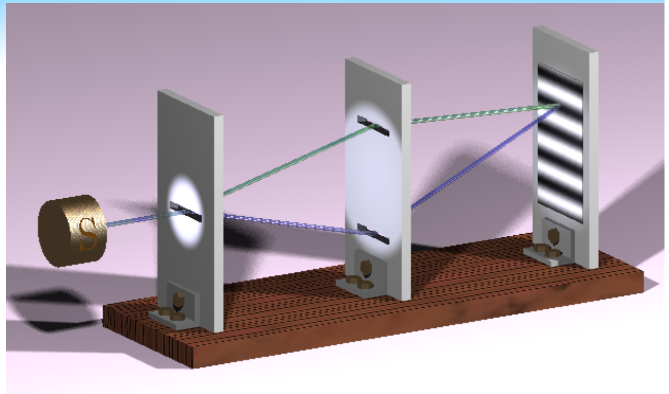


- Field amplitude is the ‘needle’ of a ‘meter’ pointing towards atomic state
  – Prototype of a quantum measurement
  – Provides a which-path information and should erase the fringes

Two limiting cases

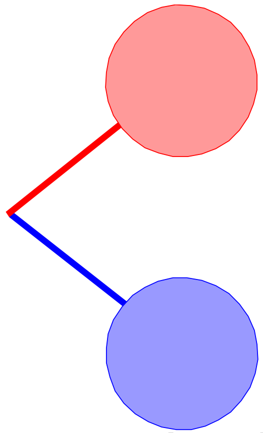
- Small phase shift (large D)
(smaller than quantum phase noise)

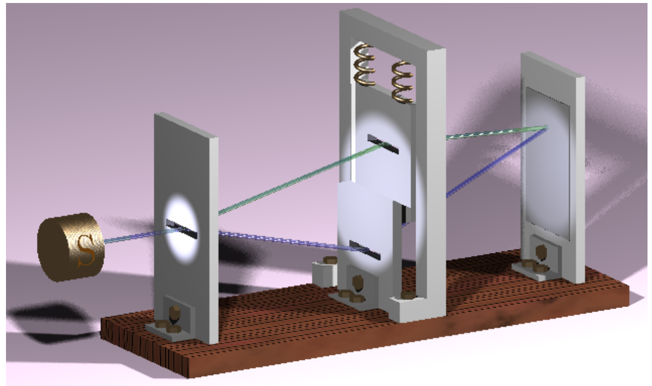




  – field phase almost unchanged
  – No which path information
  – Standard Ramsey fringes

- Large phase shift (small D)
(larger than quantum phase noise)

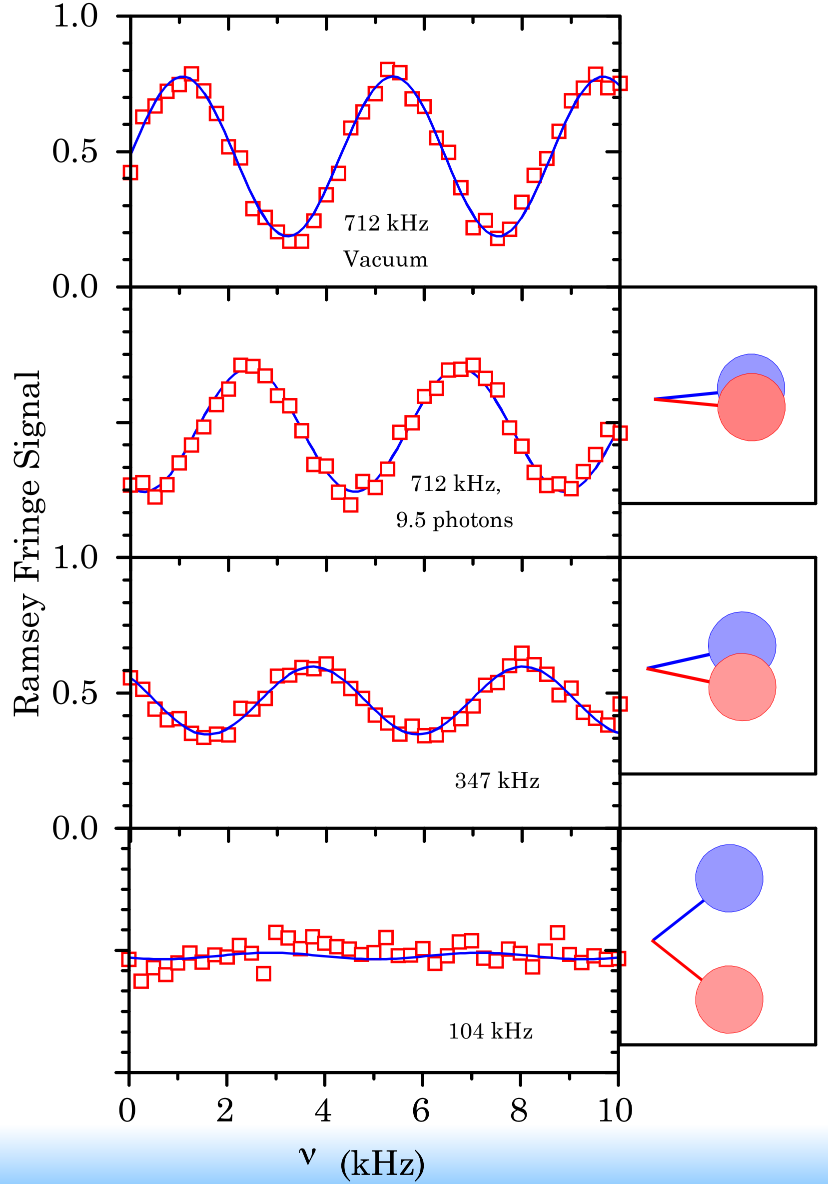




  – Cavity fields associated to the two paths distinguishable
  – Unambiguous which path information
  – No Ramsey fringes

Fringes and field state

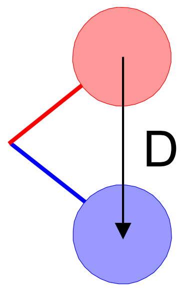
- An illustration of complementarity



Signal analysis

Fringe signal multiplied by $\langle \alpha e^{i\Phi} | \alpha e^{-i\Phi} \rangle$

- Modulus $e^{-2\bar{n}\sin^2\Phi} = e^{-D^2/2}$



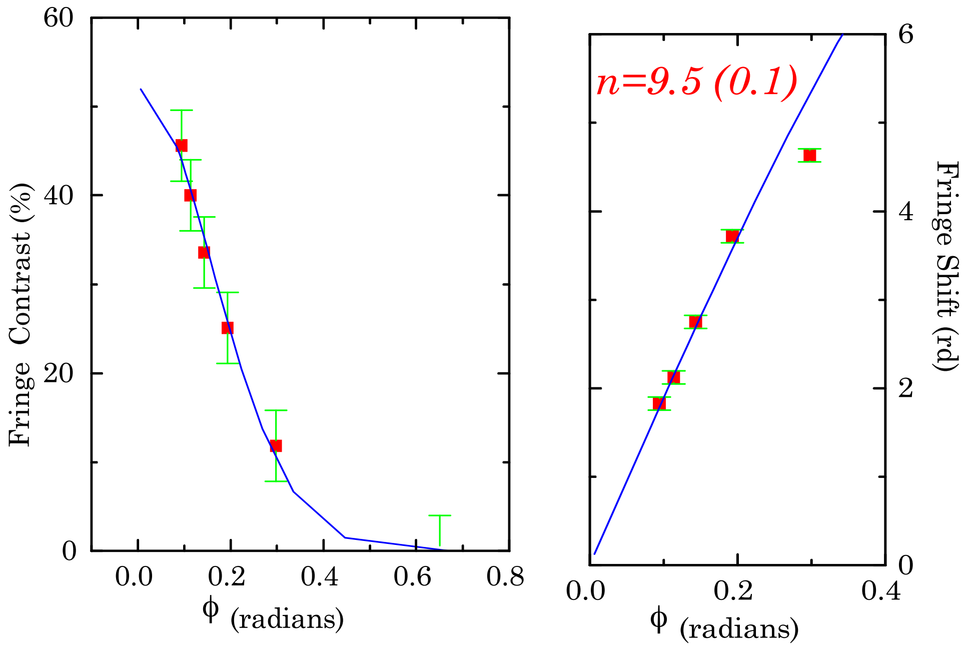
  – Contrast reduction

- Phase $2\bar{n}\sin\Phi$

  – Phase shift corresponding to cavity light shifts

Phase leads to a precise (and QND) measurement of the average photon number

Fringes contrast and phase



- Excellent agreement with theoretical predictions.
- Not a trivial fringes washing out effect

Calibration of the cavity field
9.5 (0.1) photons

A laboratory version of the Schrödinger cat

Field state after atomic detection

$$\frac{1}{\sqrt{2}}\left(|\ \rangle + |\ \rangle\right)$$

A coherent superposition of two "classical" states.

Very similar to the Schrödinger cat

Decoherence will transform this superposition into a statistical mixture

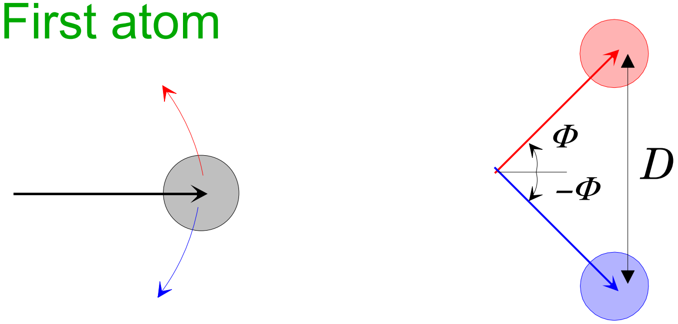
Slow relaxation: possible to study the decoherence dynamics

Decoherence caught in the act

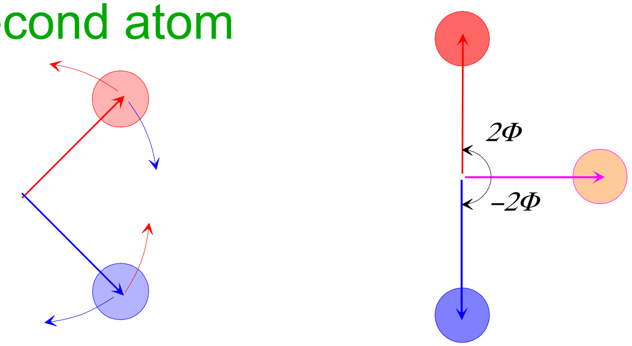
An atom to probe field coherence

Quantum interferences involving the cavity state

First atom



Second atom

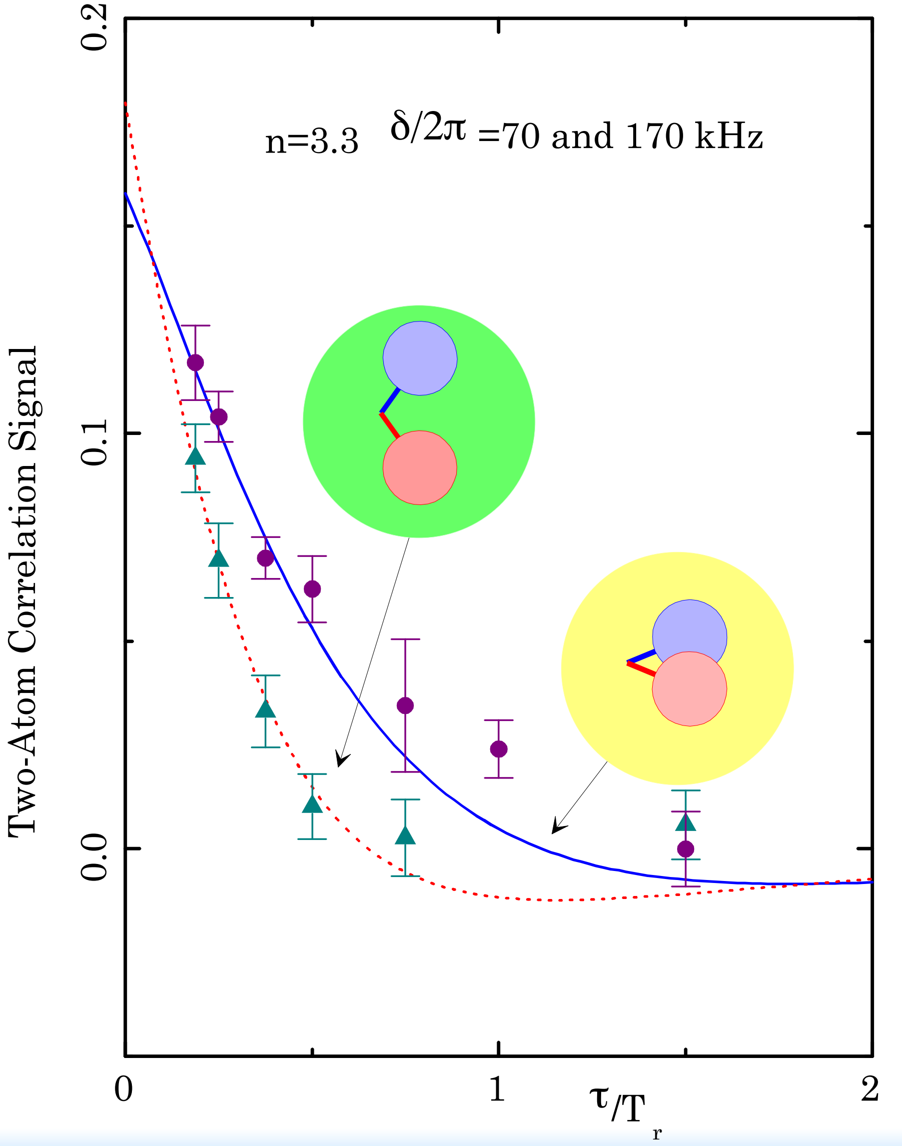


Two indistinguishable quantum paths to the same final state:

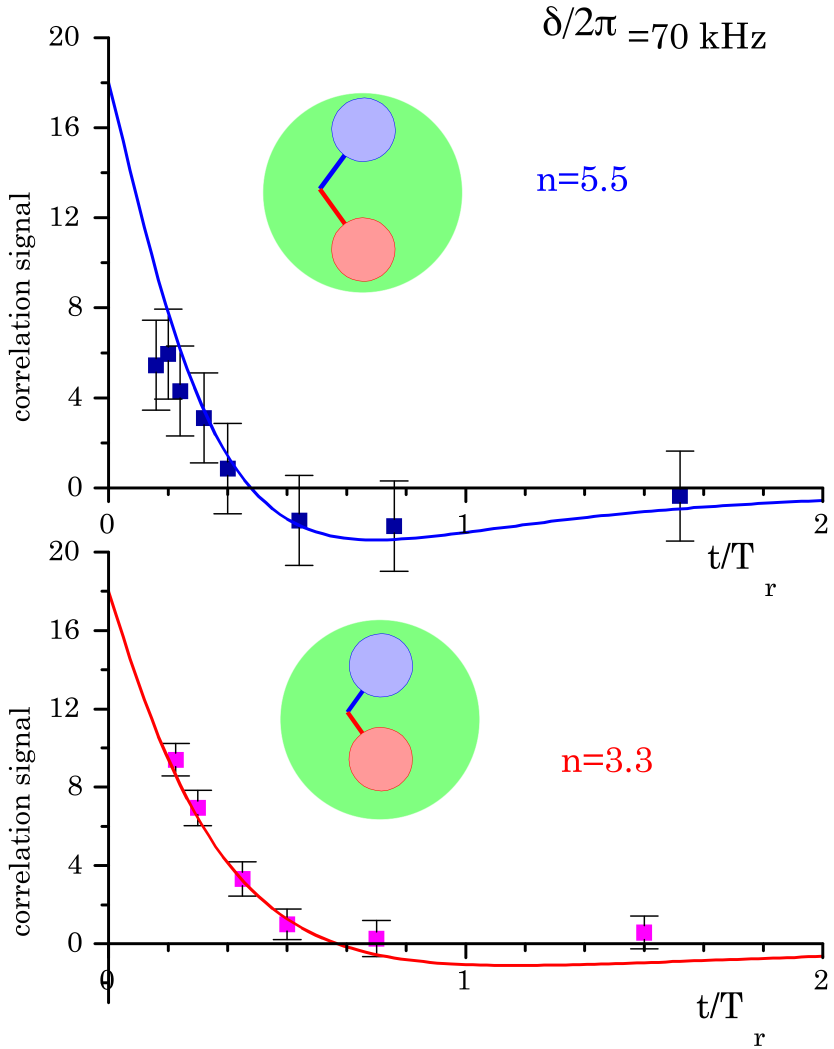
Quantum interference in a two-atom correlation signal

A decoherence study

Atomic correlation signal



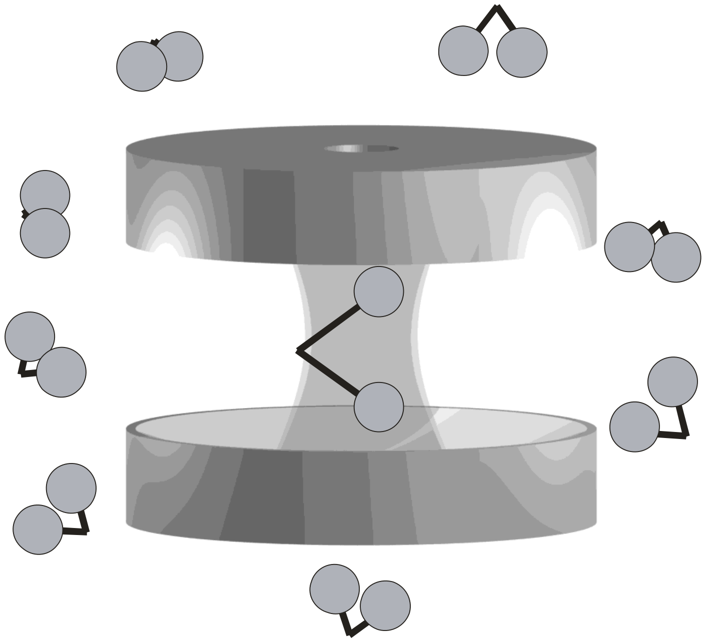
Decoherence versus size of the cat



PRL **77**, 4887 (1996)

A simple calculation of a cat's decoherence

- A cat in a cavity coupled to a bath of linear oscillators



  - Linear cavity-bath coupling: a coherent state in the cavity couples to time-dependent coherent fields in the environment modes (no cavity-environment entanglement)
  - A cat disseminates small kittens in the environment

PRL, **79**, 1964 (1997)

A simple calculation of a cat's decoherence

- Complete wavefunction at time τ:

$$\left|\alpha(\tau)e^{i\Phi}\right\rangle\prod_i\left|\beta_i(\tau)e^{i\Phi}\right\rangle+\left|\alpha(\tau)e^{-i\Phi}\right\rangle\prod_i\left|\beta_i(\tau)e^{-i\Phi}\right\rangle$$

  – Cavity state entangled with environment

- Remaining cat's coherence when tracing over the environment

$$\prod_i\left\langle\beta_i(\tau)e^{-i\Phi}\middle|\beta_i(\tau)e^{i\Phi}\right\rangle=\exp\left[-\sum_i\left|\beta_i\right|^2\left(1-e^{2i\Phi}\right)\right]$$

  – Experimental signal: 0.5x real part of this quantity

- Energy conservation

$$\sum_i\left|\beta_i(\tau)\right|^2=\overline{n}\left(1-e^{-\tau/T_r}\right)$$

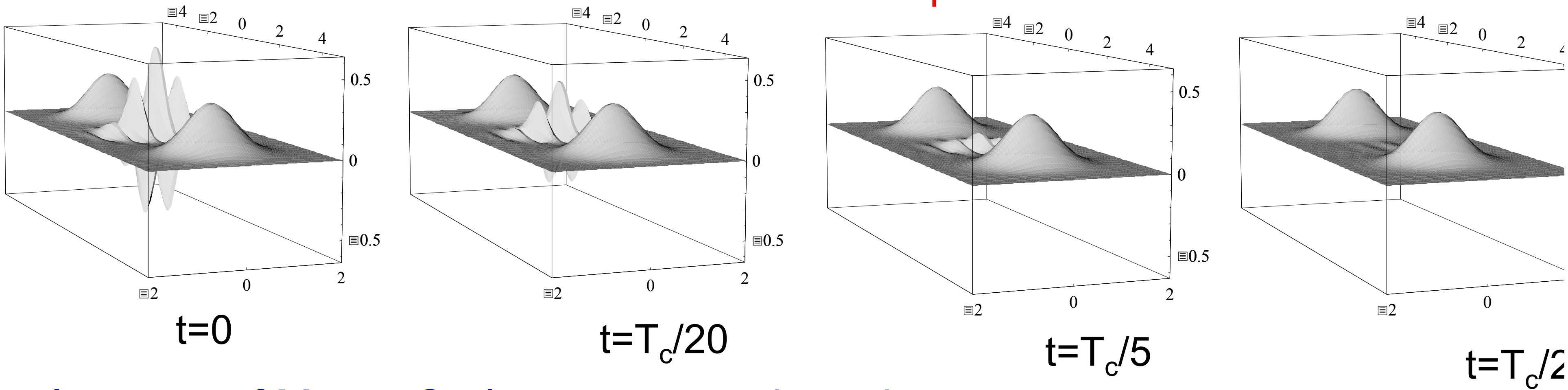
A simple calculation of a cat's decoherence

- Remaining coherence

$$\exp\left[-\bar{n}\left(1-e^{-\tau/T_r}\right)\left(1-e^{2i\Phi}\right)\right] \approx \exp\left[-2\bar{n}\tau/T_r\right] \text{ for } \Phi=\pi/2$$

- Decoherence time scale $T_r/2\bar{n} = 2T_r/D^2$ *D*: distance between cat components

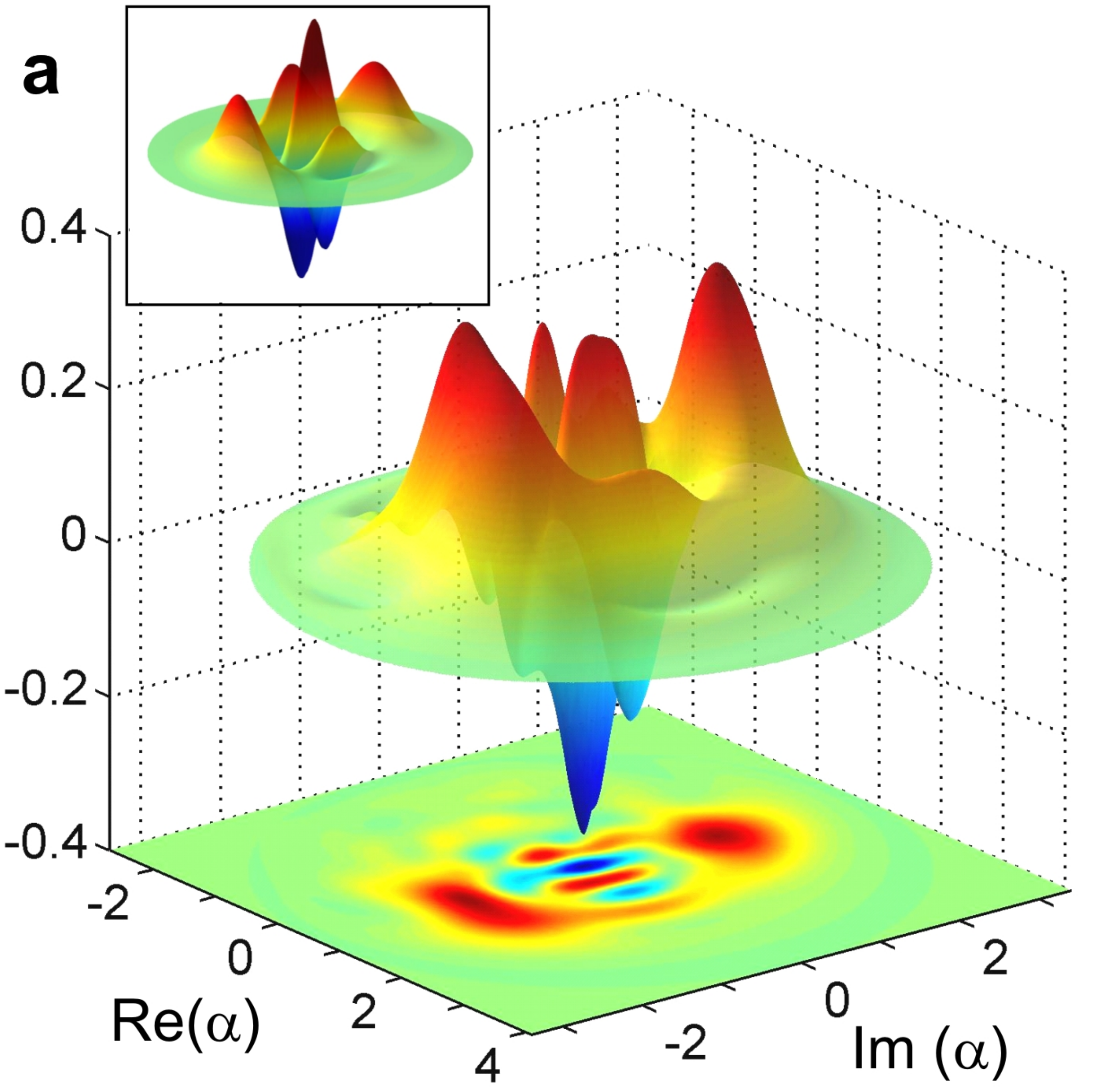
10 photons even cat



- In terms of Monte Carlo quantum trajectories
  – Cat switches parity at each photon loss
  – Parity undetermined when one photon lost on the average

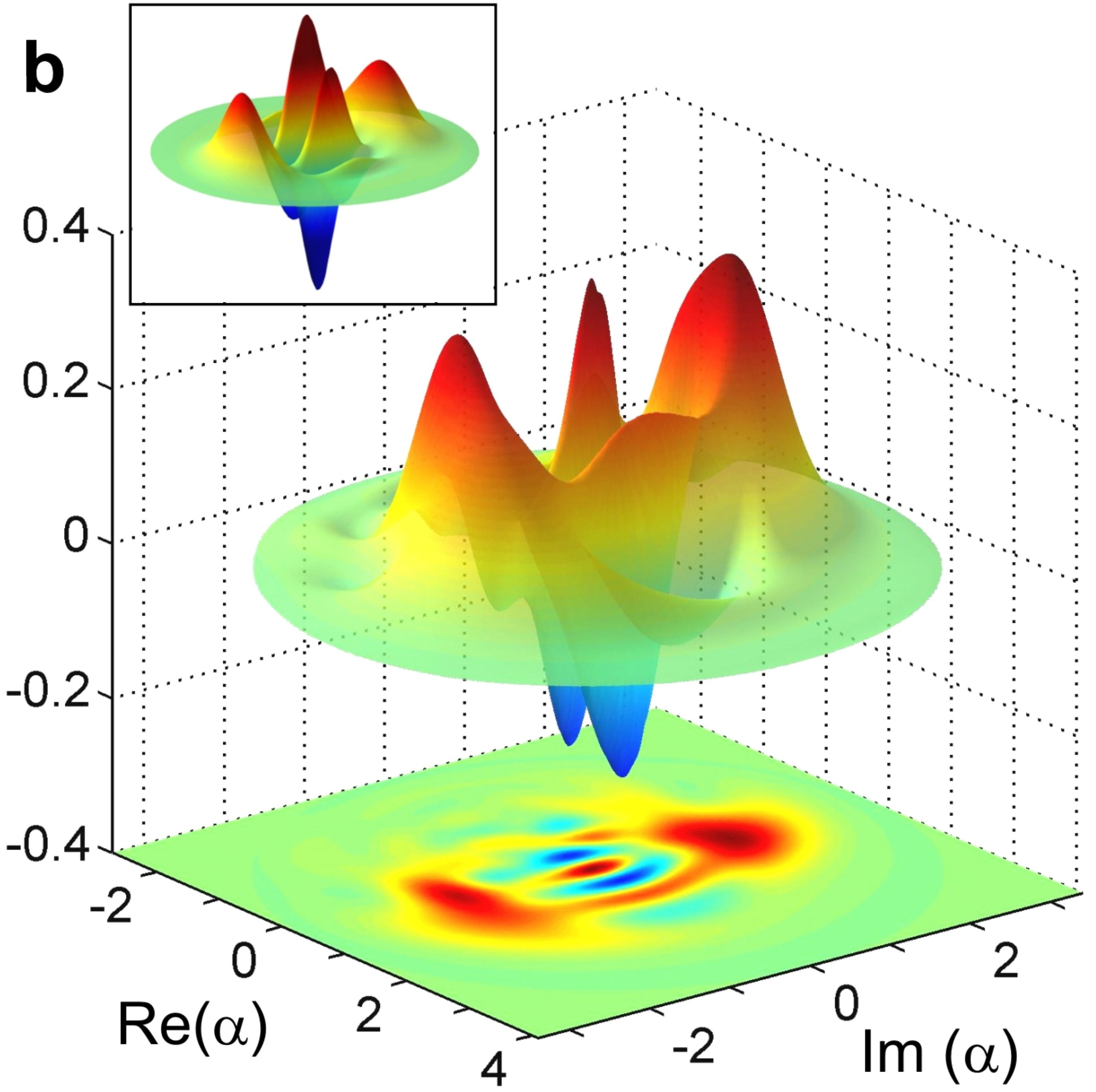
Schrödinger cat states

- Even cat
- n=3.5 photons
- $\zeta=0.37\pi$
- D^2=11.8 photons



Schrödinger cat states

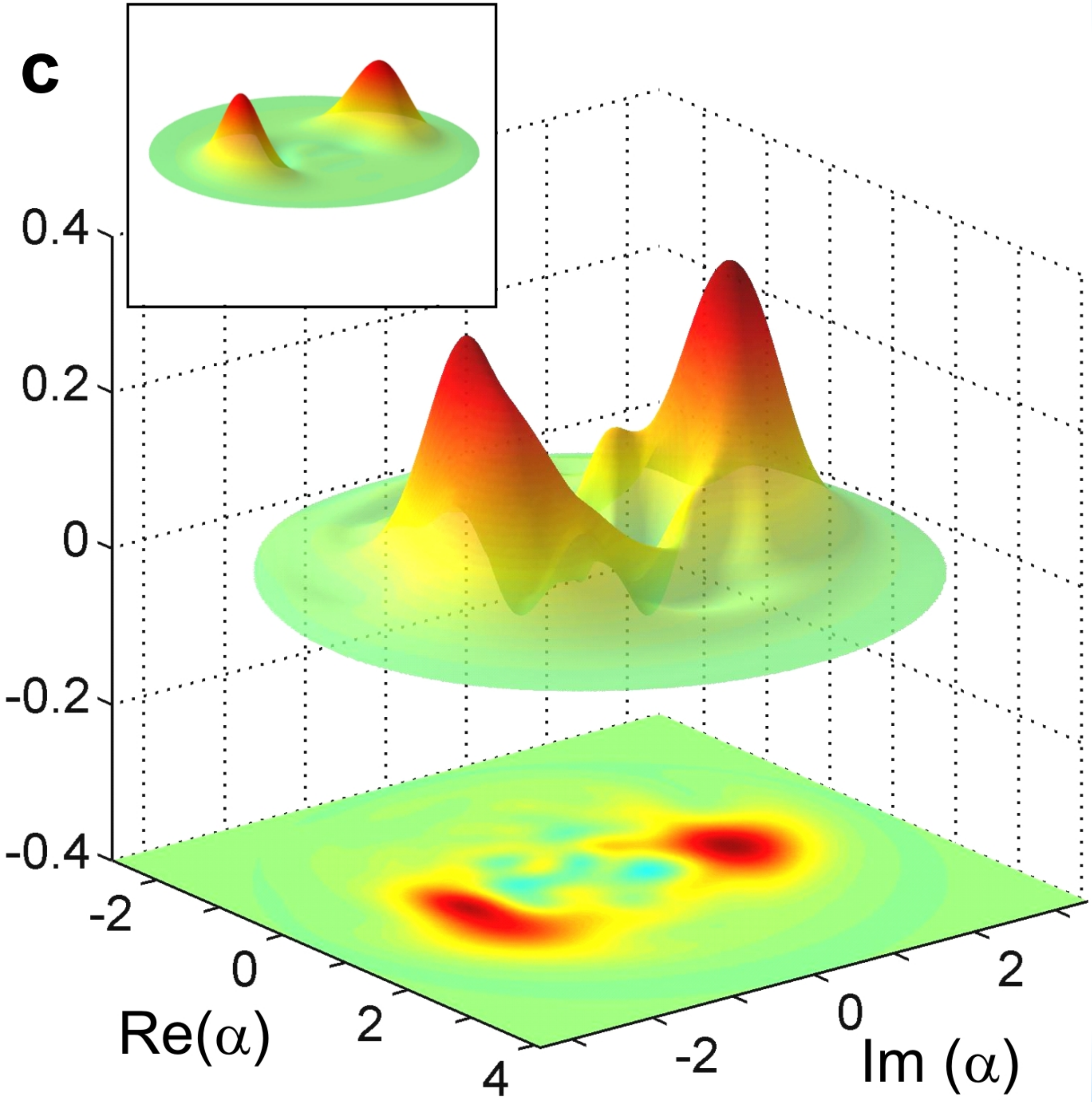
- Odd cat



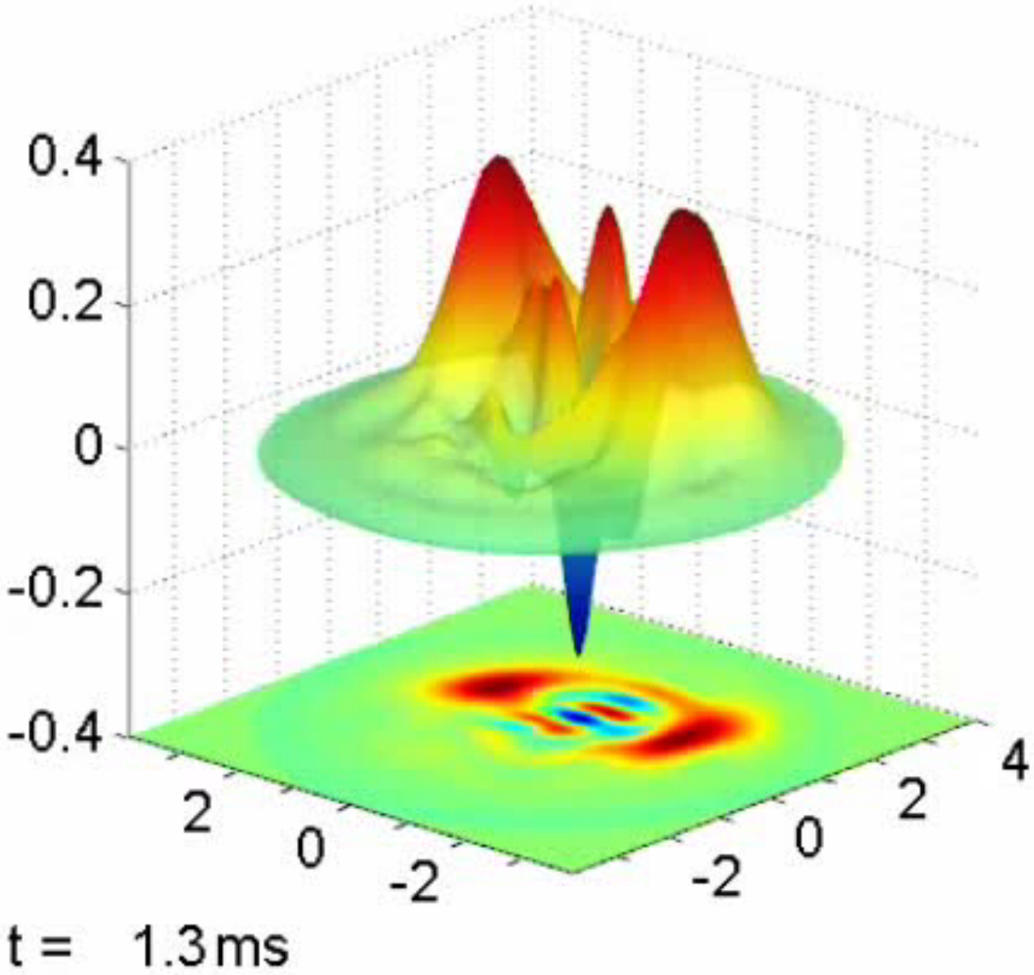
Schrödinger cat states

- Statistical mixture of cats

(or of coherent states)

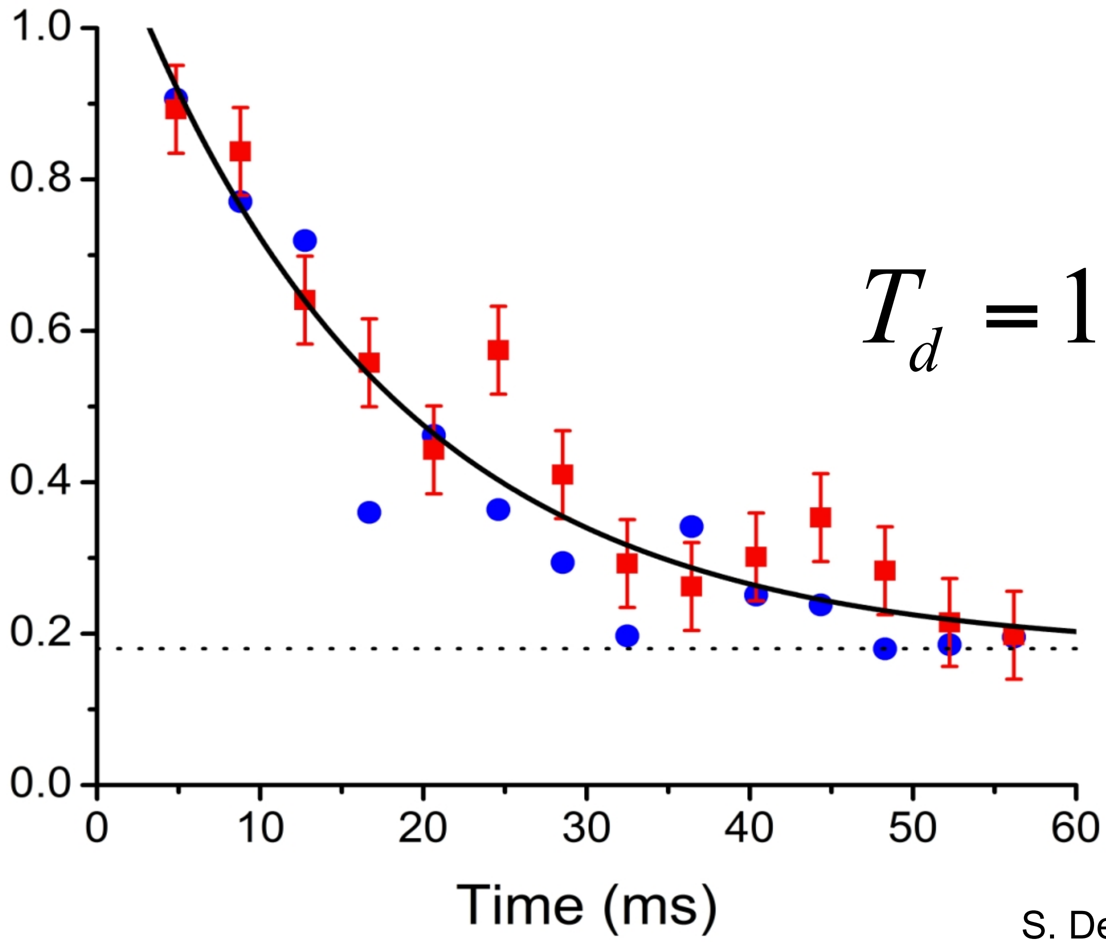


A movie of the even cat decoherence



S. Deléglise et al, Nature, **455**, 510 (2008)

Decoherence time



$$T_d = 17 \pm 3\ \text{ms}$$

S. Deléglise et al, Nature, **455**, 510 (2008)

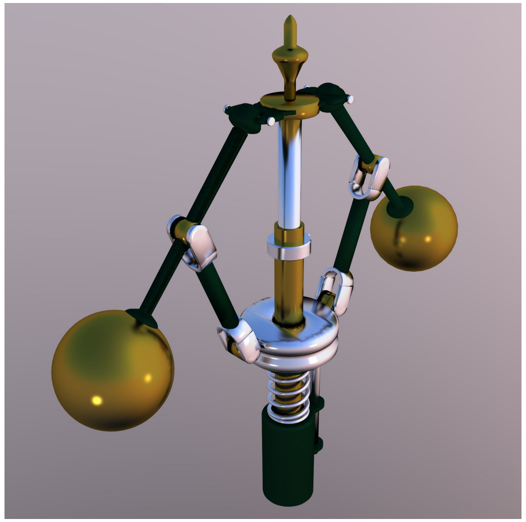
For similar work in circuit QED see Wang et al. PRL **103** 200404

Dispersive microwave CQED

1. Lamb, light and phase shifts
2. A QND measurement of the photon number
3. Zeno effect
4. Fock states reconstruction and decoherence
5. Return on Bohr's complementarity, cats and decoherence
6. Quantum feedback

Feedback: a universal technique

- Classical feedback is present in nearly all control systems
  - A SENSOR measures the system's state
  - A CONTROLLER compares the measured quantity with a target value
  - An ACTUTATOR reacts on the system to bring it closer to the target



- Quantum feedback has the same aims for a quantum system
  - Stabilizing a quantum state against decoherence
  - Must face a fundamental difficulty:
    - measurement changes the system state

Two quantum feedback experiments

- Prepare and preserve a Fock state in the cavity
  - Target state: the photon number state n_t

- Feedback loop
  - Get information on the cavity state
    - QND quantum sensor atoms sent at 82 µs time interval
  - Estimate cavity state and distance to target
    - Fast real-time computer (ADWin Pro II)
      - A complex computation taking into account all known imperfections
    - Decide upon actuator action
  - Actuator action
    - Drives the cavity state as close as possible to the target

Two experiments

- Classical actuator
  - Actuator is a coherent source
    - Displacement of the cavity field
    - Technically simple
    - Not optimal: complex procedure to correct for single photon loss
    - Preparation and protection of Fock states up to n=4

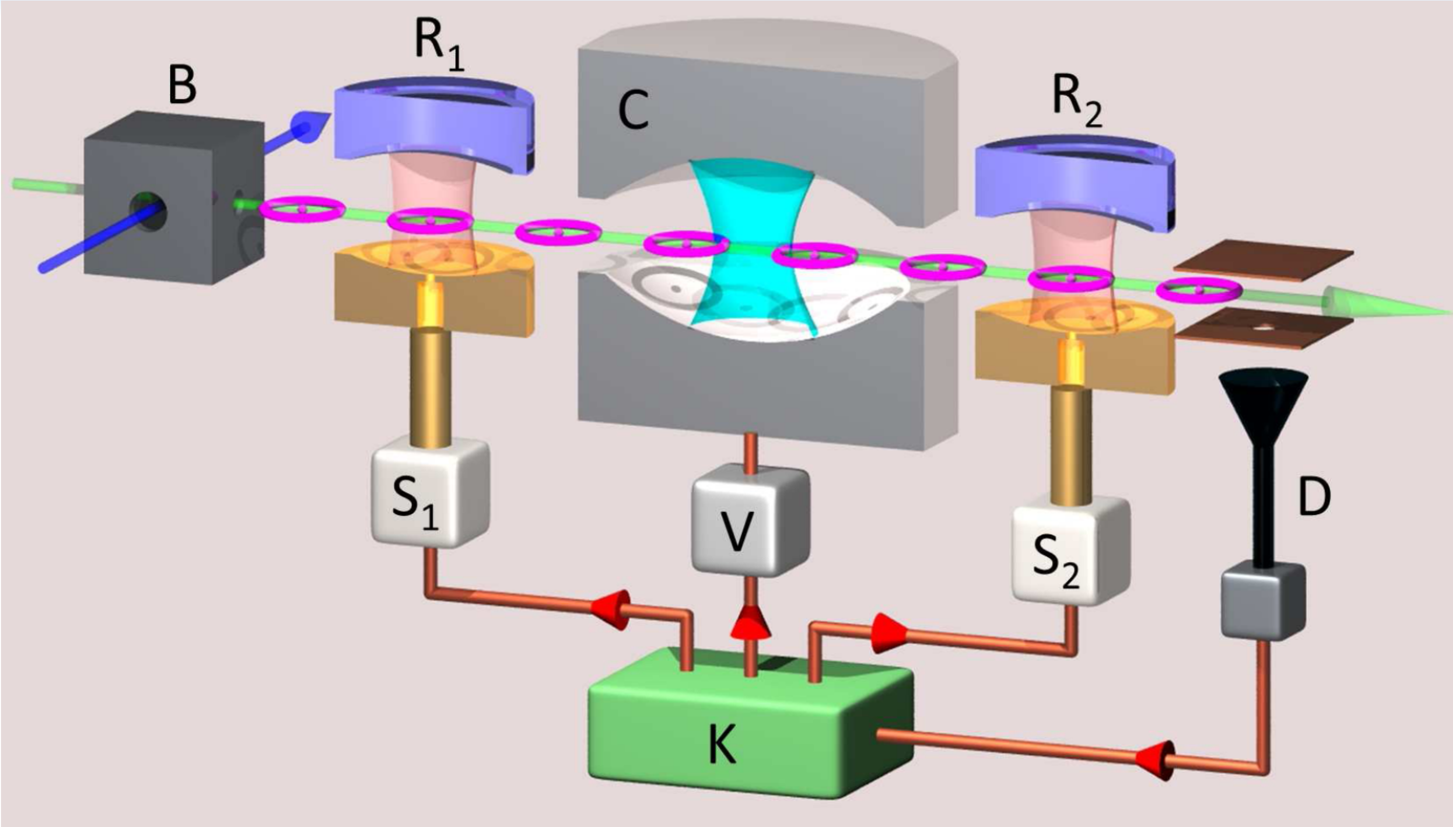
I. Dotsenko, M. Mirrahimi, M. Brune, S. Haroche, J.M. Raimond, P. Rouchon, Phys. Rev. A. 80, 013805 (2009)

C. Sayrin et al. Nature, **477**, 73 (2011)

- Quantum actuator
  - Resonant atoms used to inject/subtract photons
  - More demanding experimentally
  - Faster quantum jumps correction
  - Stabilization of Fock states up to n=7

X. Zhou *et al.,* PRL **108**, 243602 (2012)

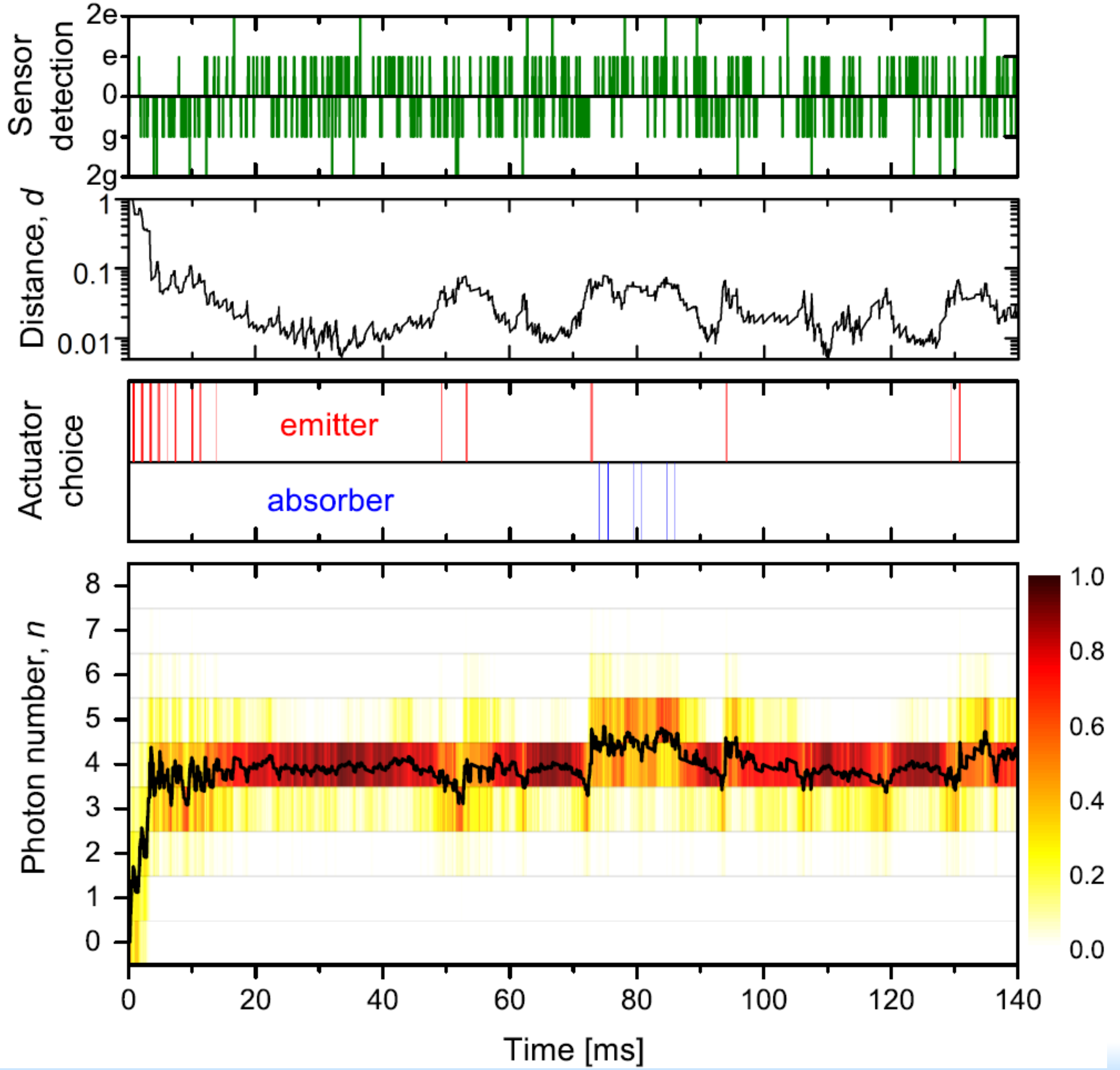
Scheme of the quantum actuator experiment



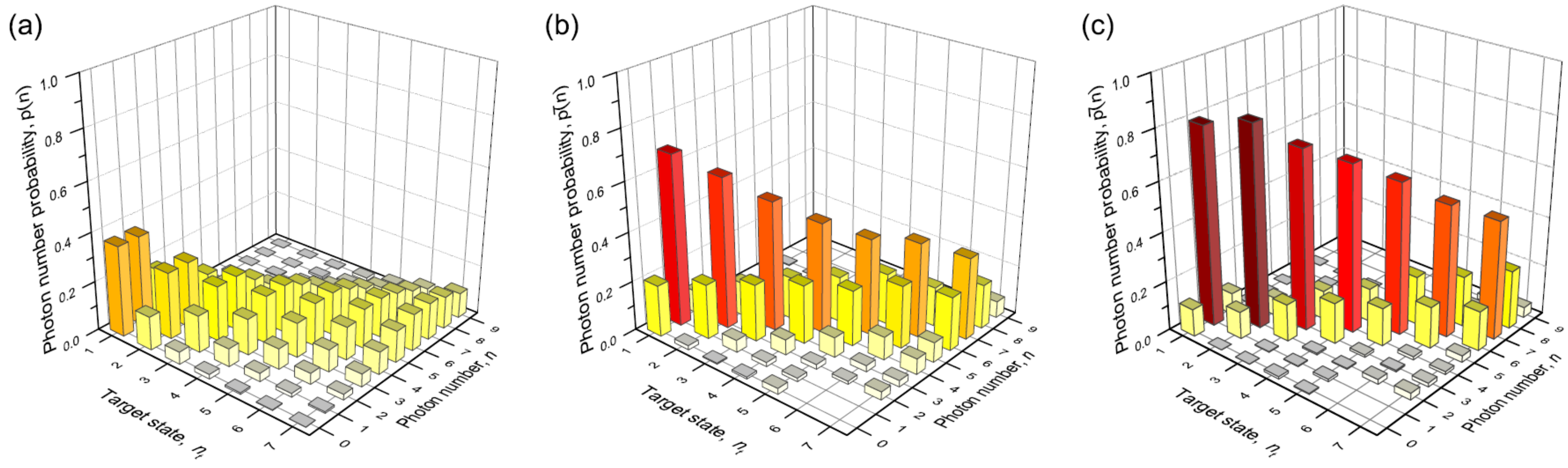
- Atomic samples
  - Sent in the cavity every 82 μs
  - Two types
    - Sensor QND samples (dispersive interaction)
    - Control samples (used by controller for feedback)
      - Absorbers, emitters or mere sensors

A single trajectory: closed loop

- Target photon number n_t=4



Feedback for high photon numbers



Reference

coherent state with n_t photons on the average

Steady state

- stops loop at 140 ms
- independent QND estimation of average photon number distribution $P(n)$

Optimal stop

- Stops loop when $p(n_t)>0.8$
- Independent QMD estimation of $P(n)$

- Stabilization of photon numbers up to 7
- Convergence twice as fast as that of the feedback with coherent source

These lectures

I) Introduction

II) Experimental tools for microwave CQED

III) Theoretical tools for microwave CQED

IV) Resonant microwave CQED

V) Dispersive microwave CQED

VI) Conclusion and perspectives

Perspectives

- Optimal QND measurements
- Reservoir engineering
- Quantum Zeno dynamics
- Non-local cats

Using feedback to optimize QND measurement

- Send atoms one by one and use previous information to optimize information brought by next atom

- A simple scheme in an ideal setting
  - Assume n<8 (0 through 7 photons)
  - First atom sent in *g* with $\phi_0=\pi$, $\phi_r=0$
    - Detected state tells the field parity
      - Detected in *e* when empty or even photon number
      - Detected in *g* when odd photon number
    - Atom gives the Least significant bit of photon number
    - Projects the field on a parity eigenstate (cat if initial state coherent)
  - Second atom sent with $\phi_0=\pi/2$
    - Phase ϕ_r adjusted to distinguish
      - 0,4 from 2,6 if parity even
      - 1,5 from 3,7 if parity od
    - Atom gives the second bit of the photon number

Using feedback to optimize QND measurement

- A simple scheme in an ideal setting
  - Third atom sent with $\phi_0=\pi/4$
    - Ramsey phase set to remove the last ambiguity
    - Atom gives the third bit of the photon number
  - Measurement of photon number from 0 to 7 with 3 atoms
    - Instead of 110

- Straigthforward generalization
  - Measurement of photon number from 0 to N-1 with $\log_2(N)$ atoms
  - Optimum set by information theory
  - An optimal quantum digital/analog converter

- Realistic setting
  - Measure photon number from 0 to 7 with ~13 atoms (instead of 110)

Perspectives

- Optimal QND measurements
- Reservoir engineering
- Quantum Zeno dynamics
- Non-local cats

Another route towards state protection

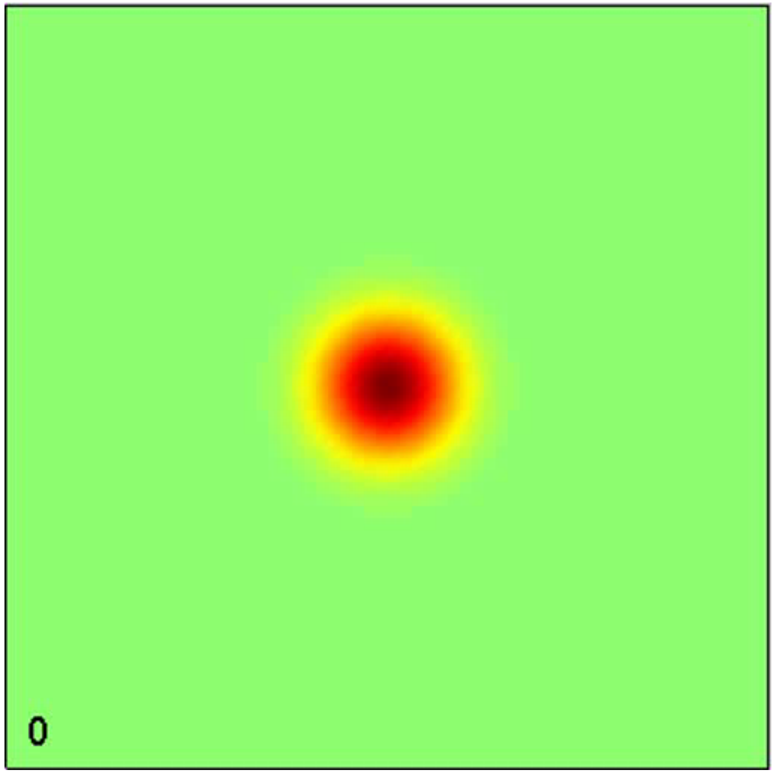
- Reservoir engineering
  - Couple the system to a controlled reservoir
    - Pointer states are the non-classical states to protect
    - Strong controlled relaxation protects these states

- Our engineered reservoir
  - A stream of atoms undergoing composite interaction with the cavity
    - Dispersive, resonant and dispersive again
  - Stabilizes all states produced by a fictitious Kerr Hamiltonian acting on a coherent state.
    - Squeezed states, cats and 'multi-legged' cats
  - An example of quantum simulation
  - An example of decoherence manipulation

A. Sarlette et al. PRL **107**, 010402

Preparation and preservation of a two-legged cat

- Ideal case

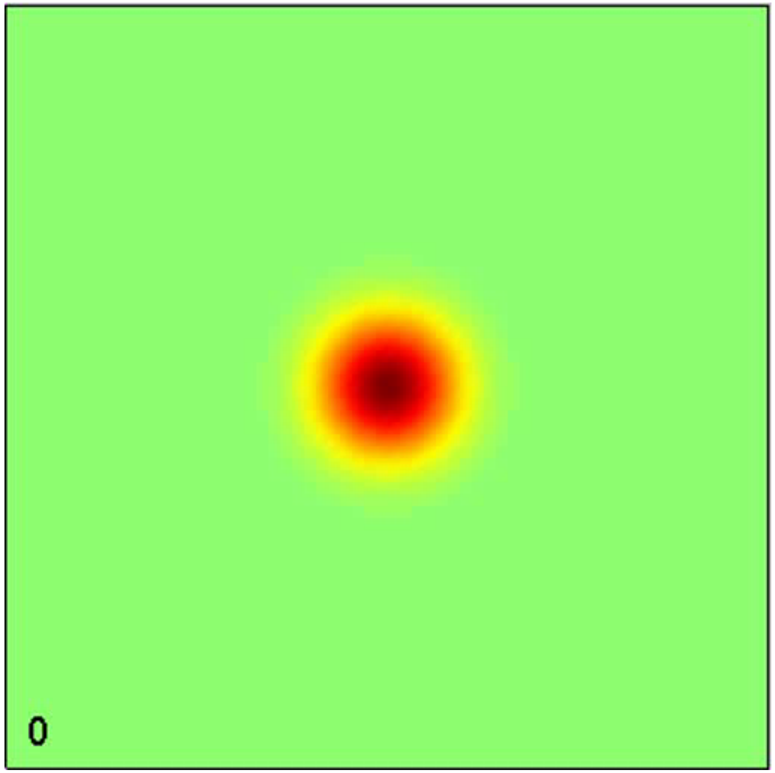


Final fidelity: 96%

u=0.45π, Θ =π/2, ϕ_0~π

Ideal cavity

- Realistic conditions



Fidelity 69%

Cavity damping time T_c=0.13 s

Thermal field n_T=0.05

v =70 m/s, Interaction time 257 μs 0.3 atom per sample, δ =2.2 Ω_0

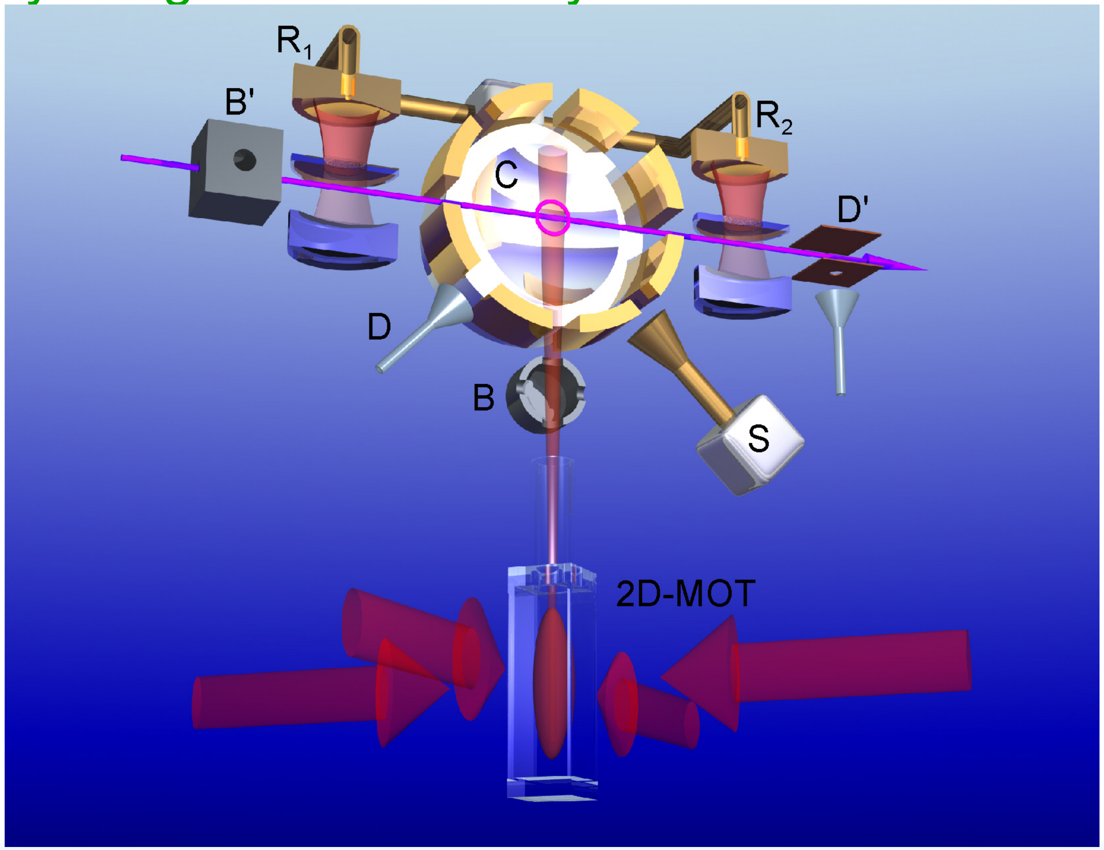
Atomic frequency tuned via the Stark effect during atomic transit

Perspectives

- Optimal QND measurements
- Reservoir engineering
- Quantum Zeno dynamics
- Non-local cats

A new cavity QED set-up

- A strong limitation of present experiments
  - Atom-cavity interaction time << both systems lifetime
    - 100 μs << 30ms, 0.13 s
- Achieving long interaction times
  - A set-up with a stationary Rydberg atom in a cavity
  - Circular state preparation and detection in the cavity
  - Interaction time ms range



A new cavity-QED set-up

- Perspectives
  - Large cats
    - tens of photons
  - Decoherence metrology
    - complete process tomography
  - Quantum random walks
    - For the phase of a coherent state (QRW in circular topology)
  - Quantum Zeno dynamics

J.M. Raimond et al PRL **105**, 213601

Quantum Zeno effect and quantum Zeno dynamics

- Quantum Zeno dynamics
  - Repeated measurement of an observable with a degenerate eigenvalue μ (eigenspace H_μ, projector P_μ)
    - State initially in H_μ remains in H_μ and evolves under the effective hamiltonian $H_\mu = P_\mu H P_\mu$
      - Restriction of evolution in a subspace may have surprising and interesting effects

  - Alternative route towards quantum Zeno dynamics:
    - Repeated actions of a unitary Kick operator U_K, with the same eigenspaces H_μ
      - Related to 'bang-bang' control techniques

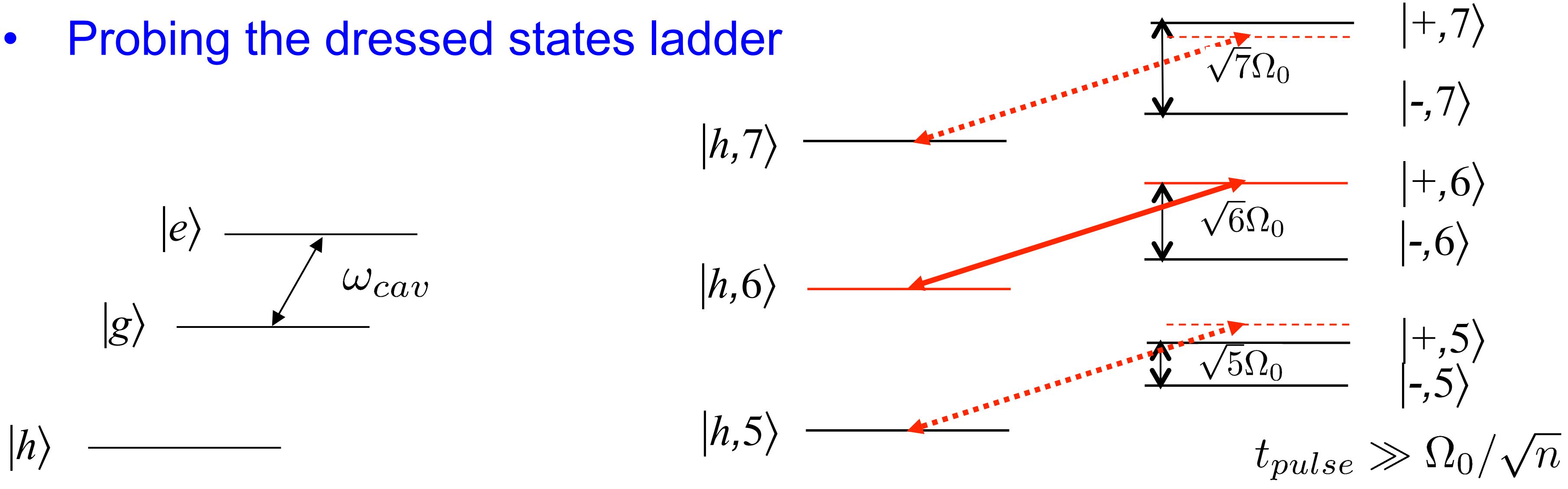
  - Our proposal:
    - Realization of a quantum Zeno dynamics for the cavity field in a subspace.

A photon number-selective measurement

- Measurement: a yes/no question
  - Are there exactly *s* photons in the cavity or not ?
- If frequently repeated
  - Confinement of the dynamics in the subspaces with less or more than *s* photons
  - Quantum Zeno dynamics in two disjoint subspaces
- Use the dressed states to implement photon-number selectivity
  - And the long interaction times to probe the dressed states with high resolution pulse

A photon number selective measurement

- Probing the dressed states ladder



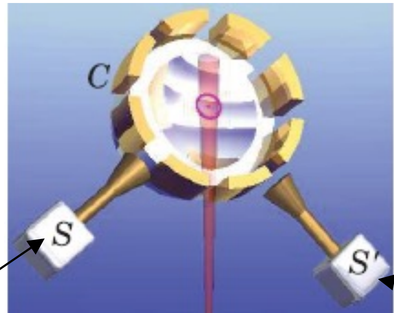
  – Resonant pulse on the $|h,s\rangle \to |+,s\rangle$ transition
    - π pulse: final atomic state [*h* or (*e,g*)] tells out the photon number
      – Atom in *e* or *g*: the photon number is exactly *s*
      – Atom in *h*: the photon number is NOT *s*
    - 2π pulse: $|h,s\rangle \to -|h,s\rangle$
      – Atom stays in *h*. Photon number selective unitary kick on the field: Uk=1-2$|s\rangle\langle s|$)
      – Same atom can be used for a new operation.
        » Focus on this situation in the following

A stroboscopic evolution

- QZD
  - Displacement performed by a coherent source

- A step process. At each step:

  - Photon number selective unitary U_k
  - Small displacement of the cavity field (amplitude β)



2π pulse on the atom.

alternate

Inject cavity field $D(\beta)$

A photon-number selective kick operation

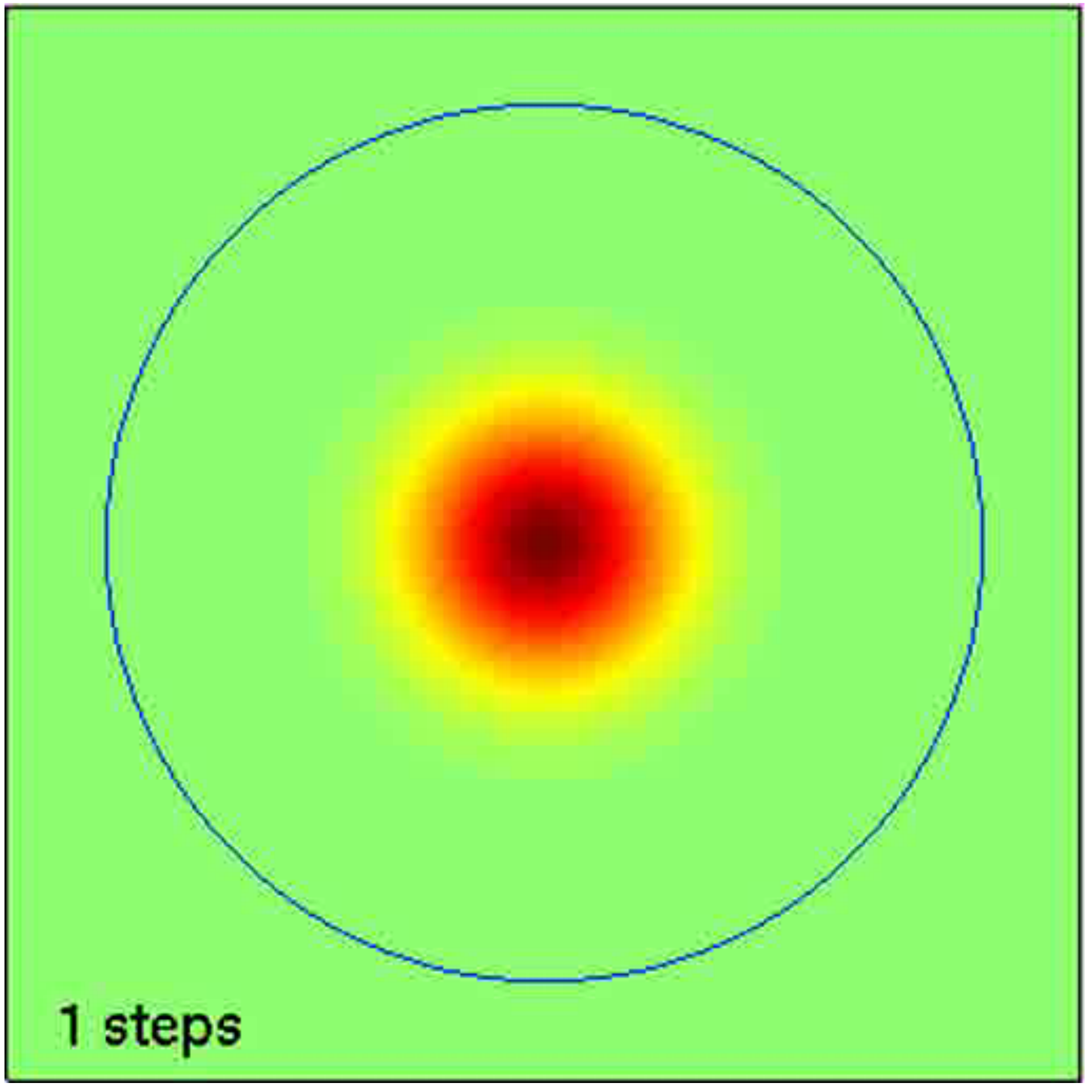
- Invariant subspaces
  - U_k has eigenvalues +1 and -1
    - -1 : associated subspace $|N_s>$
    - +1 : two degenerate eigenspaces $H_{<s}$ and $H_{>s}$
      - projectors $P_{<s}$ and $P_{>s}$
    - Effective hamiltonian for the source S':
      - $H_e = P_{<s} H P_{<s} + P_{>s}\, H\, P_{>s}$

    - Hilbert space divided in two orthogonal subspaces in which state evolution remains confined

    - $|N_s>$ is a 'hard wall' in the Hilbert space
      - Represented in phase space by a $\sqrt{N_s}$ radius 'exclusion circle' EC

Dynamics inside the exclusion circle

- 150 steps, N_s=6

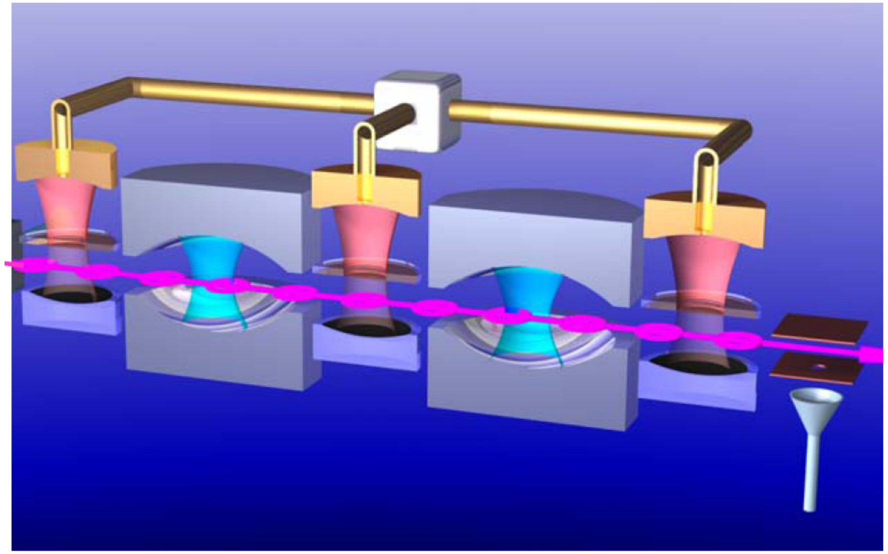


Perspectives

- Optimal QND measurements
- Reservoir engineering
- Quantum Zeno dynamics
- Non-local cats

A new breed of quantum monster

- Entangling a single atom with two mesoscopic fields



- Dispersive interaction:

no energy exchange but entanglement of the field classical phase with the atomic state (index of refraction)

Final two-cavity state

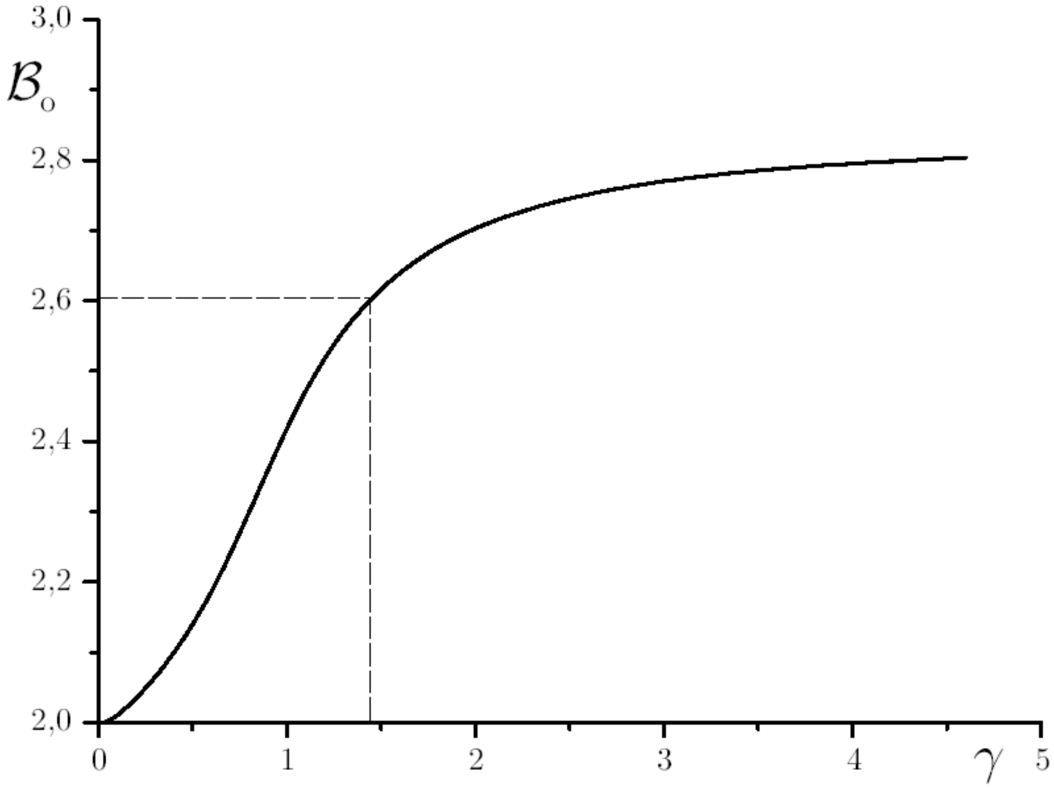
$$\left|\gamma,\gamma\right\rangle+\left|-\gamma,-\gamma\right\rangle$$

Bell inequality violation

- An adapted version of the Bell inequalities (Wodkiewicz et al. PRL **82**, 2009)

$$\mathcal{B} = |\Pi(\alpha', \beta') + \Pi(\alpha, \beta') + \Pi(\alpha', \beta) - \Pi(\alpha, \beta)| \leq 2,$$

$$\Pi(\alpha, \beta) = (\pi^2/4) W(\alpha, \beta)$$

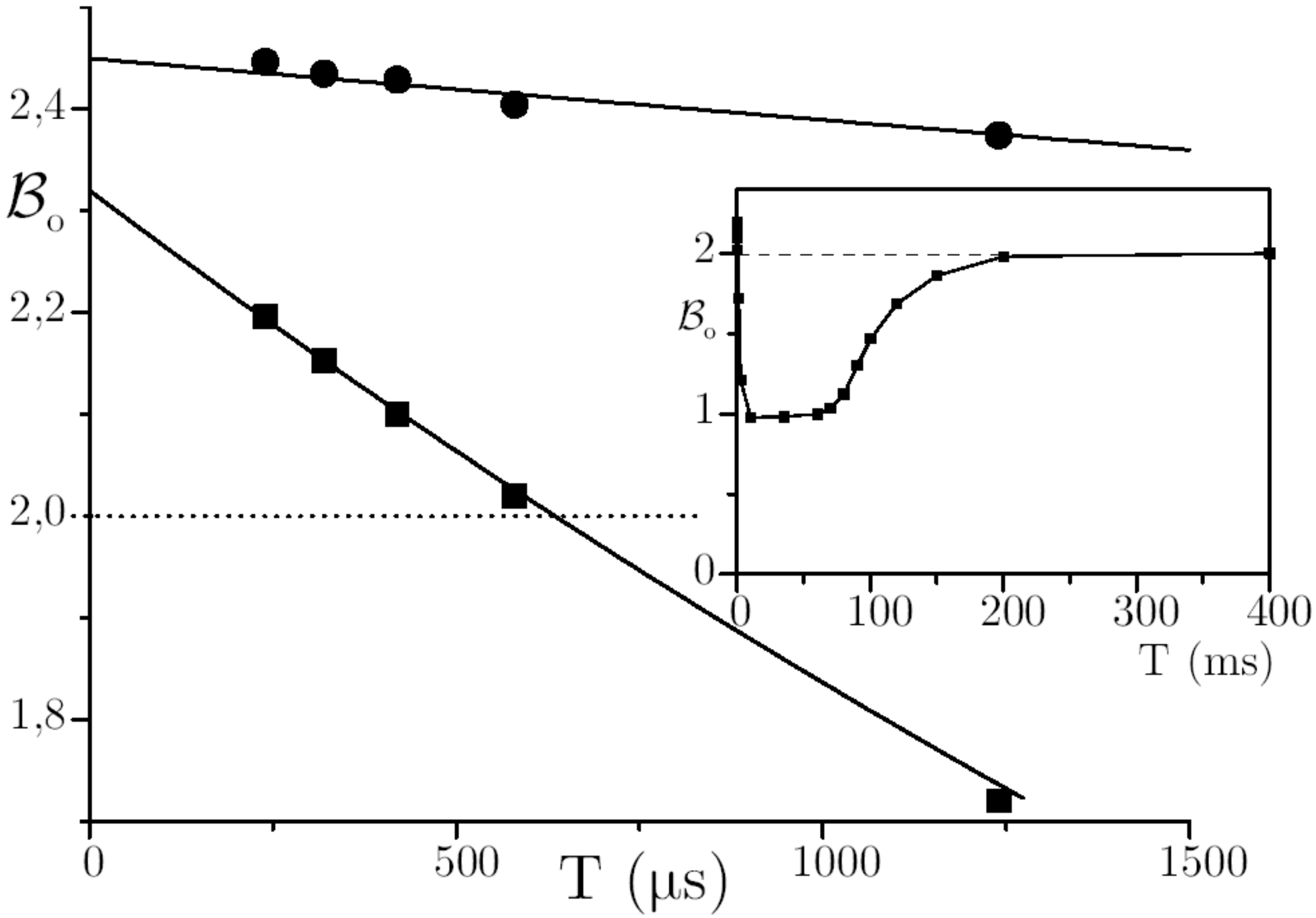


- Large violations for large fields
- Appreciable violations for a

two-photons cat.

- Joint Wigner function can be easily measured by a single probe atom adapting the single cavity protocol

Observable Bell inequality violation

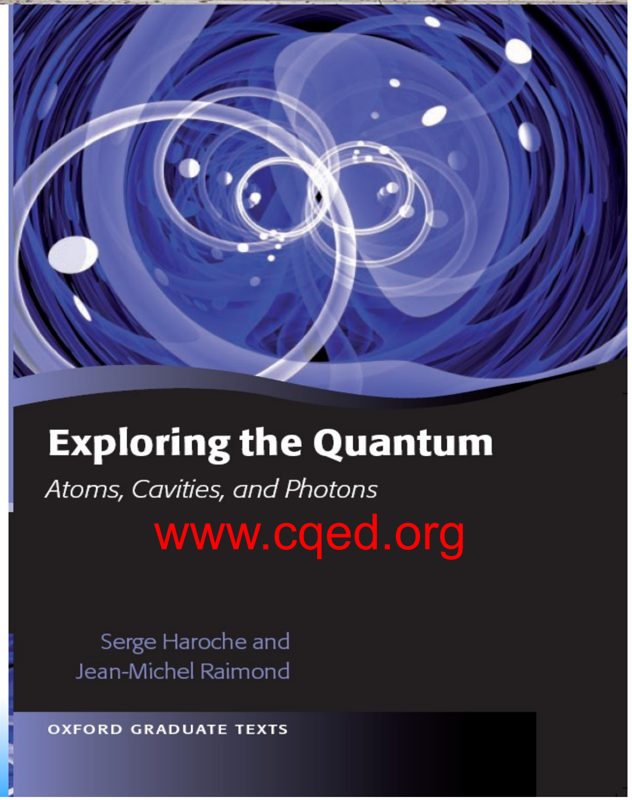


P. Milman et al EPJD, **32**, 233

- Critical parameter: cavity damping time (30 or 300 ms here)
- Requires an extremely good cavity

A team work

- S. Haroche, M. Brune,
  J.M. Raimond, S. Gleyzes, I. Dotsenko,
  C. Sayrin
- Cavity QED experiments
  - S. Gerlich
    T. Rybarczyk, A. Signoles,
    A. Facon, D. Grosso, E.K. Dietsche,
    V. Métillon, F. Assemat
- Superconducting atom chip
  - Thanh Long Nguyen, T. Cantat-Moltrecht
  - R. Cortinas
- Collaborations:
  - Cavities: P. Bosland, B. Visentin, E. Jacques
    - CEA Saclay (DAPNIA)
  - Feedback: P. Rouchon, M. Mirrahimi, A. Sarlette
    - Ecole des Mines Paris
  - QZD: P. Facchi, S. Pascazio
    - Uni. Bari and INFN
- €€:ERC (Declic), EC (SIQS, RYSQ),
  - CNRS, UMPC, ENS, CdF



A team work (1973-2014)

- Serge Haroche
- Michel Gross
- Claude Fabre
- Philippe Goy
- Pierre Pillet
- Jean-Michel Raimond
- Guy Vitrant
- Yves Kaluzny
- Jun Liang
- Michel Brune
- Valérie Lefèvre-Seguin
- Jean Hare
- Jacques Lepape
- Aephraim Steinberg
- Andre Nussenzveig
- Frédéric Bernardot
- Paul Nussenzveig
- Laurent Collot
- Matthias Weidemuller
- François Treussart
- Abdelamid Maali
- David Weiss
- Vahid Sandoghdar
- Jonathan Knight
- Nicolas Dubreuil
- Peter Domokos
- Ferdinand Schmidt-Kaler
- Jochen Dreyer
- Ed Hagley
- Xavier Maître
- Christoph Wunderlich
- Gilles Nogues
- Vladimir Ilchenko
- Jean-François Roch
- Stefano Osnaghi
- Arno Rauschenbeutel
- Wolf von Klitzing
- Erwan Jahier
- Patrice Bertet
- Alexia Auffèves
- Romain Long
- Sébastien Steiner
- Paolo Maioli
- Angie Qarry
- Philippe Hyafil
- Tristan Meunier
- Perola Milman
- Jack Mozley
- Stefan Kuhr
- Sébastien Gleyzes
- Christine Guerlin
- Thomas Nirrengarten
- Cédric Roux
- Julien Bernu
- Ulrich Busk-Hoff
- Andreas Emmert
- Adrian Lupascu
- Jonas Mlynek
- Igor Dotsenko
- Samuel Deléglise
- Clément Sayrin
- Xingxing Zhou
- Bruno Peaudecerf
- Raul Teixeira
- Sha Liu
- Theo Rybarczyk
- Carla Hermann
- Adrien Signolles
- Adrien Facon
- Eva Dietsche
- Stefan Gerlich
- Than Long Nguyen
- Mariane Penasa
- Dorian Grosso
- Tigrane Cantat
- Frédéric Assemat
- Valentin Métillon
- Rodrigo Cortinas