Cosmological Sources of Gravitational Waves

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Les Houches Summer School

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Overview

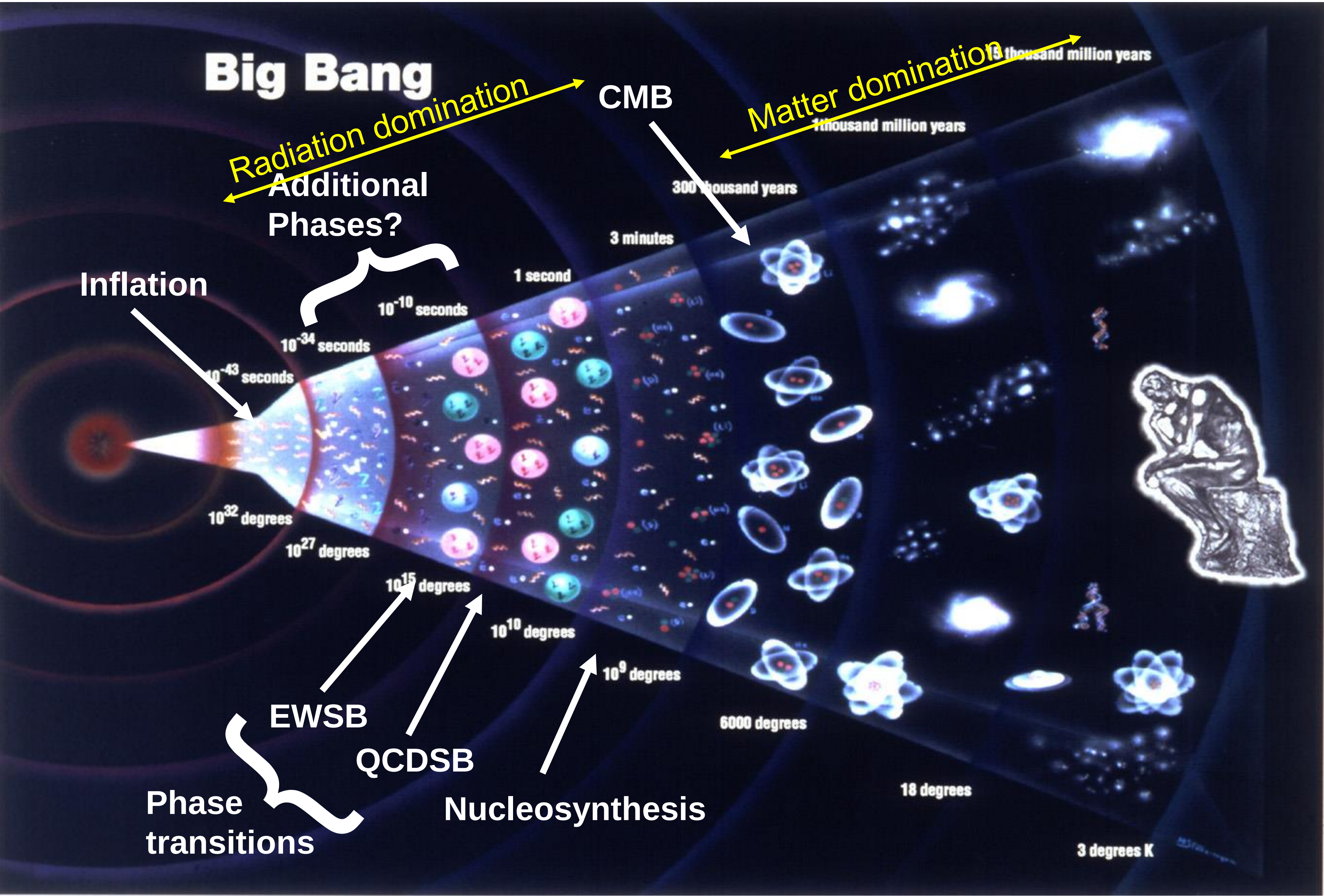
- Focus on gravitational waves produced in the early universe.
- Numerous processes are possible:
  - » Inflation
    - – Standard: amplification of vacuum modes
    - – Variations: particle production, preheating, reheating…
  - » Phase transitions
  - » Cosmic (super)strings: cusps and kinks
    - – Produced throughout the history of the universe.
  - » Additional phases in the evolution of the universe (pre-nucleosynthesis)
  - » Alternative cosmologies: pre-big-bang, …
- Common feature: no predictable waveform!
  - » Stochastic gravitational wave background (SGWB).

Overview

- Build on Joe Romano's introduction to SGWB.
- Study the standard inflationary model in some detail.
- Summarize a variety of other models and their detectability.

- Sources:
  - » B. Allen, Les Houches lectures: gr-qc/9604033
  - » M. Maggiore review: gr-qc/9909001
  - » Caprini & Figueroa review: arXiv:1801.04268
  - » B. Allen & J. Romano, Phys. Rev. D 59, 102001 (1999)
  - » M. Maggiore, *Gravitational Waves: Volume 1: Theory and Experiments*
  - » J.D.E. Creighton & W.G. Anderson, *Gravitational-Wave Physics and Astronomy*

Brief History of the Universe



Expansion of the Universe

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$$

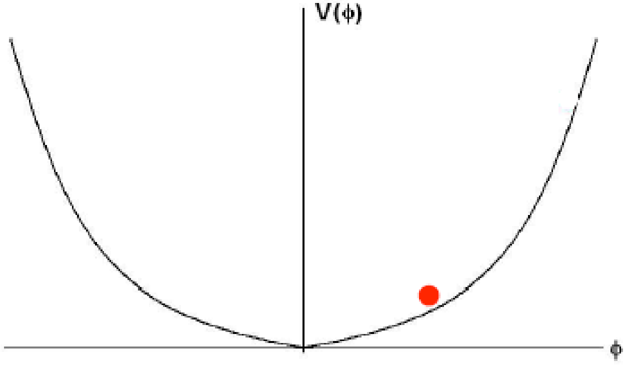
- Described by Friedmann Equation:
  - » Derived from Einstein's Equation, under the assumption of isotropic and homogeneous universe.
  - » *H* is known as the Hubble parameter.
  - » a is the scale factor of the universe (capturing its size/evolution)
  - » *K* = 0 for flat universe.
  - » The energy density depends differently on the scale factor:
    - – Matter domination: $\rho \sim a^{-3} \rightarrow a \sim t^{2/3}$
    - – Radiation domination: $\rho \sim a^{-4} \rightarrow a \sim t^{1/2}$
    - – Dark energy (and inflation): $\rho = -p \sim const$
- Note that in the case of inflation, $\dot{a} = Ca$, so $a(t) \sim e^{Ct}$ - exponential expansion of the universe.
  - » Solves several problems: flatness, horizon, monopole

Inflation

- To achieve inflation conditions, consider a potential V(ϕ) of some field ϕ.
- Can show that in this case:

$$\rho = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)$$

- In the limit $\dot{\phi} \ll V(\phi)$, we get $\rho = -p$.
- Further, if V(ϕ) is a slow function, then $\rho = const$ and we get exponential expansion.
- This is the *slow-roll* inflation model.



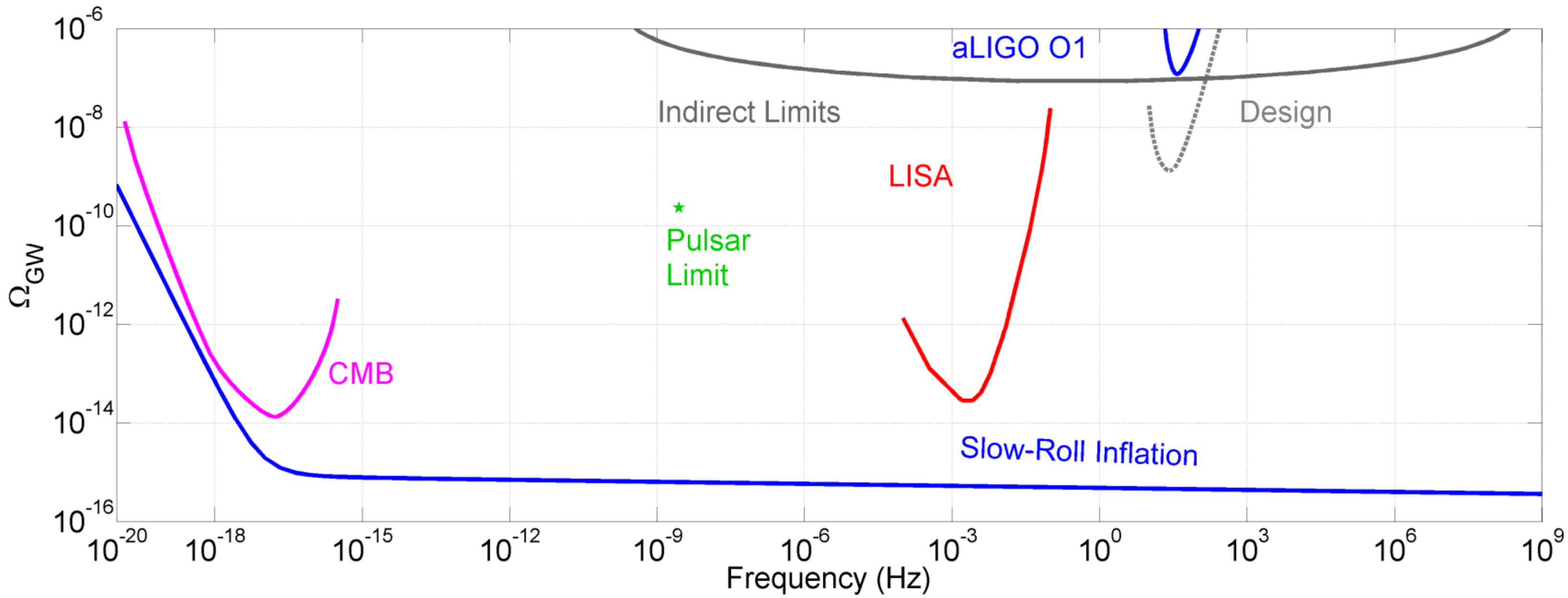
General Expectation

- Gravitational waves produced in the big bang decouple early from the primordial plasma.
  - » Gravity is a weak force…
- Specifically, at the time of GW decoupling, the rate of GW interactions Γ is similar to the rate of the expansion of the universe (Hubble rate, H).
- $\Gamma = n\, \sigma\, v \sim G^2 T^5$
  - » The number density scales as $n \sim T^3$ in radiation domination.
  - » Cross section, $\sigma \sim G^2 T^2$ (for neutrino interactions, $\sigma \sim G_F^2 E^2 \sim G_F^2 T^2$; here, we replace Fermi's constant with the Newton's constant)
  - » $v \sim c$
- $H \sim T^2 / M_{pl}$ (Friedmann Equation)
- Bring together: $\Gamma / H \sim G^2 T^3 / M_{pl} \sim (T / M_{pl})^3$
- So, GWs decoupled at the Planck scale, 10^{19} GeV ~ 10^{32} K.
- They therefore carry unique, unperturbed signatures of the physics of very high energy scales, not accessible in laboratories!

Notes

- Please see typed notes.

Inflationary Background



Slow-Roll Inflationary Model:

- Very flat (but still sloped) potential.
- Hubble parameter is not constant during inflation.
- Leads to a small slope in the GW energy density, $n_t = - r / 8$.
- Still very weak background, very difficult to detect with current or projected technologies.

Beyond the Simplest Model

Anber & Sorbo, PRD 81, 043534 (2010)
Barnaby & Peloso, PRL. 106, 181301 (2011)
Barnaby et al, JCAP 1104, 009 (2011)

- We still know little about inflationary physics.
  - » Numerous models exist in the literature.
- One way to ensure flatness of the inflaton potential is to impose shift symmetry, $\phi \to \phi$ + constant.
  - » Can be achieved with an axion field.
- Assume Lagrangian of the form: $\Delta\mathcal{L} \sim \frac{C}{f_\phi}\phi F\tilde{F}$

Axion decay constant

Gauge field strength

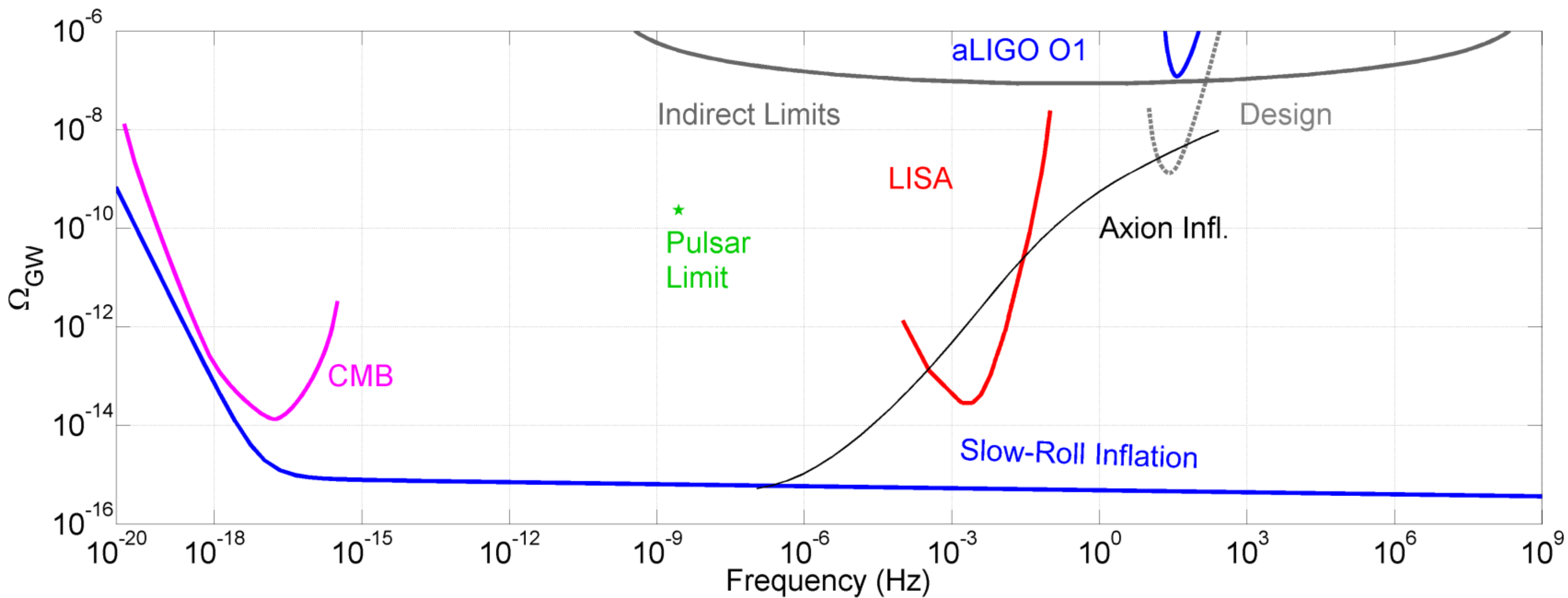
- Modifies dispersion relation:

$$\omega_\pm^2 = k^2 \mp 2kaH\xi \quad , \quad \xi \equiv \frac{C\,\dot{\phi}}{2f_\phi H}$$

- The scale factor grows exponentially, so one solution becomes tachyonic.
  - » One helicity survives, parity violation!

Axion Inflation

- Furthermore, the Lagrangian supports $A_+A_+ \rightarrow \delta\phi$
  - » Extends inflation!
- And it supports $A_+A_+ \rightarrow h$
  - » Produces additional gravitational waves, preferentially at smaller scales (higher frequencies) – and polarized!

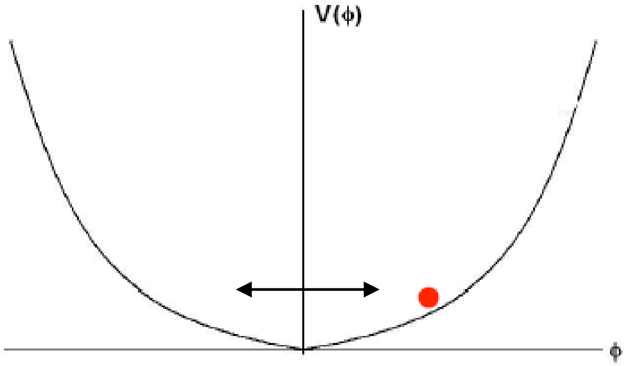


Reheating

- Due to rapid expansion, whatever particles were initially produced at the big bang are now diluted.
  - » Except for the inflaton itself.
- There must be a mechanism for efficiently converting the inflaton energy into particles we know.
  - » Reheating!
  - » Expect almost all particles to be formed then.
- Need a particle physics model, coupling inflaton to other particles.
  - » Numerous models exist.
  - » Could be non-perturbative, including resonant behavior: Preheating

Preheating

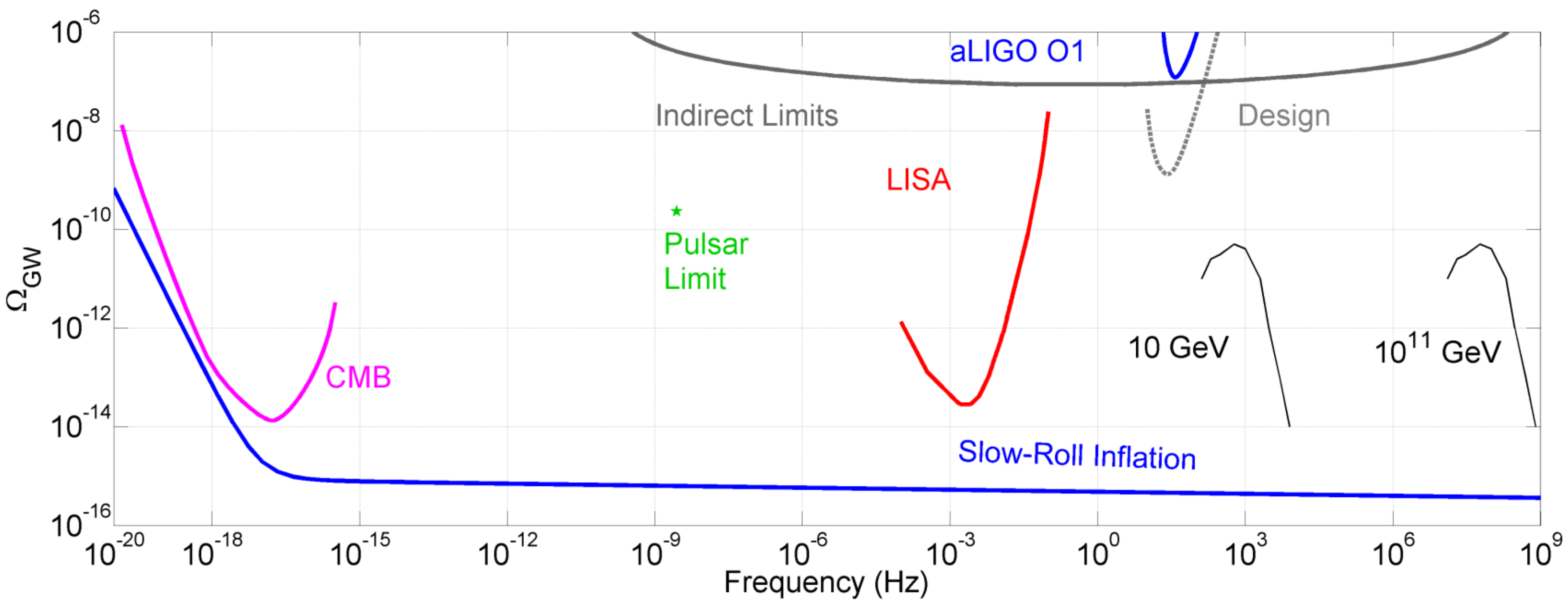
- For example, inflaton field could be coupled to some bosonic or fermionic fields.
- As the inflaton field oscillates at the bottom of the inflaton potential (at the end of inflation), it produces bosonic fields in bursts.
- Resonance can be set up, with the bosonic field growing exponentially.
- Field gradients → anisotropic stress → GW's!
- Frequency of GWs is set by the energy scale of the preheating process.
  - » Typically a high energy process, high-frequency GWs.
  - » There are exceptions: hybrid inflation, could be as low as MeV.



Easther, Giblin, Lim '06-'08
Figueroa, Bellido, et al '07-'10
Kofman, Dufaux et al '07-'09

Preheating

- Numerical simulations used to predict GW spectrum.
- Strongly dependent on the energy scale and mechanism of preheating.
- Spectrum typically peaks at high frequencies, where we have no detectors.



Additional Phases?

- The consistency relation need not be correct:

$$n_t = -r/8$$ (*r* is the tensor to scalar ratio)

- Should measure it!

- Relationship between Ω_{GW} and *r* depends strongly on the unknown physics in the early universe.
    - Ω_{GW} could increase with frequency.
    - Potentially detectable by advanced detectors.

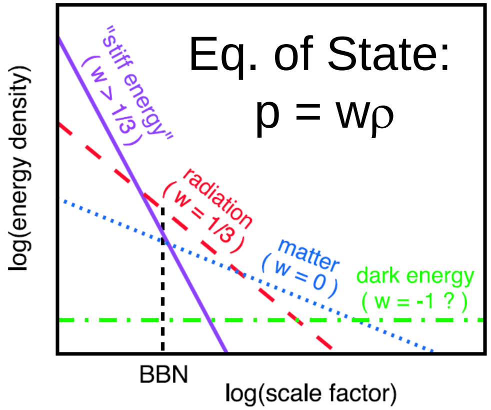
- Essentially no data exists about the universe before big bang nucleosynthesis (~ 1 minute after the big bang).
- It is possible that additional phases existed in the history of the universe, in addition to inflation, radiation, and matter dominated phases.
    - Additional amplification at transitions!

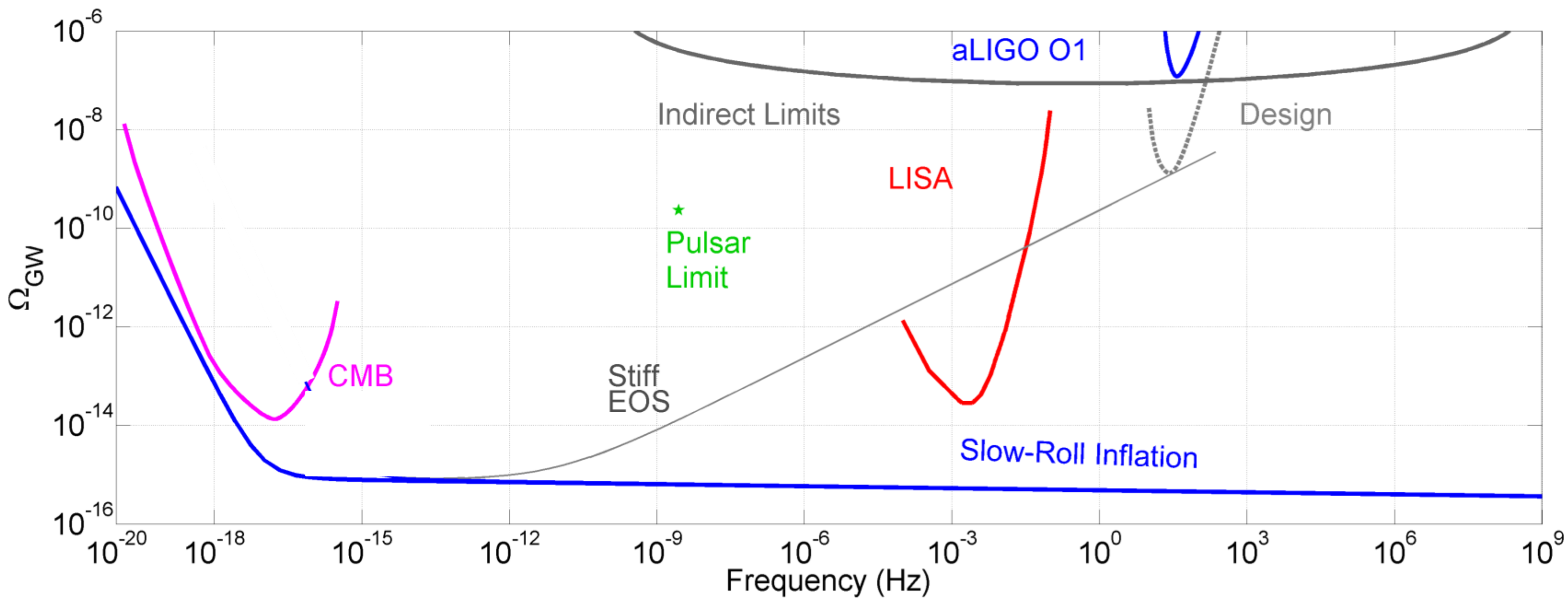
Equation of State in the Early Universe

- Boyle & Buonanno PRD 78, 043531 (2008).
  - » Derived a general relationship:

$$\Omega_{GW}(f) \propto r f^{\alpha(\hat{w}, f) + \hat{n}_t}$$

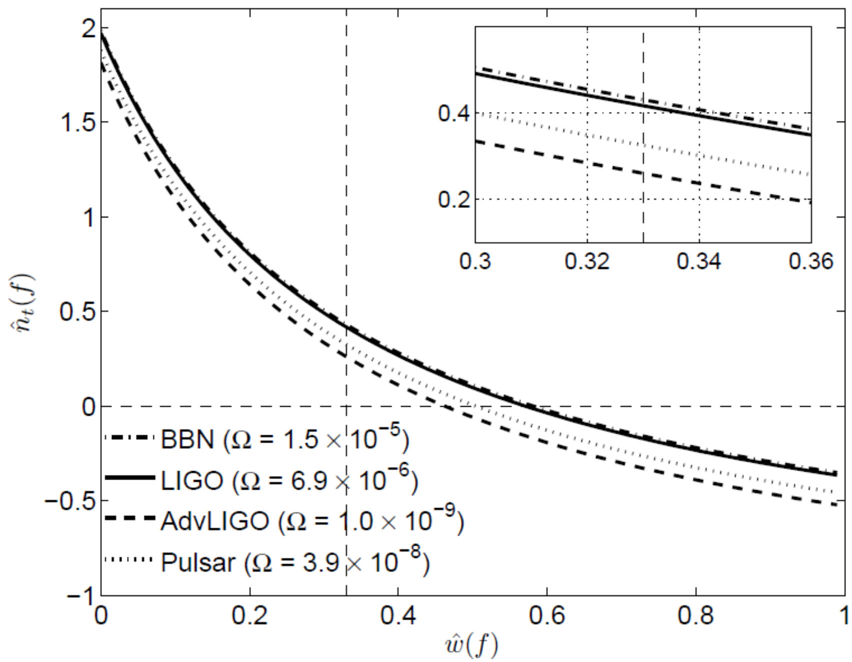
- The equation of state between inflation and radiation era could be stiff ($w > 1/3$).
  - Results in increasing spectrum with frequency.





Equation of State in the Early Universe

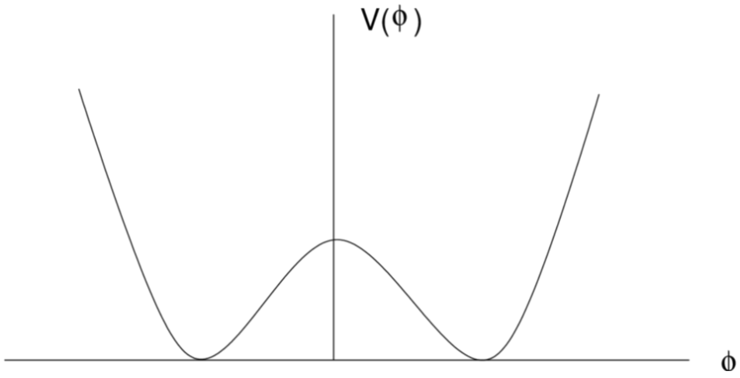
- For a given *r*, SGWB can be used to constrain the equation of state and the tensor tilt.
- Can be probed at different frequencies.
  - » Complementarity of detector technologies.



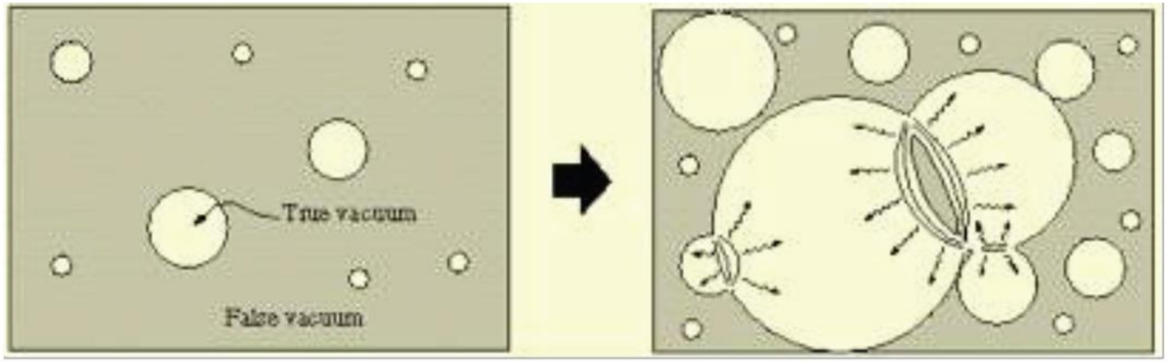
Nature 460, 990 (2009)

Phase Transitions

- At high temperature, details at the bottom of the potential might not matter.
  - » Symmetric (false) vacuum.
- As the universe cools, true vacuum seeds can be initiated at various points in space.
- Bubbles of the true vacuum grow, due to pressure difference acting on their walls.
- Eventually, they interfere with each other.
- 1st order phase transition.



Bubble walls collisions can be violent, perturbing the plasma inside the bubbles.



Phase Transitions

- At least 2 phase transitions happened in the early universe:
  - » Electroweak PT at ~1 TeV energy scale:
    - – Symmetry breaking (due to Higgs potential) separates the EM and weak nuclear force.
    - – Expected to be 2^{nd} order in the Standard Model of Particles, unless there is new physics…
  - » QCD PT also expected to be 2^{nd} order based on lattice QCD calculations, unless there is new physics (such as lepton asymmetry).
- Additional PT at higher energies possible:
  - » Signatures of new physics.
    - – Extra dimensions models.
    - – Dark matter models (only gravitationally interacting with ordinary matter).

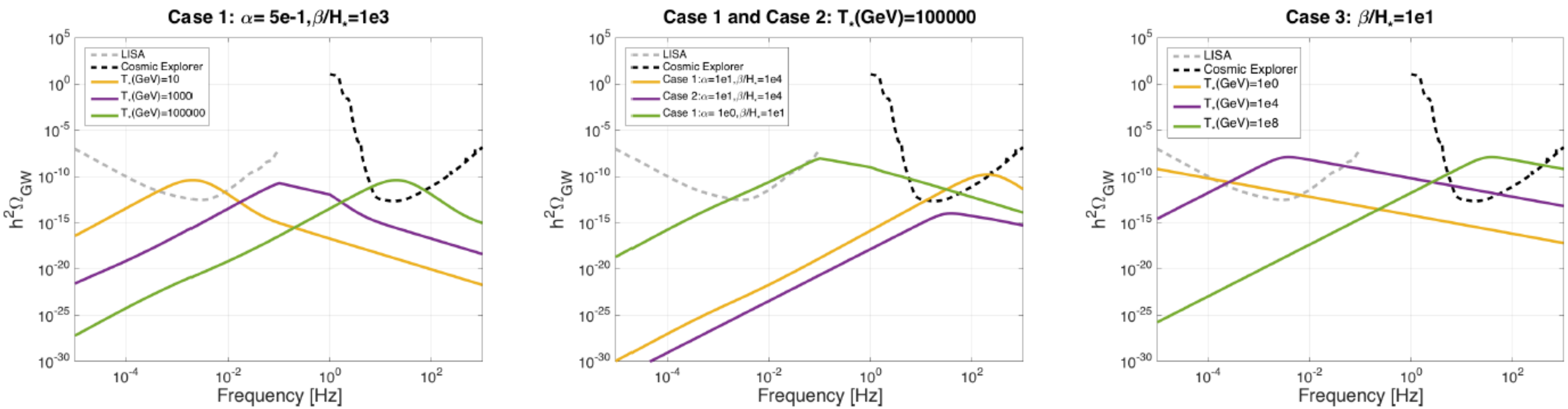
PT: GW Production

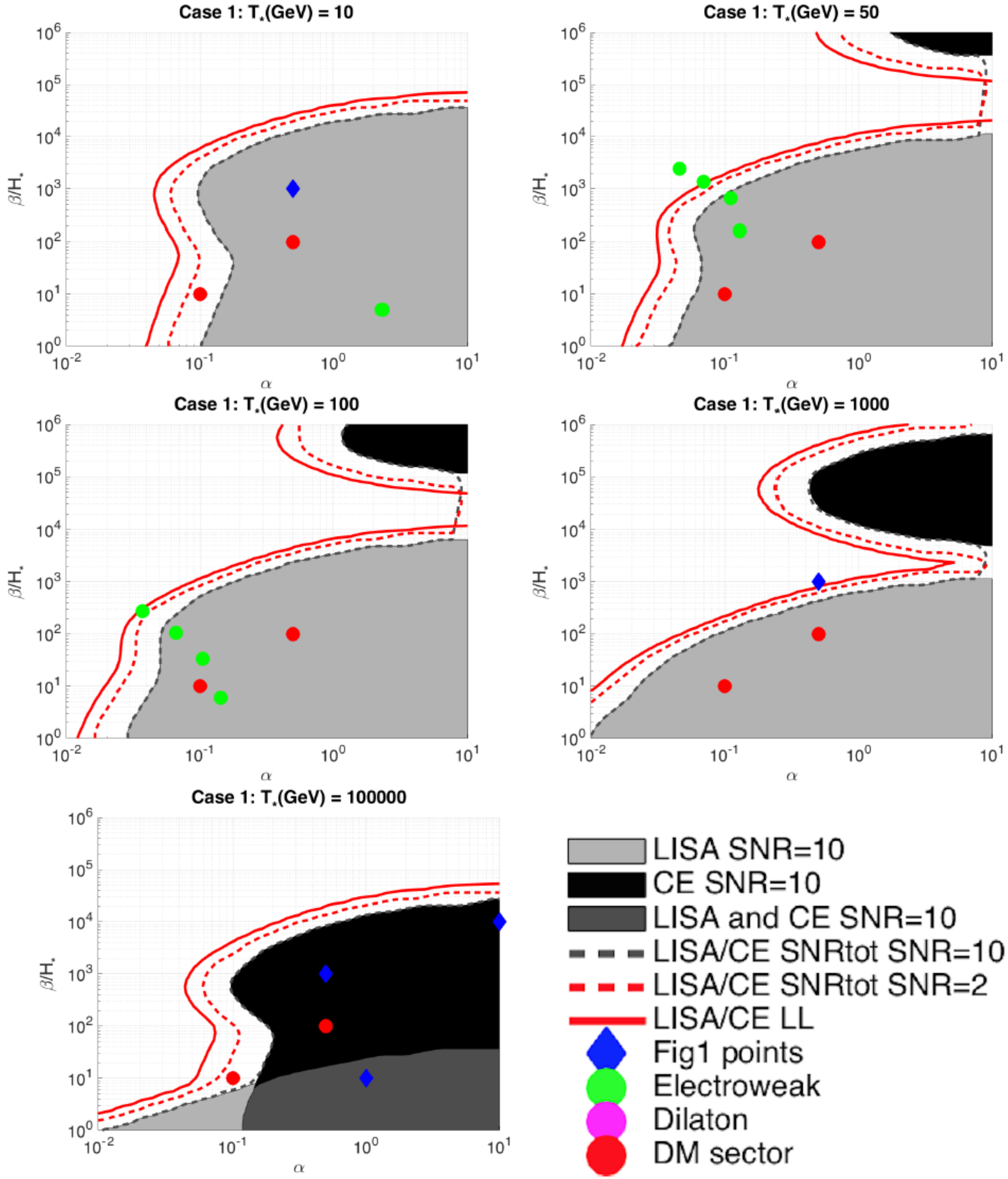
- At least 3 mechanisms for GW production exist:
  - » Collisions of bubble walls
  - » Sound waves set up in the plasma, as bubble walls sweep through it.
  - » Magneto-hydrodynamic turbulence in the plasma.
- GW spectral shape and amplitude, and the relative contributions of the 3 mechanisms depend on several parameters:
  - » β^{-1} – duration of the phase transition
  - » H_* - Hubble parameter at the time of PT
  - » T_* - Temperature at the time of PT
  - » v – bubble wall speed
  - » Fractions of the energy density in the form of kinetic/vacuum/radiation energy.
- Excellent review: Caprini et al., JCAP 4, 001 (2016).

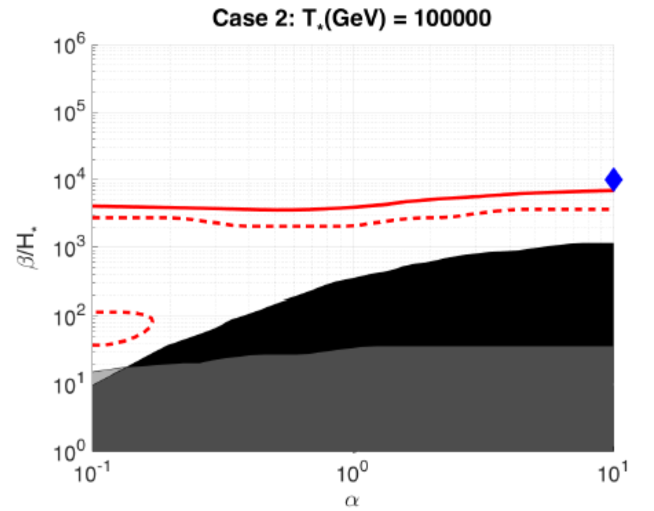
PT: GW Spectra

- EWPT: TeV energy scale roughly corresponds to the LISA frequency band.
  - » Tail might be detectable by 3rd generation GW detectors.
- Higher energy 1st order PT could be detectable by 3rd generation terrestrial GW detectors.

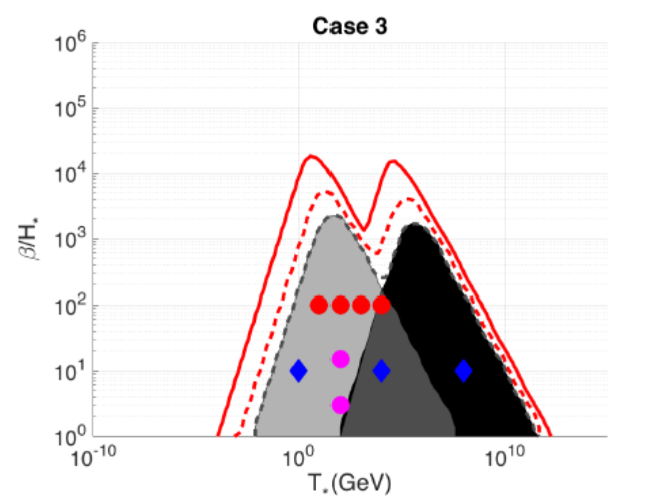
Fitz Axen et al., arXiv:1806.02500





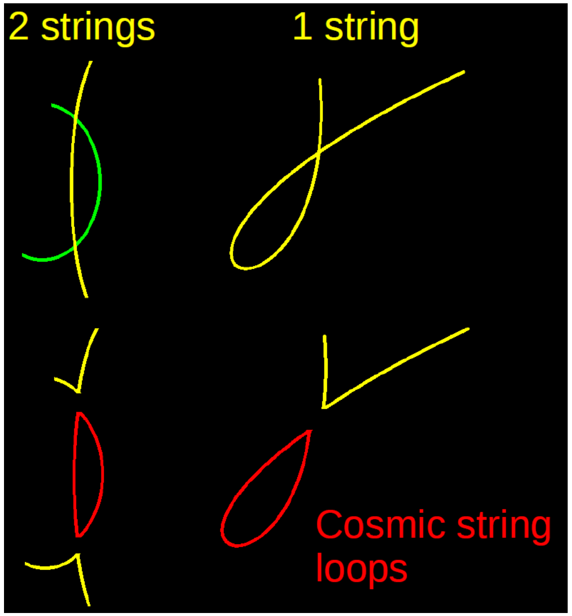


Accessible regions identified for 3 model cases: Fitz Axen et al., arXiv:1806.02500



Cosmic (Super)strings

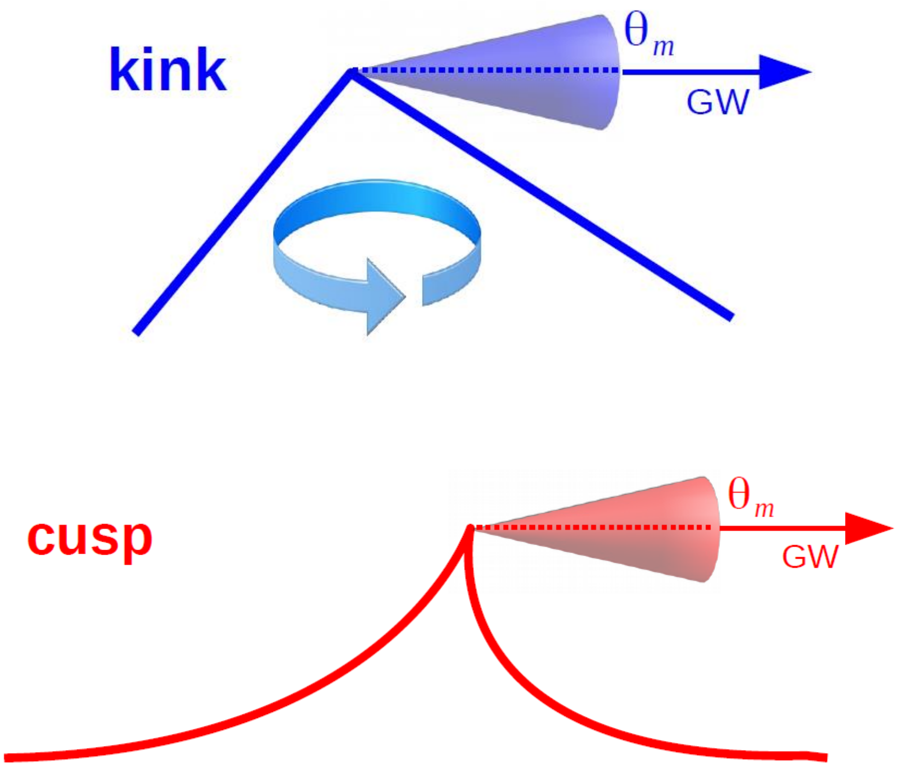
- Cosmic strings can form as topological defects during phase transitions in the early universe.
- Or, they could exist as fundamental strings (string theory), enlarged to cosmological scales.
- Strings can reconnect when they intersect (with some probability *p*).
  - » Create loops, which decay away via GW emission.
- Expect a network of strings to exists, likely with a wide distribution of string length scales.
- Network expands with the universe.
- Details are complex – need numerical simulations to understand the number and length distribution of strings in a network.



Vilenkin & Shellard, *Cosmic Strings and Other Topological Defects*

Cusps and Kinks on Cosmic Strings

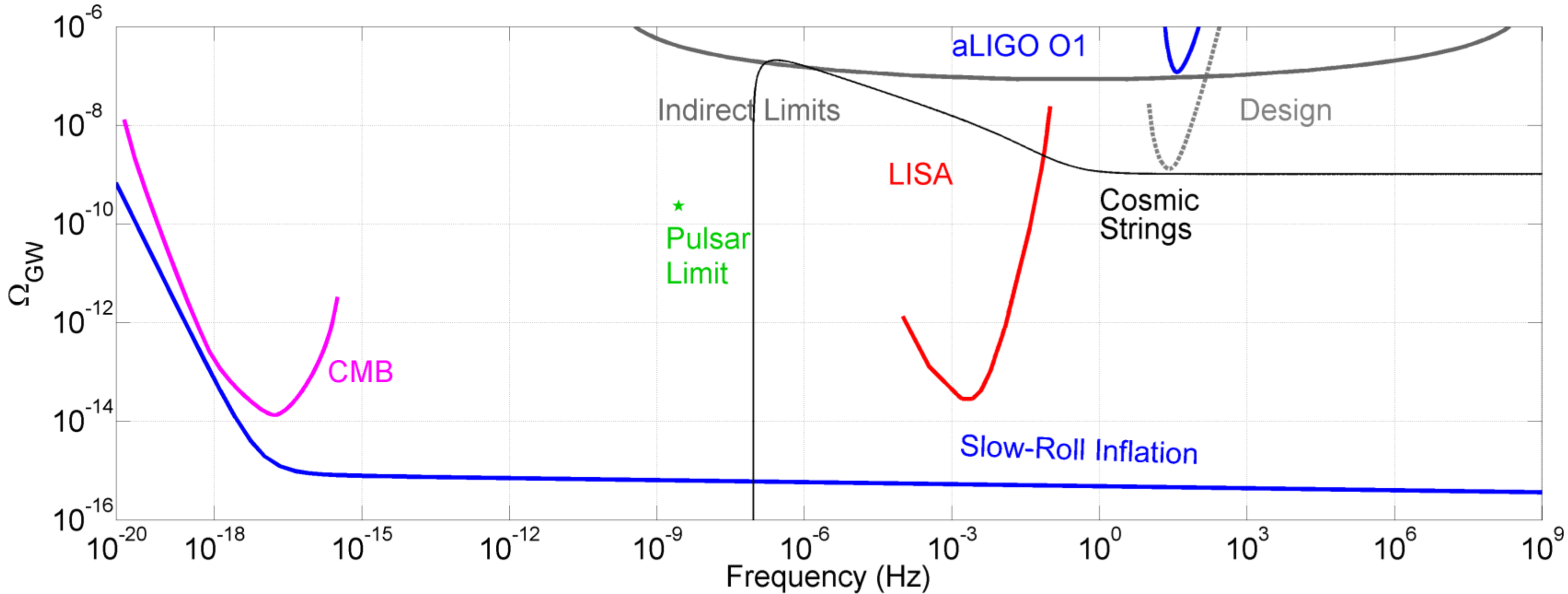
- Strings are characterized by mass density $G\mu$.
- Cusps or kinks on the strings would generate GWs.
- If boosted in the direction of the Earth, cusps/kinks appear as bursts of GWs.
- Or, sum up over all cusps and kinks in the string network and generate a stochastic GW background.



Berezinsky et al., PRL 64, 043004 (2001)
Damour & Vilenkin, PRL 85, 3761 (2000)
Damour & Vilenkin, PRD 71, 063510 (2005)

Cosmic String GW Spectrum

- Stochastic background due to cusps/kinks is continuously produced, from the early universe to today.
- Spectrum strongly depends on parameters $G\mu$ and *p*.



Cosmic Strings vs Searches

Phys. Rev. D 97, 102002 (2018)

3 models of cosmic string network:

M1: PRD 73, 105001 (2006)
 - old, reference

M2: PRD 89, 023512 (2014)
 - loop size at formation determined by simulations.

M3: JCAP 1010, 003 (2010)
 - loop distribution (not only at formation) determined by simulations.

Significant regions already excluded.