Modes & States in Quantum Optics

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Physique quantique et applications

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Introduction:
modes in Quantum Optics

What is a mode ?

mode :

a normalized solution $\boldsymbol{f}_1(\boldsymbol{r},t)$ of Maxwell equations

mode basis :

a complete orthonormal set $(\boldsymbol{f}_n)$ of modes

Description of a classical multimode field

general expression for the complex electric field:

$$\mathbf{E}^{+}(\mathbf{r},t) = \sum_n E_n \, \mathbf{f}_n(\mathbf{r},t)$$

complex amplitude

$$\mathbf{E}^{+}(\mathbf{r},t) = \sum_n (E_{n,X} + iE_{n,P}) \, \mathbf{f}_n(\mathbf{r},t)$$

"quadrature components"

Description of a **quantum** multimode field

$$\mathbf{E}^+(\mathbf{r},t) = \sum_n E_n\,\mathbf{f}_n(\mathbf{r},t)$$

$$\mathbf{E}^+(\mathbf{r},t) = \sum_n (E_{n,X} + iE_{n,P})\,\mathbf{f}_n(\mathbf{r},t)$$

general expression for the complex electric field operator:

$$\hat{\mathbf{E}}^+(\mathbf{r},t) = \sum_n E_0\,\hat{a}_n\,\mathbf{f}_n(\mathbf{r},t)$$

$$[\hat{a}_n, \hat{a}^\dagger_{n'}] = \delta_{n,n'}$$

$$\hat{\mathbf{E}}^+(\mathbf{r},t) = \sum_n (\hat{E}_{n,X} + i\hat{E}_{n,P})\,\mathbf{f}_n(\mathbf{r},t)$$

$$[\hat{E}_{nX}, \hat{E}_{n'P}] = 2iE_0^2\delta_{n,n'}$$

Description of a **quantum** multimode field

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QUANTUM

$$[\hat{E}_{nX}, \hat{E}_{n'P}] = 2iE_0^2\delta_{n,n'}$$

Description of a **quantum** multimode field

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$$\hat{\mathbf{E}}^{+}(\mathbf{r},t) = \sum_n (\hat{E}_{n,X} + i\hat{E}_{n,P}) \, \mathbf{f}_n(\mathbf{r},t)$$

QUANTUM OPTICS

$$[\hat{E}_{nX}, \hat{E}_{n'P}] = 2iE_0^2 \delta_{n,n'}$$

the double linearity of quantum optics

$$\hat{\mathbf{E}}^{+}(\mathbf{r},t) = \sum_n (\hat{E}_{n,X} + i\hat{E}_{n,P})\, \mathbf{f}_n(\mathbf{r},t)$$

-linearity of Quantum Mechanics
-linearity of Maxwell equations

Two Hilbert spaces to consider

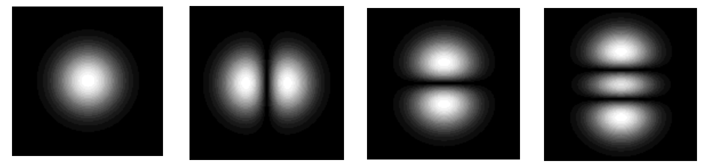
- the quantum Hilbert space H^q of quantum states of light
- the modal Hilbert space H^{mod} of solutions of Maxwell equations

Different mode bases

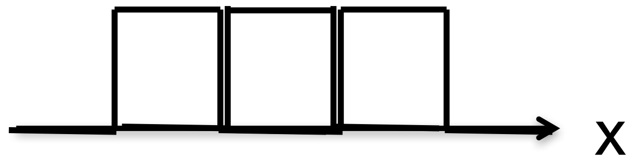
- the travelling plane wave $e^{i(\mathbf{k}.\mathbf{r}-\omega t)}$

spatial modes

- Hermite Gauss modes

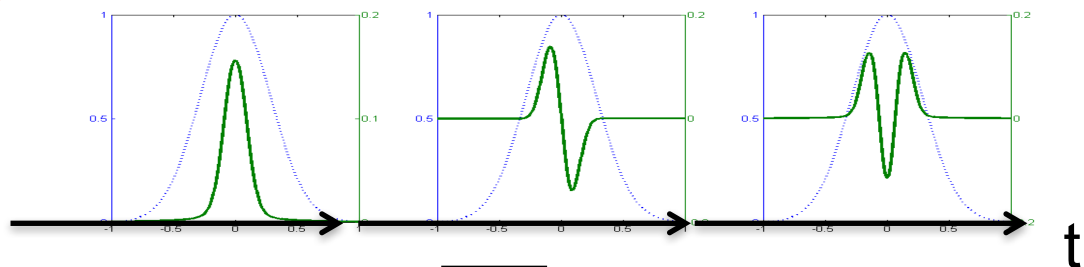


- the pixel modes

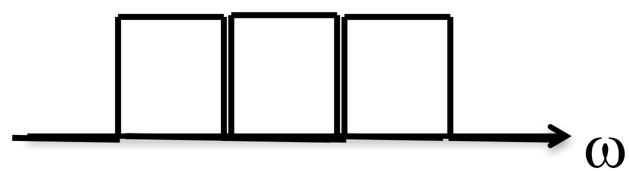


temporal/frequency modes

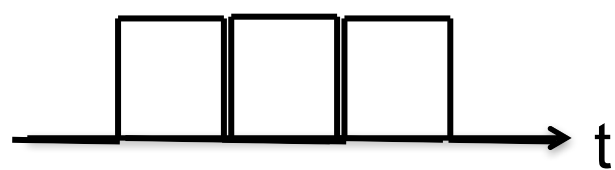
-the Hermite Gauss pulses



-the "frequency band" modes



-the time bin modes



Quantum state in mode basis change

$$\{\mathbf{f}_n\} \xleftrightarrow{\mathbf{U}_{mod}} \{\mathbf{g}_\ell\}$$

$$\{\hat{a}_n\} \longleftrightarrow \{\hat{b}_\ell\}$$

$$|\Psi\rangle = \sum_{p_1} \sum_{p_2} \dots A_{p_1,p_2,\dots} |p_1 : \mathbf{f}_1, p_2 : \mathbf{f}_2, \dots\rangle$$

$$= \sum_{q_1} \sum_{q_2} \dots B_{q_1,q_2,\dots} |q_1 : \mathbf{g}_1, q_2 : \mathbf{g}_2, \dots\rangle$$

the same quantum state $|\Psi\rangle$ has different expressions in different mode bases

two-mode example:

$$|\Psi\rangle = |1 : \mathbf{f}_1 \, , \, 1 : \mathbf{f}_2\rangle$$ **factorized**

$$|\Psi\rangle = |2 : \mathbf{g}_+ \, , \, 0 : \mathbf{g}_-\rangle - |0 : \mathbf{g}_+ \, , \, 2 : \mathbf{g}_-\rangle$$ **entangled**

with $\mathbf{g}_\pm = (\mathbf{f}_1 \pm \mathbf{f}_2)/\sqrt{2}$

Quantum state in mode basis change

$$\{\mathbf{f}_n\} \longleftrightarrow \{\mathbf{g}_\ell\}$$

$$\mathbf{U}_{mod}$$

$$\{\hat{a}_n\} \longleftrightarrow \{\hat{b}_\ell\}$$

$$|\Psi\rangle = \sum_{p_1}\sum_{p_2} ... A_{p_1,p_2,...}|p_1 : \mathbf{f}_1, p_2 : \mathbf{f}_2, ...\rangle$$

$$= \sum_{q_1}\sum_{q_2} ... B_{q_1,q_2,...}|q_1 : \mathbf{g}_1, q_2 : \mathbf{g}_2, ...\rangle$$

the same quantum state $|\Psi\rangle$ has different expressions in different mode bases

two-mode example:

$$|\Psi\rangle = |squeezed\,vac : \mathbf{f}_1\rangle \otimes |squeezed\,vac : \mathbf{f}_2\rangle$$ **factorized**

$$|\Psi\rangle = |EPR\,entangled\,state\rangle$$ **entangled**

on basis $\mathbf{g}_\pm = (\mathbf{f}_1 \pm \mathbf{f}_2)/\sqrt{2}$

mode basis-independent, or "intrinsic", quantities ?

- total photon number:

$$\hat{N}_{tot} = \sum_{\ell} \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell} = \sum_{n} \hat{a}_{n}^{\dagger} \hat{a}_{n}$$

- Wigner function values:

$$W_b(\beta_1, ...) = W_a(\alpha_1, ...)$$

with $$(\beta_1, ...)^T = U(\alpha_1, ...)^T$$

W(0) does not depend on mode basis

"negativity" of W is intrinsic

- P function values:

"non-classicality " is intrinsic

the number of excited modes is depends on the mode basis

is there a minimum number of modes in which a state lives?

to count modes: use the coherence matrix

$$C^{i,j}_{coherence} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle$$

two-mode case:

$$C_{coherence} = \begin{bmatrix} \langle \hat{a}_1^\dagger \hat{a}_1 \rangle & \langle \hat{a}_1^\dagger \hat{a}_2 \rangle \\ \langle \hat{a}_2^\dagger \hat{a}_1 \rangle & \langle \hat{a}_2^\dagger \hat{a}_2 \rangle \end{bmatrix}$$

the matrix can be diagonalized
by a **mode basis change**

for example, case of two non-zero eigenvalues

$$C_{complex} = \begin{bmatrix} \boxed{\begin{matrix} N_1 & 0 \\ 0 & N_2 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix}$$

$$|\Psi\rangle = \sum_{p_1,p_2} A_{p_1,p_2} |p_1 : \mathbf{g}_1, p_2 : \mathbf{g}_2, 0 : \mathbf{g}_3\rangle$$ intrinsic two-mode state

the number of non-zero eigenvalues (rank) of covariance matrix is the minimum number of modes needed to describe the quantum state

the corresponding Hilbert space is the smallest space in which the state is living

useful for example to make full tomography of the state

intrinsic **single mode** state

its coherence matrix has only one nonzero eigenvalue

there is a mode basis $\{\mathbf{g}_\ell\}$
in which it is single mode

$$|\Psi\rangle = \left(\sum A_{q_1} |q_1 : \mathbf{g}_1\rangle\right) \otimes |0 : \mathbf{g}_2\rangle \otimes |0 : \mathbf{g}_3\rangle \otimes \ldots$$

example: the **single photon** state

defined as eigenstate of $\hat{N}_{tot}$ with eigenvalue 1

$$|\Psi_1\rangle = \sum_n A_n |1 : \mathbf{f}_n\rangle$$

can be written: $|\Psi_1\rangle = |1 : \mathbf{g}_1\rangle \otimes |0, 0, ...\rangle$

with $$\mathbf{g}_1 = \sum_n A_n \mathbf{f}_n$$

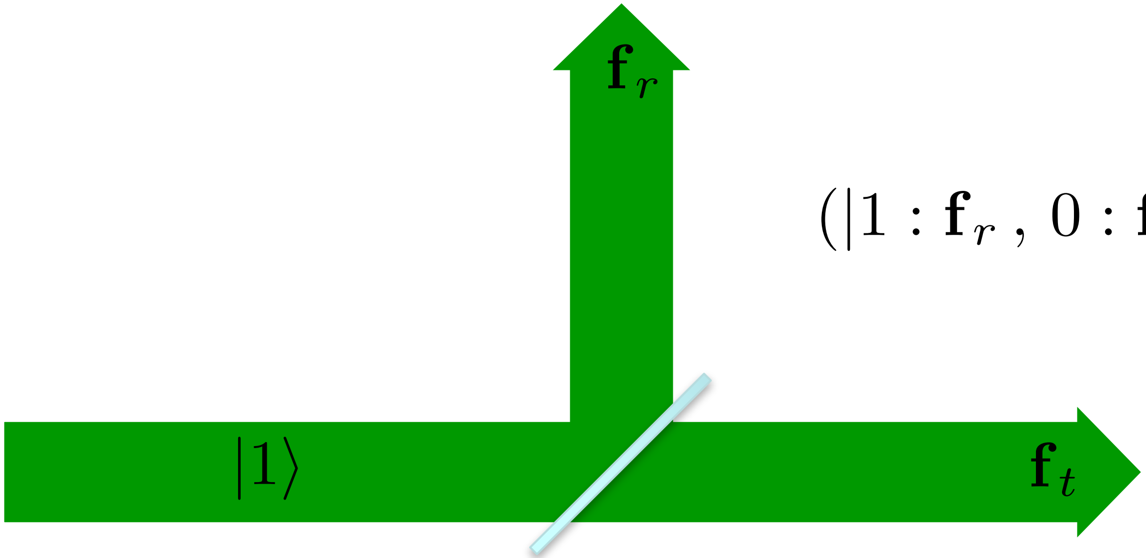
a single photon state is always a single mode state

a single photon state cannot be defined independently of the mode in which it is defined

its properties depend on this mode

example:

single photon through beamsplitter



$$(|1:\mathbf{f}_r\,,\,0:\mathbf{f}_t\rangle + |0:\mathbf{f}_r\,,\,1:\mathbf{f}_t\rangle)/\sqrt{2}$$

single photon in single mode $\quad \mathbf{g}_1 = (\mathbf{f}_r + \mathbf{f}_t)/\sqrt{2}$

single mode states and optical coherence

in any single mode state :

$$|g^{(1)}| = 1$$

with

$$g^{(1)} = \frac{< E^{(-)}(\mathbf{r},t)E^{(+)}(\mathbf{r}',t') >}{(< E^{(-)}(\mathbf{r},t)E^{(+)}(\mathbf{r},t) >< E^{(-)}(\mathbf{r}',t')E^{(+)}(\mathbf{r}',t') >)^{1/2}}$$

interferences are of perfect visibility
whatever the quantum state

interference visibility: a mode property, not a state property

(R. Glauber)

Complete characterization of multimode Gaussian states

the *coherence* matrix can be used to count modes on any quantum state, but gives partial information

in contrast Gaussian states are completely characterized by the *covariance* matrix

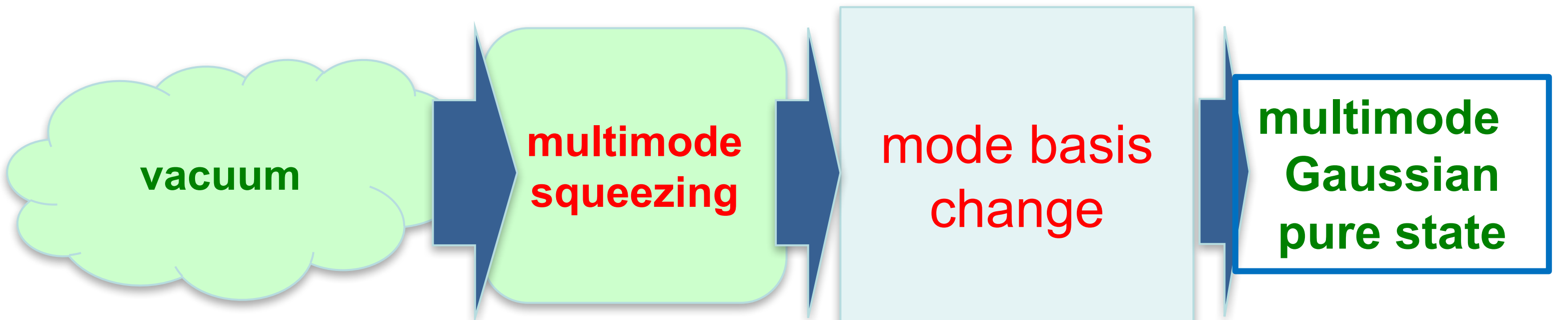
$$\begin{bmatrix} \Delta^2 E_{X1} & < E_{X1} E_{X2} > & < E_{X1} E_{P1} > & < E_{X1} E_{P2} > \\ < E_{X2} E_{X1} > & \Delta^2 E_{X2} & < E_{X2} E_{P1} > & < E_{X2} E_{P2} > \\ < E_{P1} E_{X1} > & < E_{P1} E_{X2} > & \Delta^2 E_{P1} & < E_{P1} E_{P2} > \\ < E_{P2} E_{X1} > & < E_{P2} E_{X2} > & < E_{P2} E_{P1} > & \Delta^2 E_{P2} \end{bmatrix}$$

of dimension $(2N_{modes}) \times (2N_{modes})$

it can be diagonalized, but the corresponding linear transformation is not a mode basis change

characterization
of a multimode Gaussian quantum state:

for a pure state:



uncorrelated squeezed mode basis

turns squeezed mixed states into entangled states

$\{\mathbf{g}_\ell\}$

$\mathbf{U}^2_{mod}$

Bloch Messiah (or Singular Value) decomposition

characterization of a multimode Gaussian quantum state:

for a mixed state:

uncorrelated thermal states → mode basis change → **multimode squeezing** → mode basis change → **multimode Gaussian mixed state**

uncorrelated thermal mode basis

$\{\mathbf{f}_n\}$

produces correlated classical noise

$\mathbf{U}^1_{mod}$

uncorrelated squeezed mode basis

$\{\mathbf{g}_\ell\}$

turns squeezed mixed states into entangled states

$\mathbf{U}^2_{mod}$

two mode basis

Bloch Messiah Williamson reduction

characterization
of a multimode Gaussian quantum state:

- all pure multimode Gaussian states are **factorizable**
  there is a mode basis in which:

$$W(x_1, p_1, ...x_n, p_n) = W_1(x_1, p_1) \times ... \times W_n(x_n, p_n)$$

- all mixed multimode Gaussian states are **separable**
  there is a mode basis in which

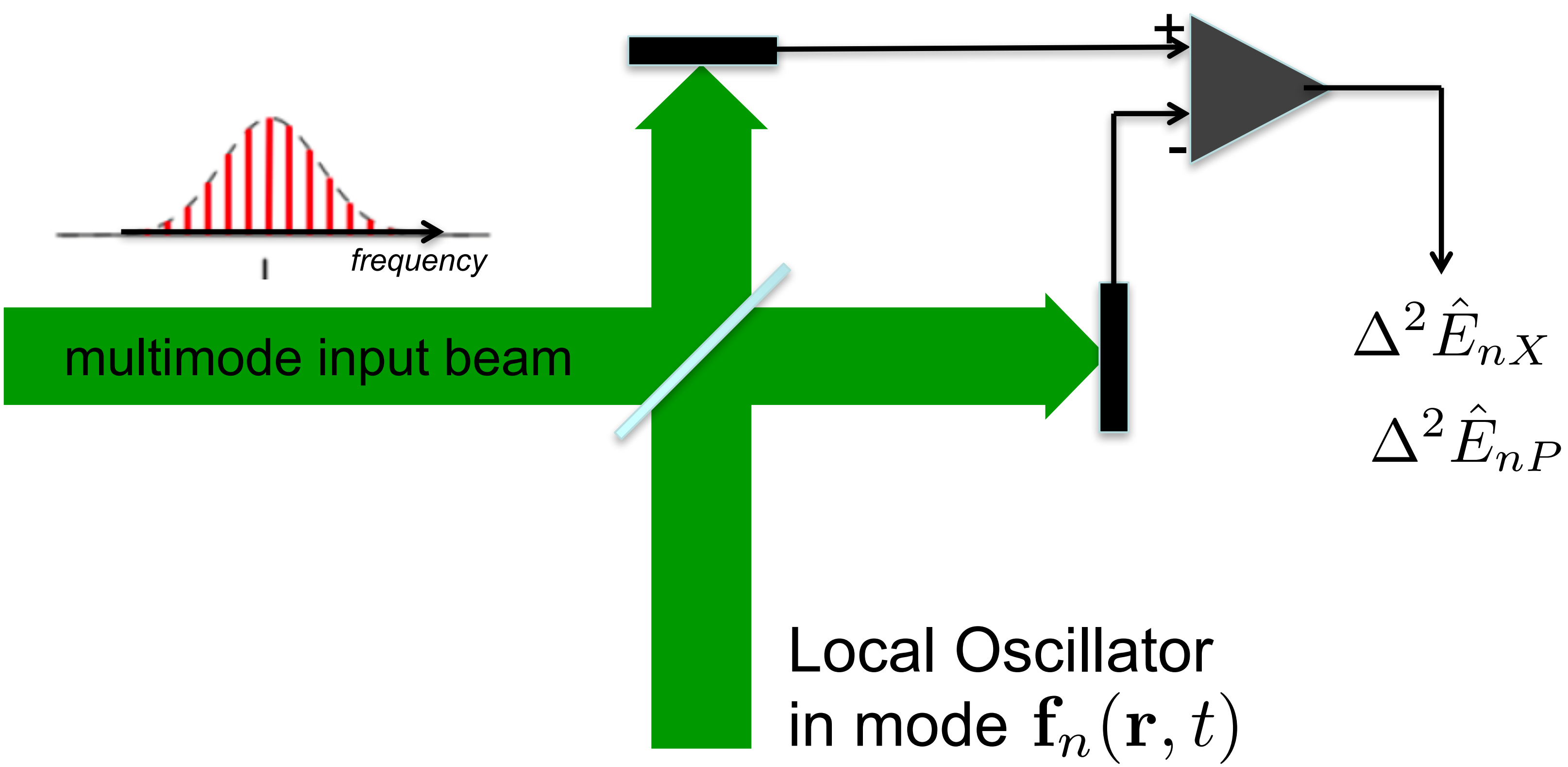
$$W(x_1, p_1, ...x_n, p_n) = \int d\lambda p(\lambda) W_{1,\lambda}(x_1, p_1) \times ... \times W_{n,\lambda}(x_n, p_n)$$

there are no intrinsically entangled Gaussian states

How to measure the
quadrature covariance matrix?

How to measure the covariance matrix?

balanced homodyne detection gives information about the **projection** of the multimode state on the local oscillator mode

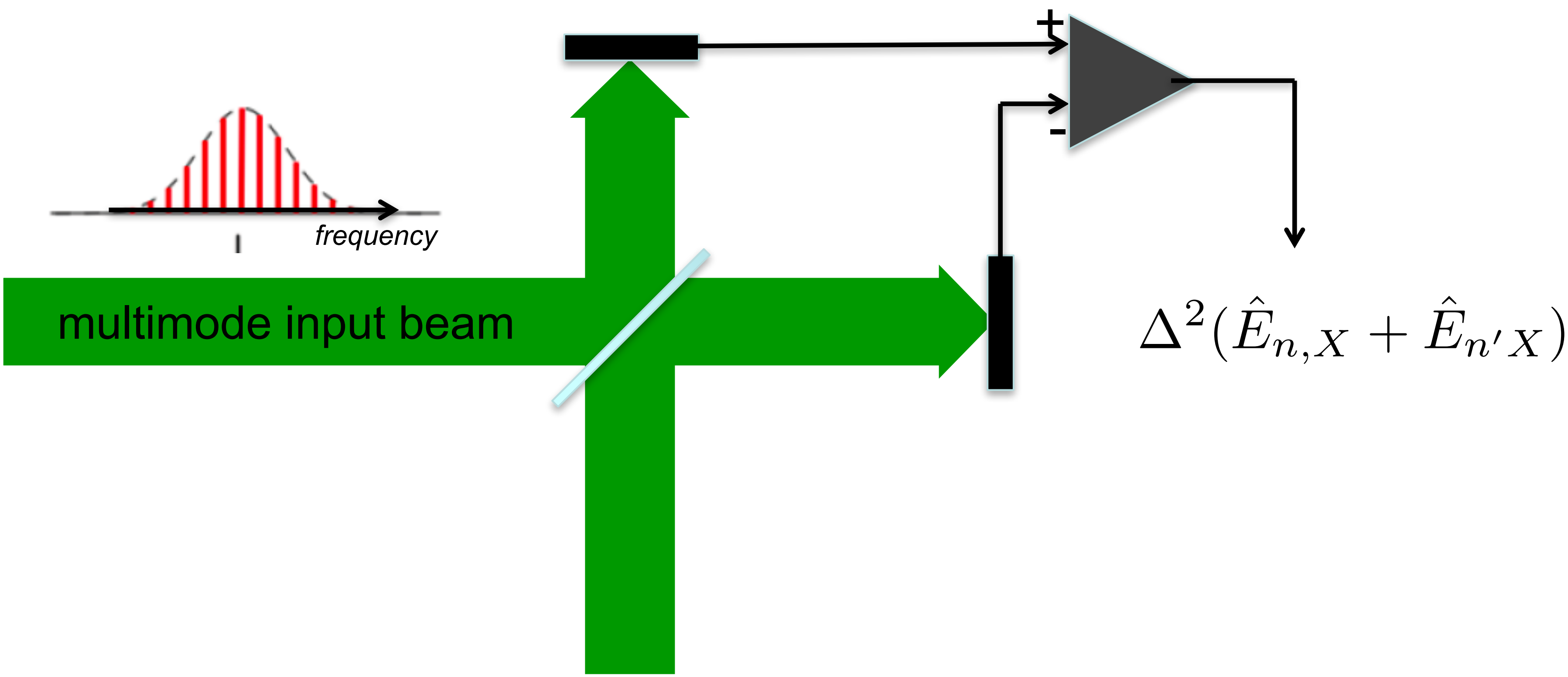


make a series of homodyne measurements using a set of orthogonal modes $\mathbf{f}_n(\mathbf{r},t)$

How to measure the off-diagonal part of the covariance matrix?

make homodyne measurements
using the sum of two modes

$$\langle \hat{E}_{nX} \hat{E}_{n'X} \rangle = (\Delta^2(\hat{E}_{nX} + \hat{E}_{n'X}) - \Delta^2 \hat{E}_{nX} - \Delta^2 \hat{E}_{n'X})/2$$

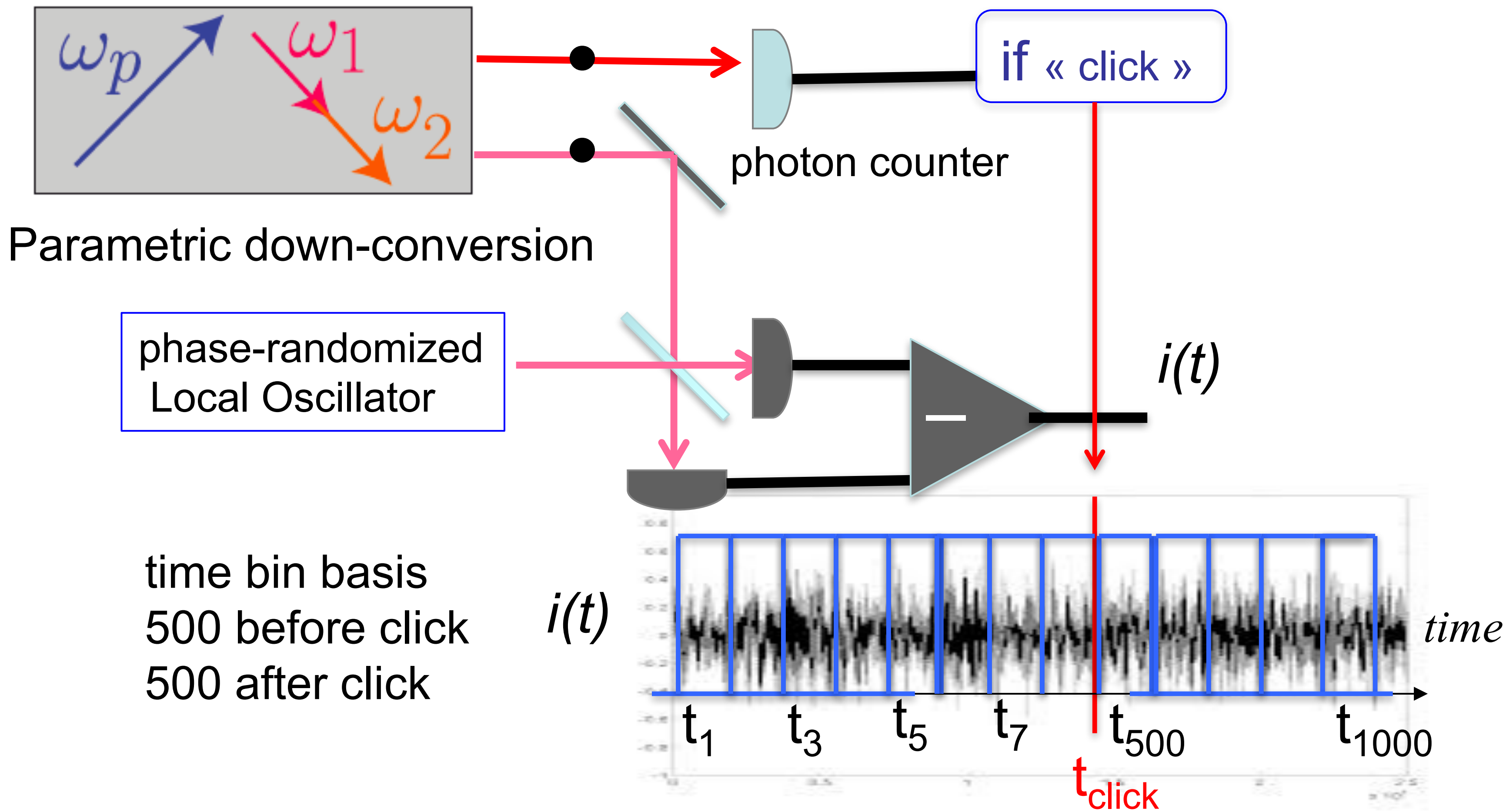


Local Oscillator
in mode $\mathbf{f}_n + \mathbf{f}_{n'}$

experimental determination of the mode of an intrinsic single photon

determination of the coherence matrix of a heralded single photon

O. Morin, C. Fabre, J. Laurat PRL **111**, 213602 (2013)



A. Mc Rae et al PRL **109**, 033601 (2012)

averaged over
a great number of clicks

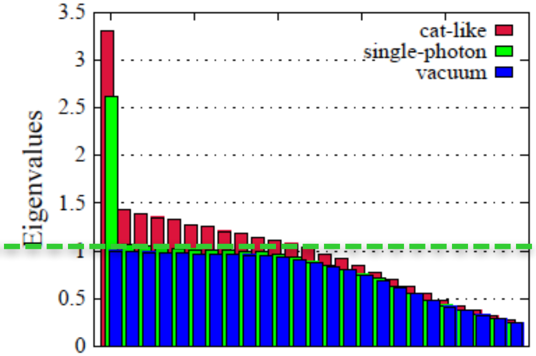
one gets a 1000×1000 matrix :

$$C^{exp}_{n,n'} = \overline{i(t_n)i(t_{n'})}$$
$$= 2\,C_{coherence} + 1$$

diagonalisation of C^{exp} :

(a) (b)

vacuum level



eigenvalues

only one eigenvalue different from vacuum fluctuations

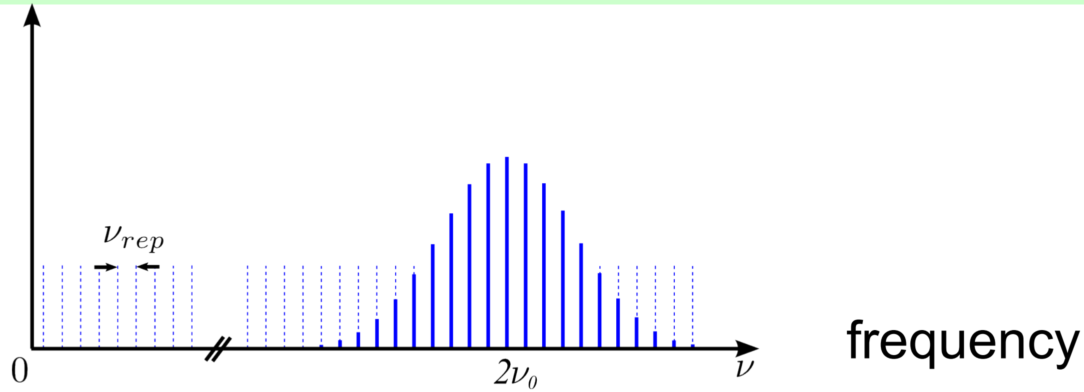
the generated state is indeed single mode

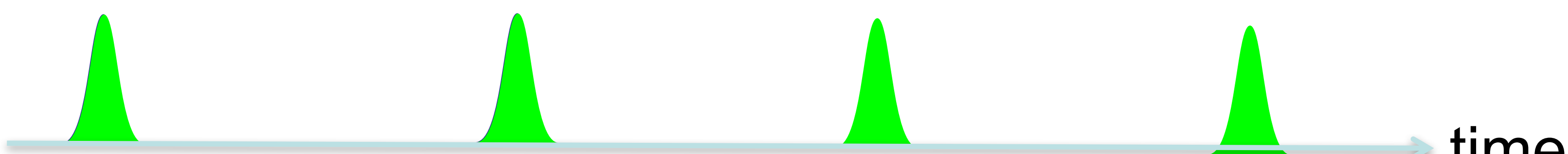
the corresponding eigenstate gives the
shape of the temporal mode of the single photon

Characterization of a highly multimode Gaussian non-classical state

A multi-frequency-mode light:
the frequency comb generated by a mode-locked laser

optical frequency comb:

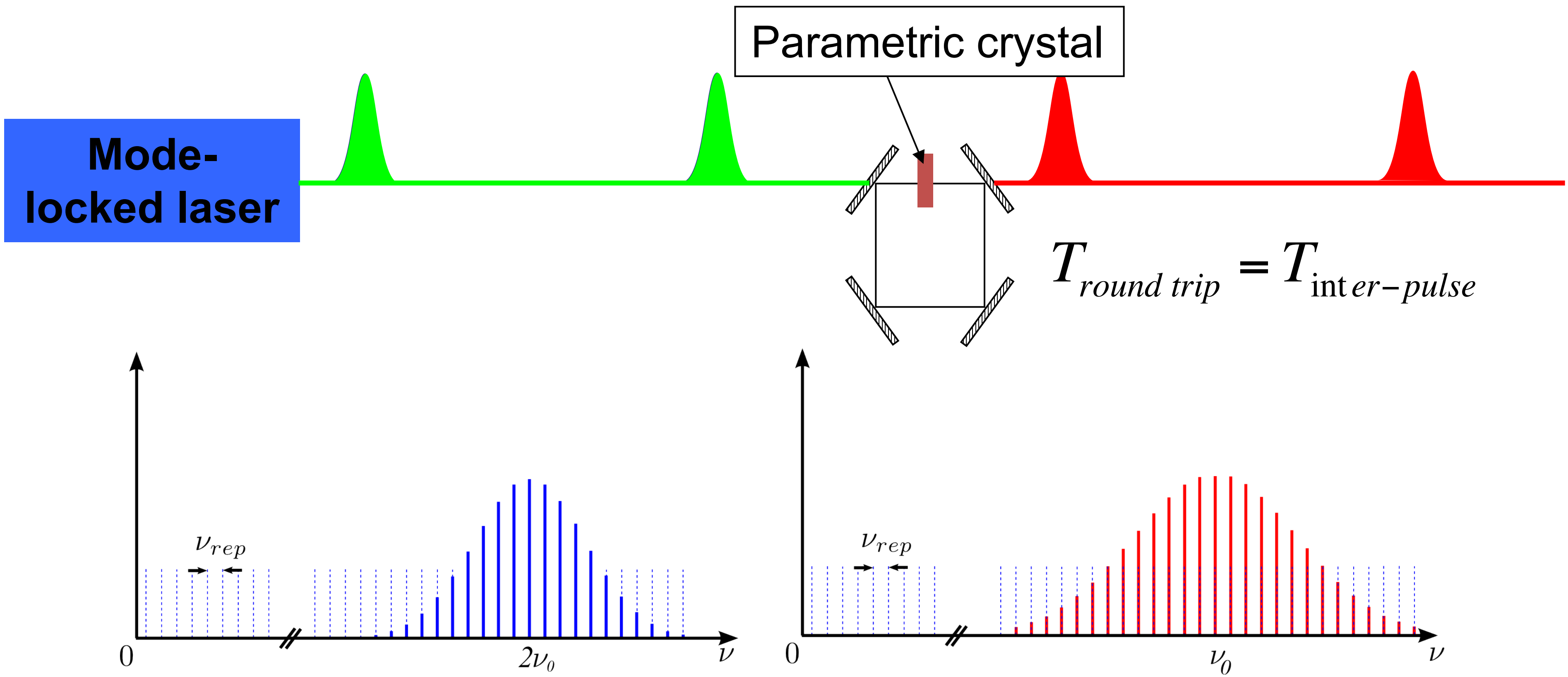




Frequency modes of a mode-locked laser: **about 100.000**

a quantum frequency comb ?

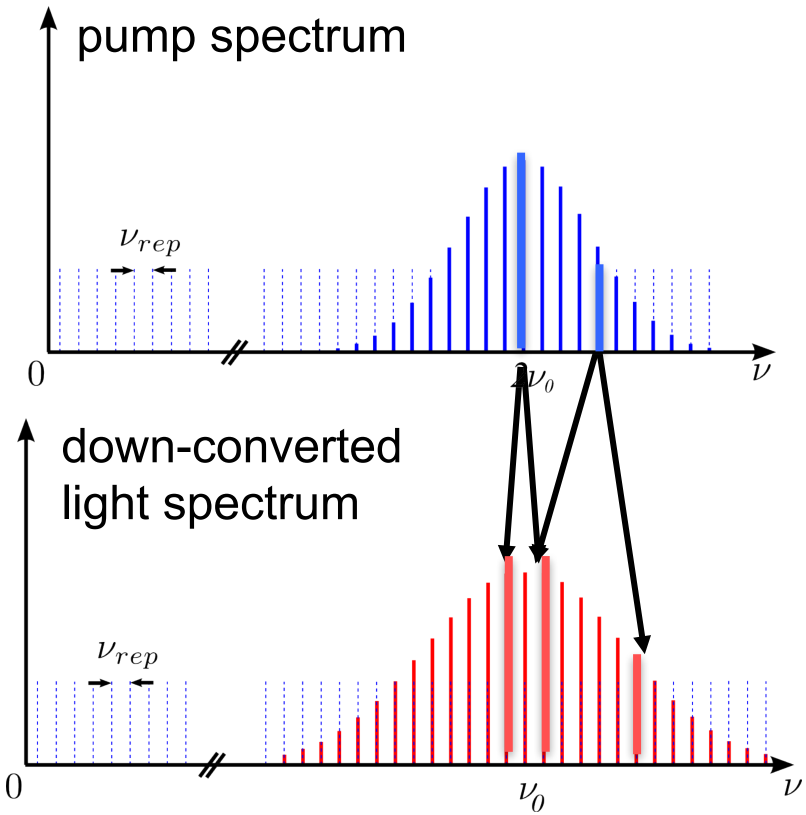
generation by parametric down conversion
of a mode-locked laser



the 10^5 frequency modes : are they entangled ?

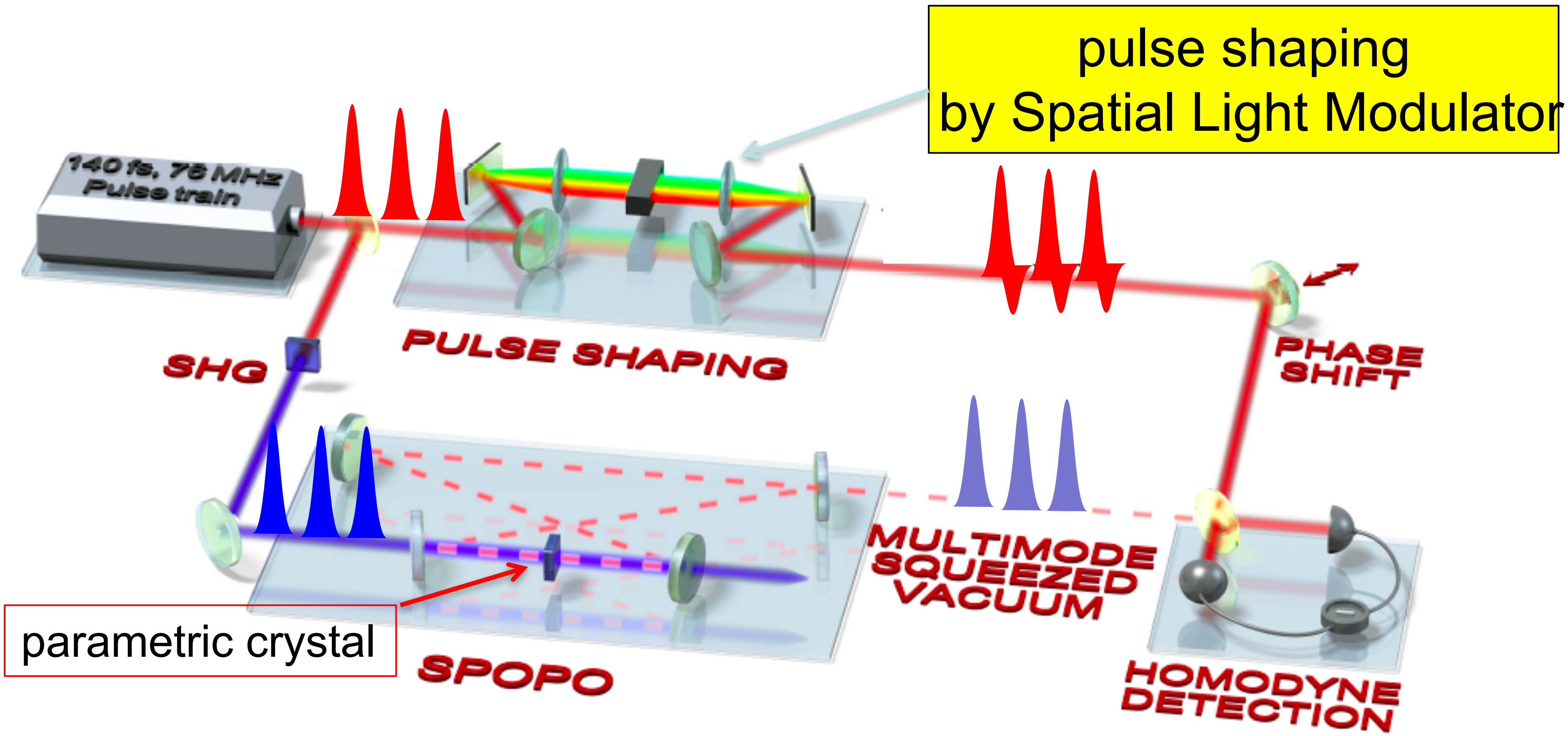
Generation of a multimode quantum state from multimode pump

parametric down conversion of a monochromatic pump gives rise to EPR entangled signal and idler beams



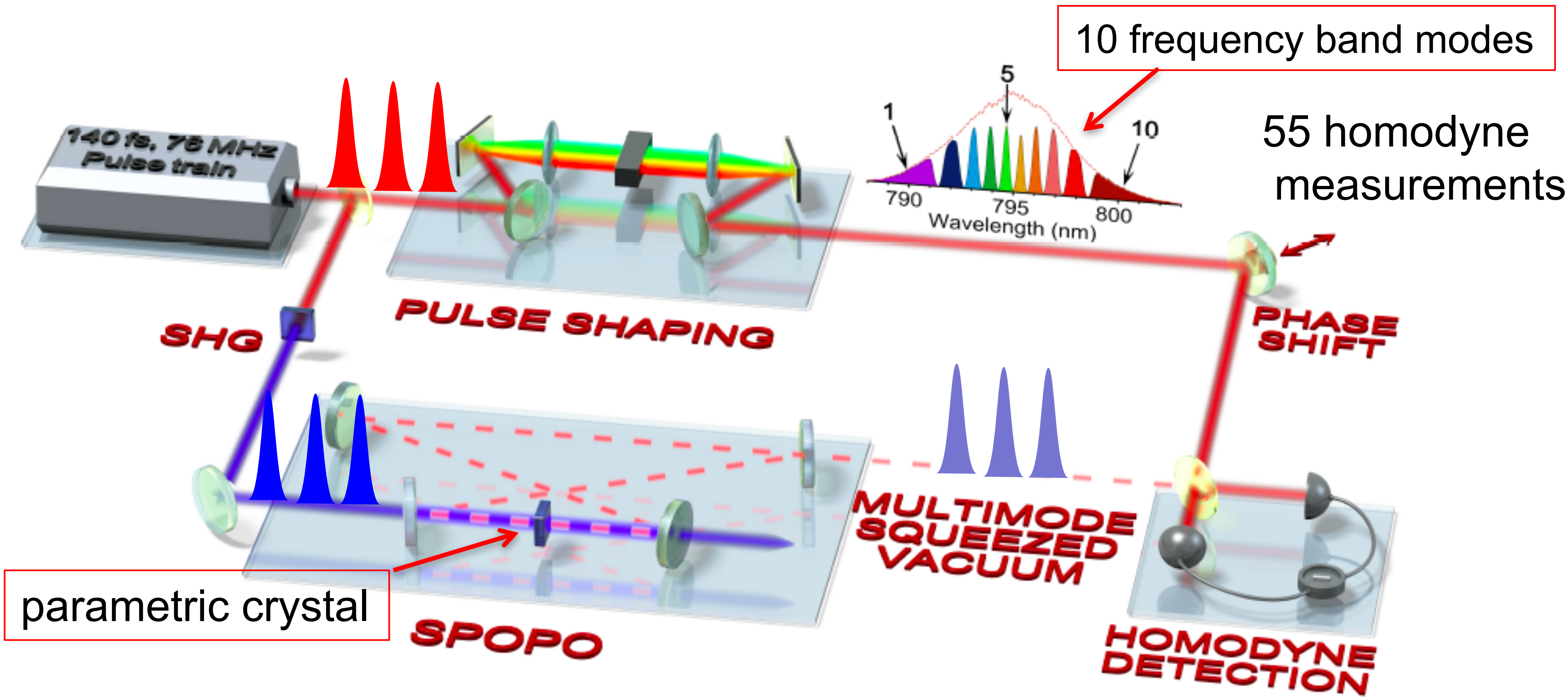
all couples of frequencies modes should be entangled !

Experimental set-up



O. Pinel et al, Phys. Rev. Letters **108,** 083601 (2012)
J. Roslund et al, Nature Photonics, **8**, 109 (2014)
R. Medeiros de Araujo et al, Phys Rev A**89,** 053828 (2014)
Yan Cai et al, Nature Com **8**, 15645 (2017)

Experimental set-up

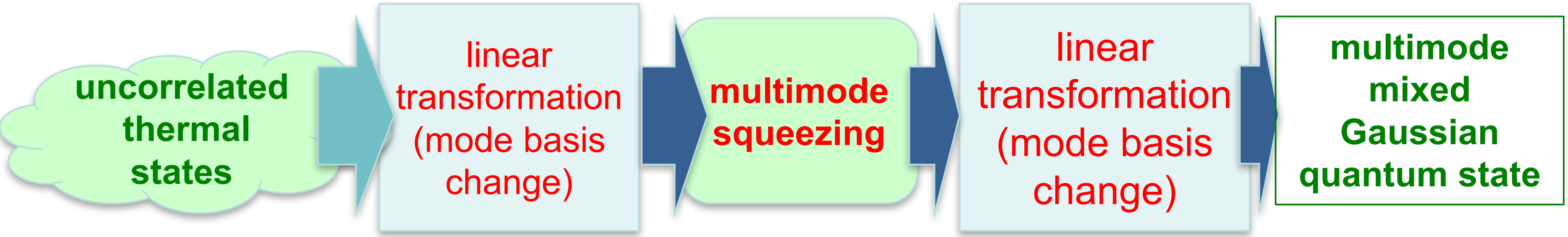


experimental value of the 20 by 20 covariance matrix

x quadratures $\qquad\qquad$ p quadratures

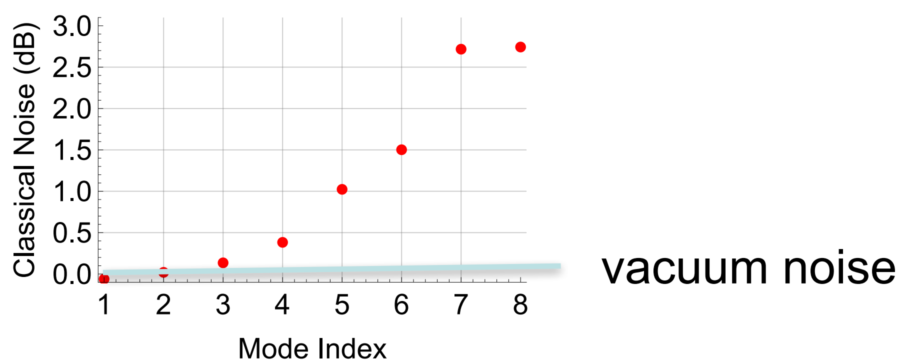
$$
\begin{array}{cc}
\begin{pmatrix}
1.66 & 0.25 & 0.09 & 0.08 & -0.03 & -0.05 & -0.08 & -0.11 & -0.42 & -1.23 \\
0.25 & 1.33 & 0.19 & 0.15 & 0.06 & -0.06 & -0.17 & -0.33 & -0.59 & -0.51 \\
0.09 & 0.19 & 1.22 & 0.07 & -0.01 & -0.08 & -0.21 & -0.34 & -0.44 & -0.20 \\
0.08 & 0.15 & 0.07 & 1.08 & -0.02 & -0.12 & -0.21 & -0.24 & -0.25 & -0.07 \\
-0.03 & 0.06 & -0.01 & -0.02 & 0.92 & -0.13 & -0.14 & -0.11 & -0.09 & 0.01 \\
-0.05 & -0.06 & -0.08 & -0.12 & -0.13 & 0.89 & -0.07 & 0.02 & 0.02 & 0.04 \\
-0.08 & -0.17 & -0.21 & -0.21 & -0.14 & -0.07 & 1.02 & 0.12 & 0.10 & 0.20 \\
-0.11 & -0.33 & -0.34 & -0.24 & -0.11 & 0.02 & 0.12 & 1.14 & 0.29 & 0.22 \\
-0.42 & -0.59 & -0.44 & -0.25 & -0.09 & 0.02 & 0.10 & 0.29 & 1.36 & 0.40 \\
-1.23 & -0.51 & -0.20 & -0.07 & 0.01 & 0.04 & 0.20 & 0.22 & 0.40 & 1.88
\end{pmatrix} & [0] \\
[0] &
\begin{pmatrix}
1.66 & 0.25 & 0.09 & -0.02 & 0.04 & 0.00 & 0.01 & 0.09 & 0.39 & 1.19 \\
0.25 & 1.33 & 0.19 & 0.05 & 0.13 & 0.12 & 0.14 & 0.28 & 0.50 & 0.53 \\
0.09 & 0.19 & 1.22 & 0.17 & 0.21 & 0.28 & 0.32 & 0.37 & 0.39 & 0.26 \\
-0.02 & 0.05 & 0.17 & 1.31 & 0.35 & 0.38 & 0.37 & 0.33 & 0.26 & 0.12 \\
0.04 & 0.13 & 0.21 & 0.35 & 1.38 & 0.47 & 0.43 & 0.31 & 0.22 & 0.10 \\
0.00 & 0.12 & 0.28 & 0.38 & 0.47 & 1.42 & 0.42 & 0.30 & 0.19 & 0.10 \\
0.01 & 0.14 & 0.32 & 0.37 & 0.43 & 0.42 & 1.34 & 0.27 & 0.21 & 0.08 \\
0.09 & 0.28 & 0.37 & 0.33 & 0.31 & 0.30 & 0.27 & 1.30 & 0.22 & 0.15 \\
0.39 & 0.50 & 0.39 & 0.26 & 0.22 & 0.19 & 0.21 & 0.22 & 1.36 & 0.40 \\
1.19 & 0.53 & 0.26 & 0.12 & 0.10 & 0.10 & 0.08 & 0.15 & 0.40 & 1.88
\end{pmatrix}
\end{array}
$$

characterization of the multimode state by Williamson Bloch Messiah reduction

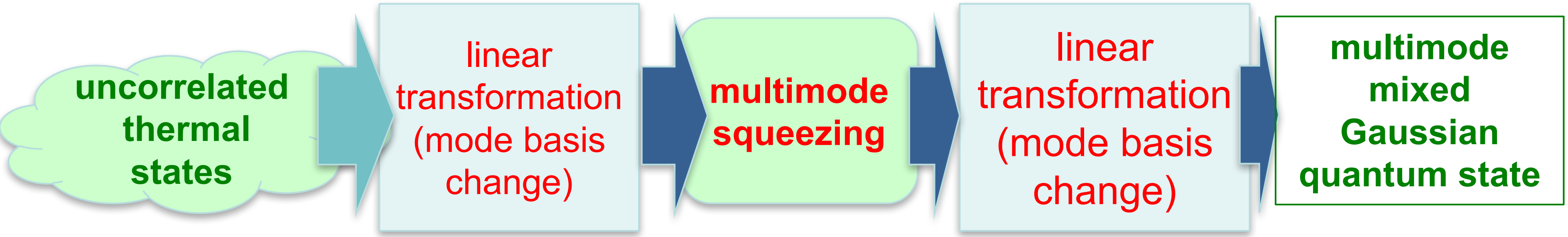


Determinant of covariance matrix gives purity: $\mathrm{Tr}\rho^2 = 0.14$

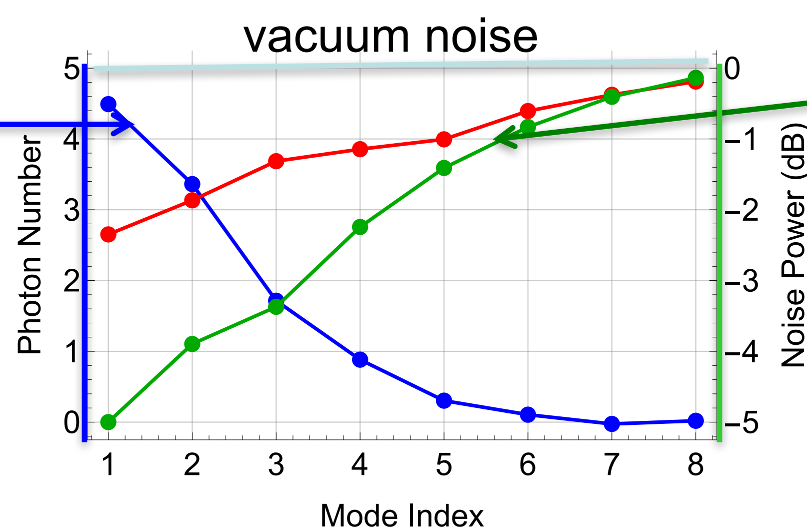
excess noise of input thermal modes:



characterization of the multimode state by Williamson Bloch Messiah reduction

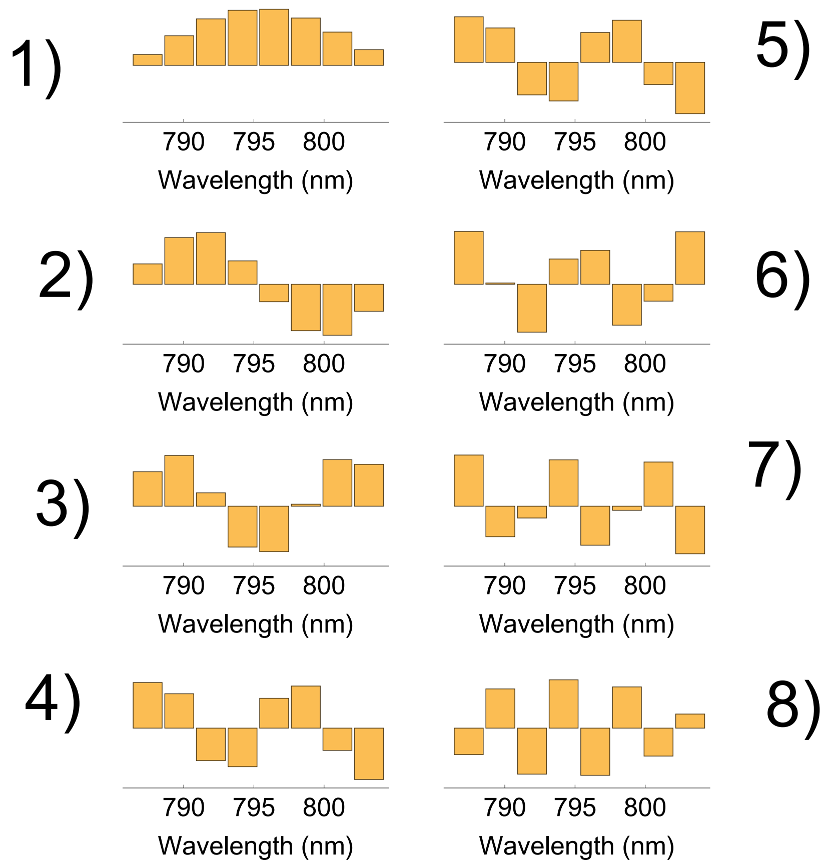


mean photon number in each eigenmode of the **coherence matrix**

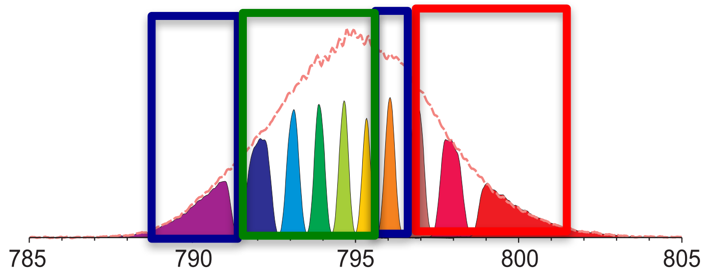


squeezing in the different modes

frequency shape of the squeezed eigenmodes



multi-partite entanglement



115 974
possible multipartitions
with 10 bands

multipartite optimal entanglement witnesses from covariance matrix:

J. Sperling and W. Vogel, Phys. Rev. Lett. **111**, 110503 (2013).

S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, C. Fabre, Phys. Rev. Letters **114,** 050501 (2015)

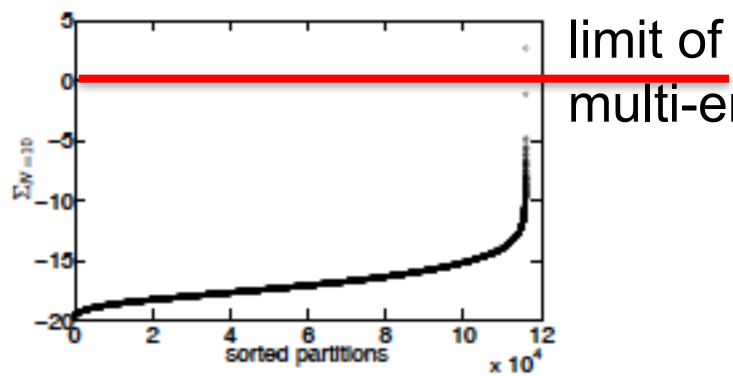


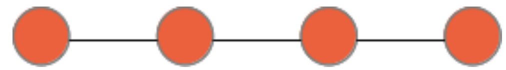
FIG. 3. The verified entanglement for all 115974 nontrivial partitions – sorted by significance Σ – for the 10-mode frequency-comb Gaussian state.

all 115 974 multipartitions are entangled !

Application to measurment-based quantum computing

Measurement Based Quantum Computing

Briegel et al



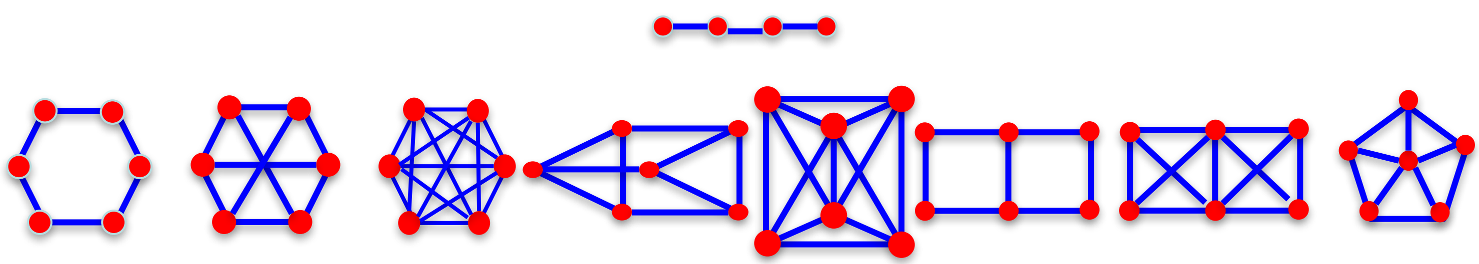
Starting element : the **cluster state**,
An entangled quantum state spanning on several « **nodes** »

Nodes can be light modes

(Furusawa, Van Loock, Menicucci, Pfister, …)

New possibilities offered by manipulation of modes,
much simpler than manipulation of qubits in different ions
more easily scalable to many nodes

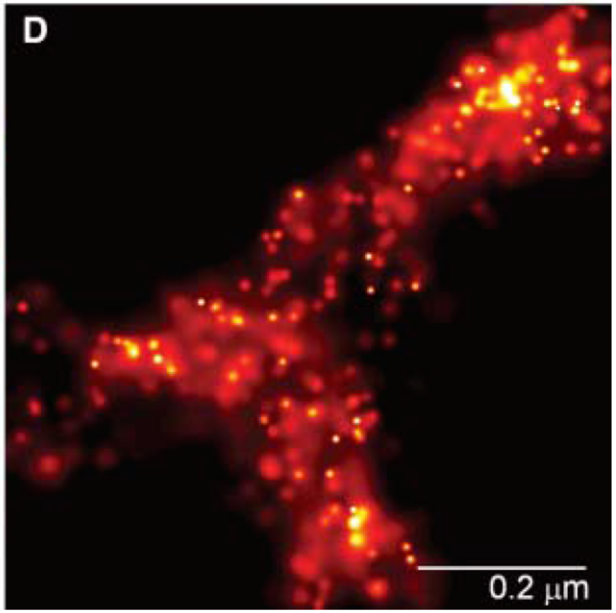
SPOPO: no manipulation of experimental set-up needed



Mode optimization to reduce quantum limits in parameter estimation

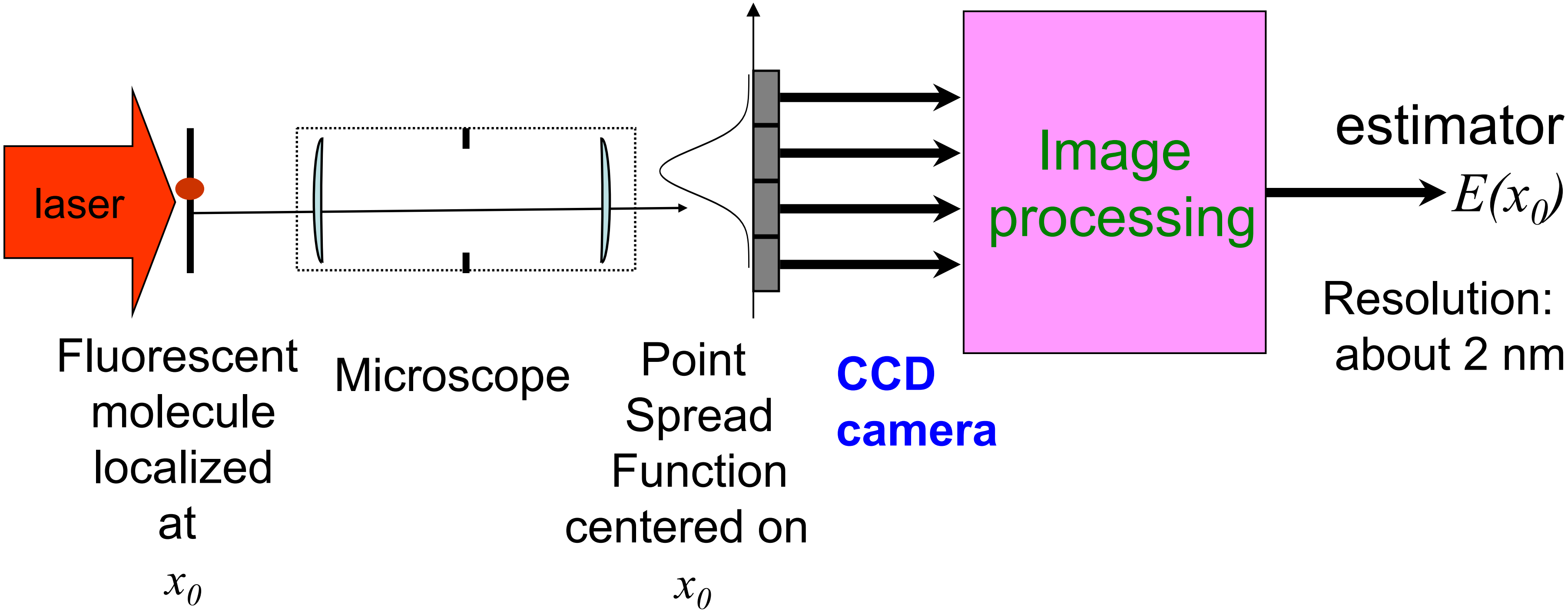
Very sensitive optical measurements:

- Interferometry: estimation of phase shift
- Ranging : estimation of time delay
- Imaging : estimation of **transverse position** x_0, y_0 of point-like sources
- …



Imaging Intracellular Fluorescent Proteins at Nanometer Resolution
E. Betzig et al Science **313** 1642 (2006)

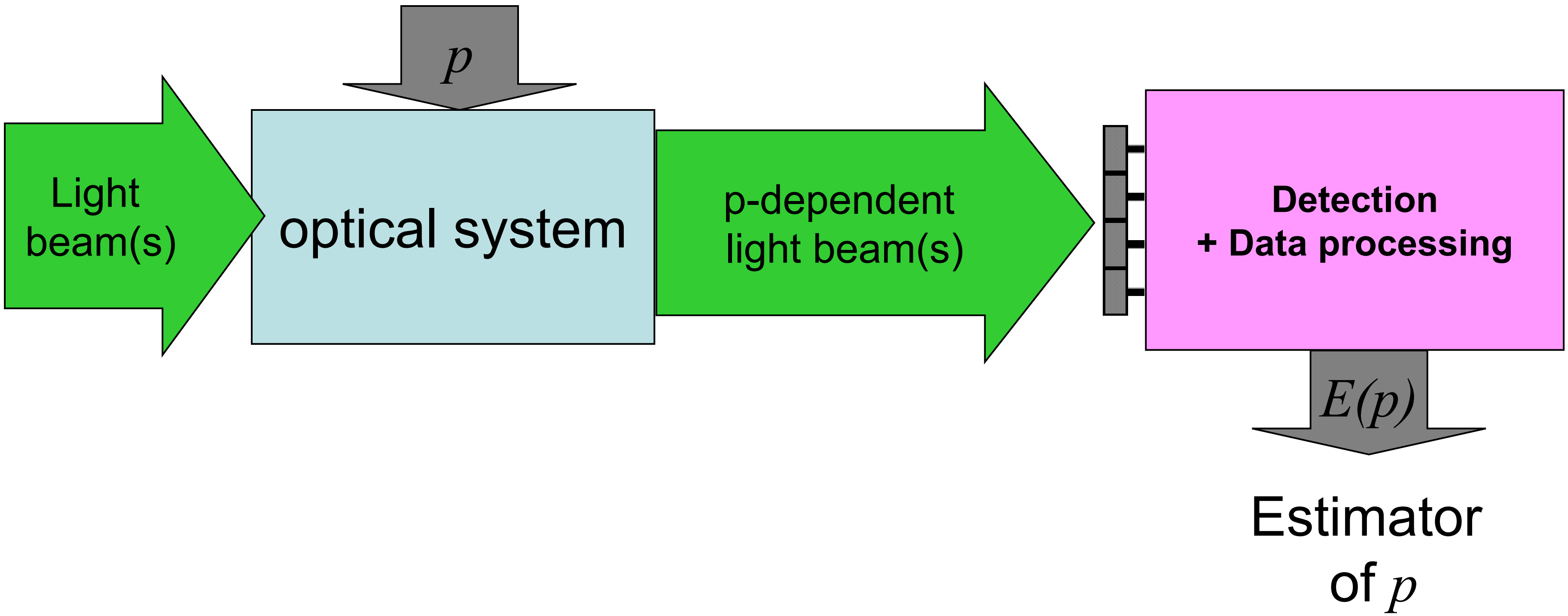
Strategy of measurement:



total image recorded by **CCD camera** :
the information needed for the estimation of x_0
is **distributed over many pixels**

data processing is necessary to extract the estimator

General scheme for estimating a parameter p using information carried by light



What is the smallest measurable variation of p around value p_0, for a given mean photon number ?

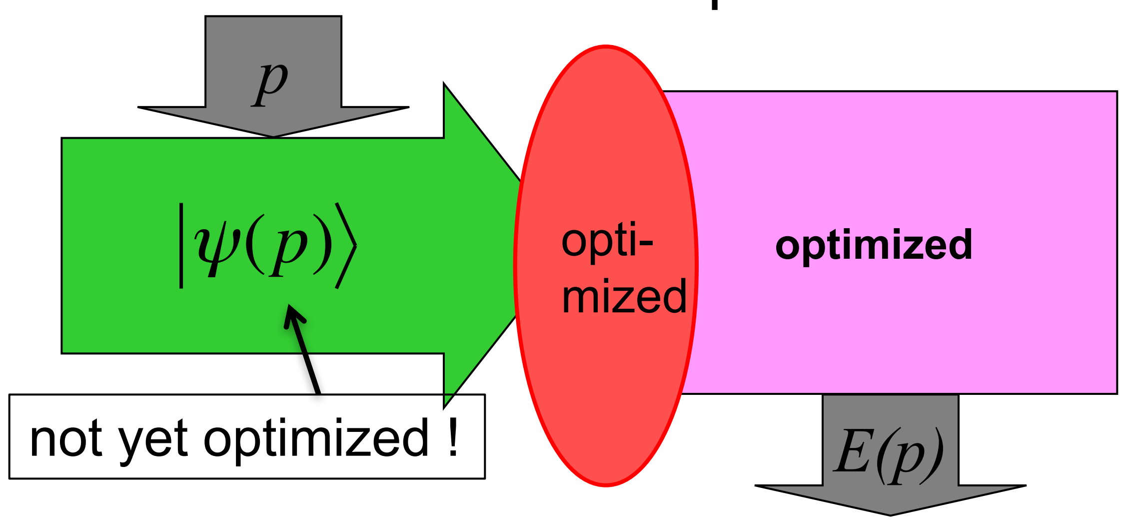
Quantum Cramer Rao Bound

Helstrom
Caves, Braunstein

Gives the minimum variance of the estimator

the bound is valid
independently of the detection device used
and of the precise strategy used in data processing

It depends only
on the characteristics of the quantum state of light



choice of light state $|\psi(p)\rangle$?

best choice for an experimentalist in Quantum Optics:

- large mean photon number N

  quantum limits scale as $1/N^x$

- **multimode state**

  more adapted to a multiplexed measurement

$$\hat{\mathbf{E}}^+(\mathbf{r},t) = \sum_n E_0\, \hat{a}_n\, \mathbf{f}_n(\mathbf{r},t)$$

QUANTUM **OPTICS**

choice of multimode quantum state

choice of mode shape

our choice : the multimode Gaussian pure state

choice left for :
- the number of modes,
- the spatio-temporal shape of modes
- the Gaussian quantum state of light

includes **a wide class of non-classical states**

- single and multimode squeezed states
- Einstein Podolsky Rosen (EPR) state
- multipartite quadrature entangled state
- + coherent state

$$\boxed{\langle N \rangle \approx 10^{15}}$$

excludes states which are « more quantum »,
but not scalable to very large N value

Useful mode 1: the « mean field mode »

$$u_{mean}(x,y,t,p) = \frac{1}{\sqrt{N}} \left\langle \psi\left(p,t\right) \middle| \hat{E}^{(+)}(x,y) \middle| \psi(p,t) \right\rangle$$

contains the spatio-temporal dependence of the mean field.

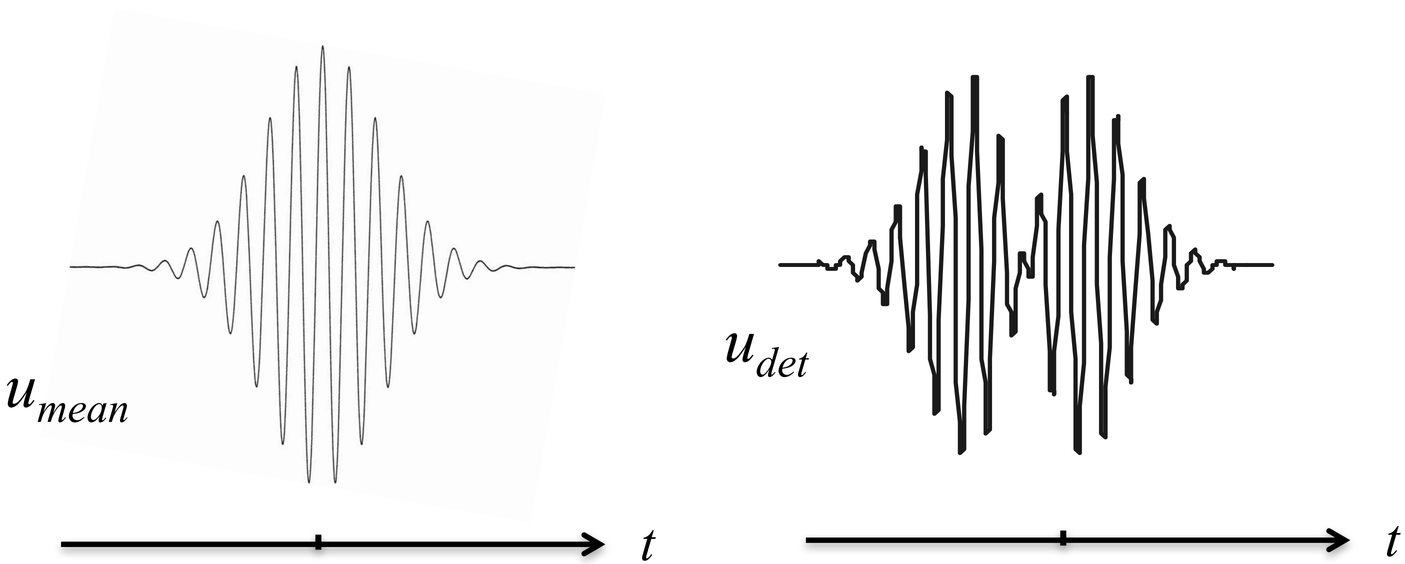
Useful mode 2: the «detection mode »

$$u_{\det}(x,y,t) = p_c \left. \frac{\partial u_{mean}}{\partial p} \right|_{p=p_0}$$

normalizing factor

Laser ranging

delay in the arrival of a light pulse



Quantum Cramer Rao bound for Gaussian pure states

p-sensitivity

expression in the high N limit:

$$\Delta p_{QCRb} = \frac{p_c}{2\sqrt{N}} \frac{1}{\sqrt{\left(\Gamma^{-1}\right)_{11}}}$$

« shot noise »

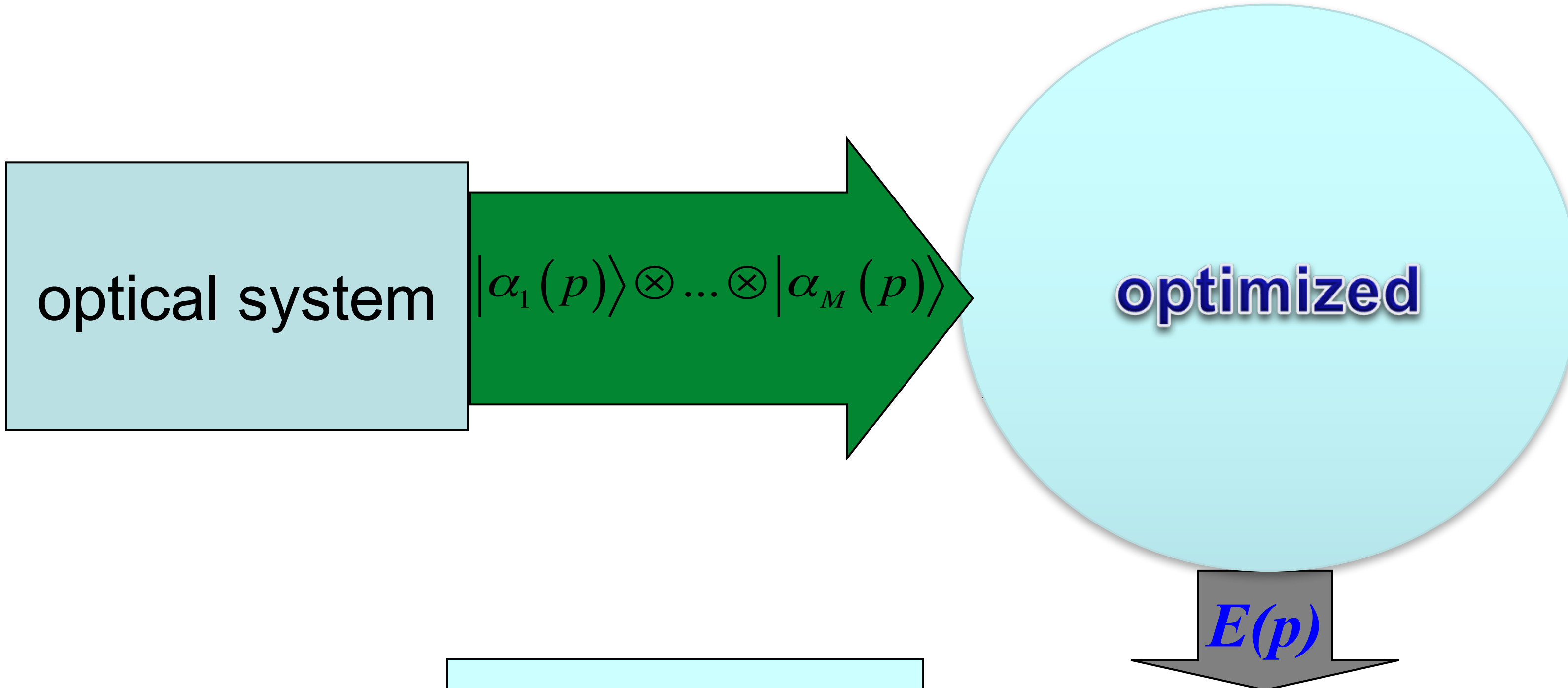
noise term

Diagonal element of the inverse covariance matrix in the **detection mode** u_{det}

value independent of the fluctuations of all other modes

Standard Quantum Cramer Rao bound

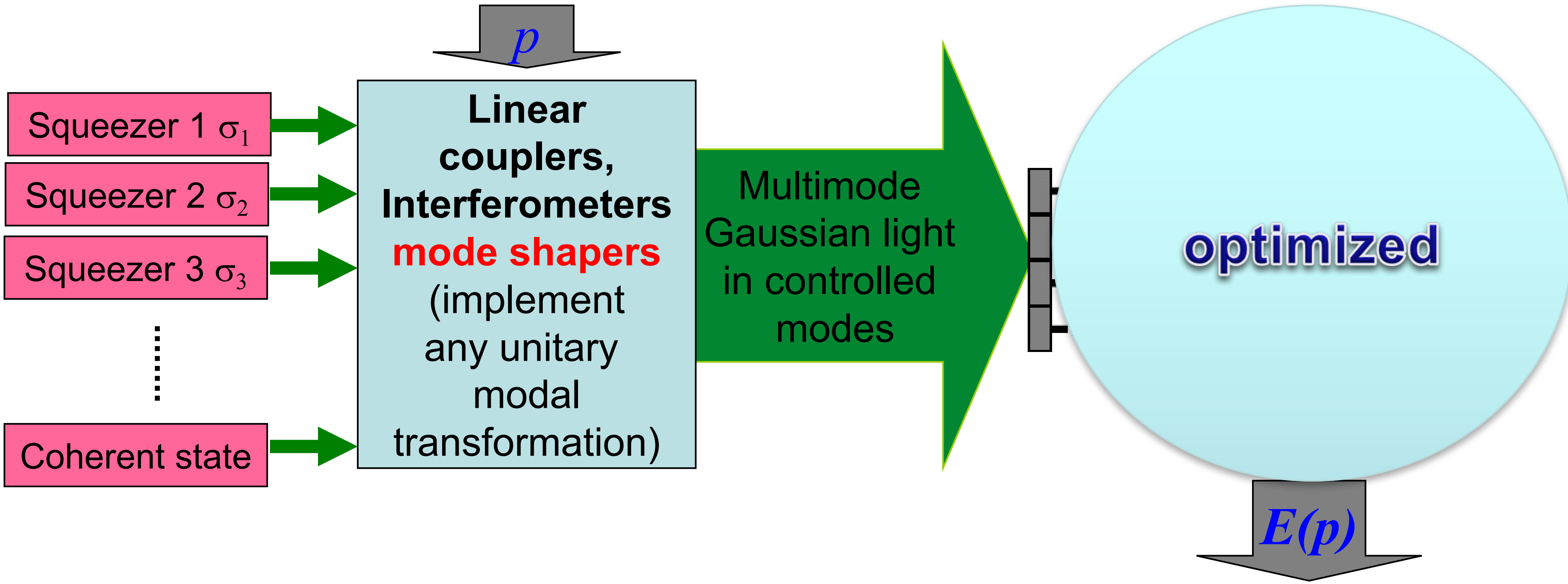
When only coherent states are used



$$\Delta p_{SCRb} = \frac{p_c}{2\sqrt{N}}$$

optimized shot noise limit

Quantum Cramér Rao bound using several vacuum squeezed states



$$\Delta p_{CRb} = \frac{p_c}{2\sqrt{N}}\sqrt{\sigma_{\min}}$$

$$\sigma_{\min} = Min\{\sigma_1, \sigma_2, \ldots, \sigma_s\}$$

Conclusions for the experimentalist

To get the lowest possible Quantum Cramer Rao bound

- Put maximum power in coherent mode

- Squeeze one mode only

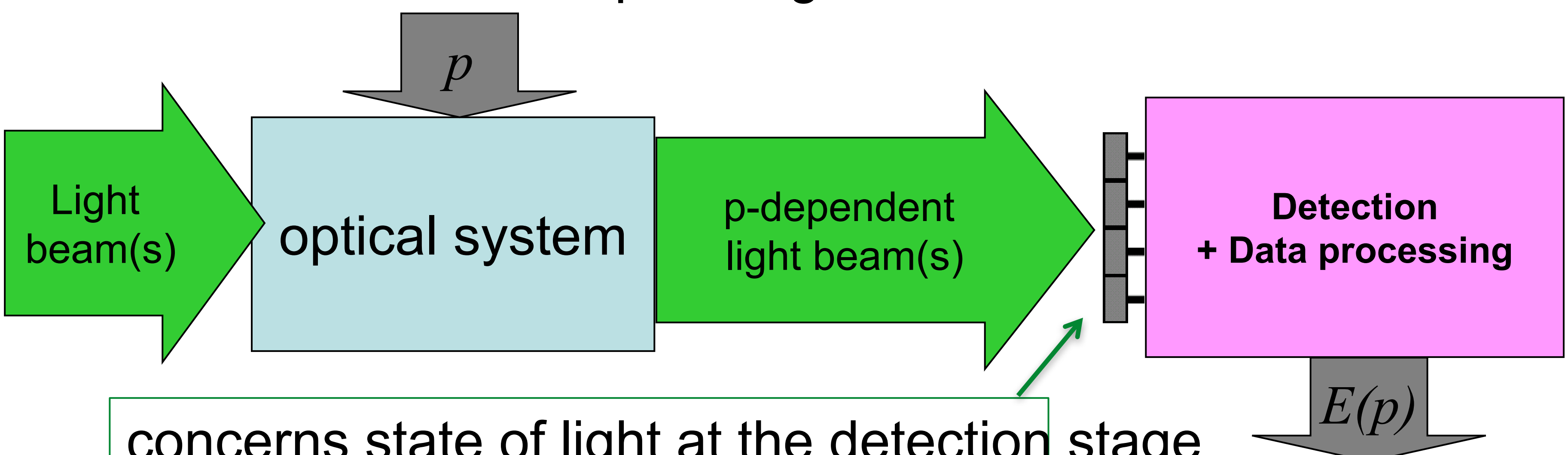
squeezing is not « additive »

- Squeeze the right mode

squeeze the detection mode

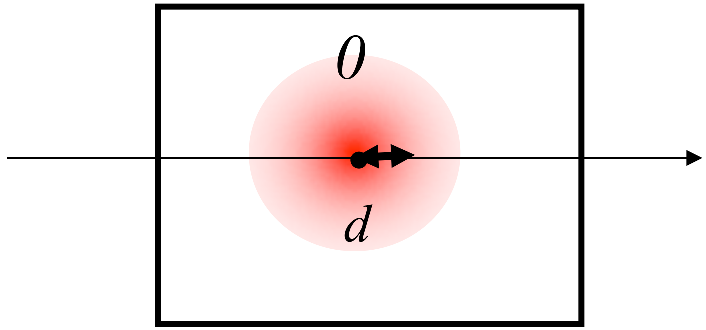
- Do not entangle detection mode with other modes

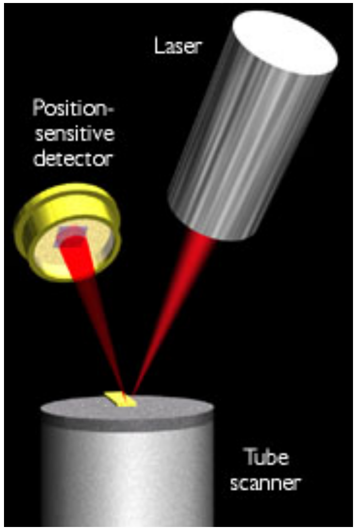
entangling modes actually
reduces the squeezing



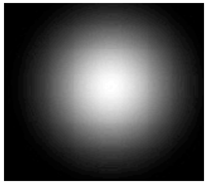
concerns state of light at the detection stage

Example: beam displacement



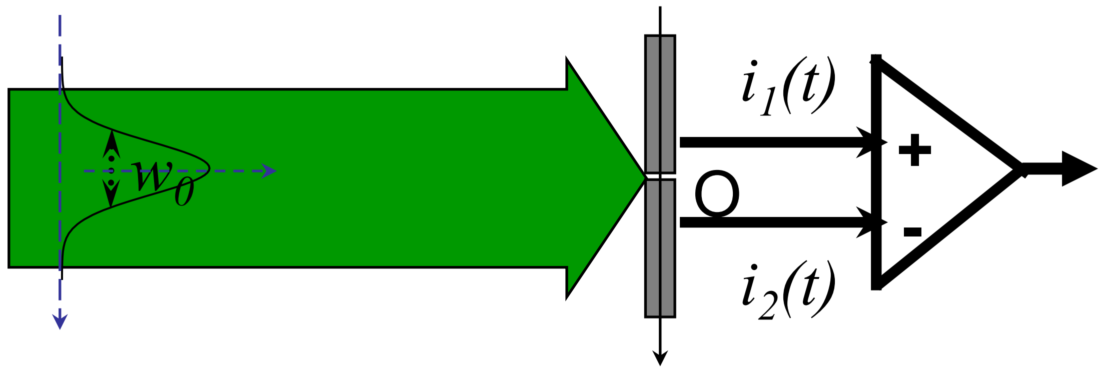


Quantum Cramer Rao bound for a coherent TEM $_{00}$ beam:



$$\left(\Delta p\right)_{S-CRb} = \frac{w_0}{2\sqrt{N}}$$

The usual technique: the split detector method



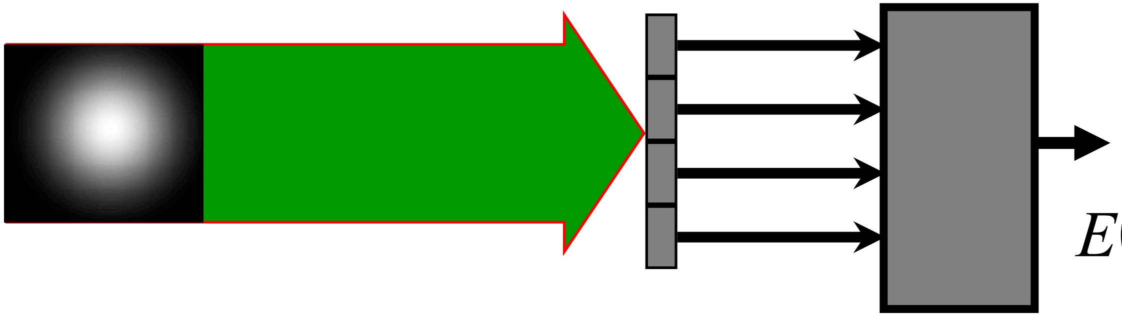
$$i_1(t) - i_2(t) = E(p)$$

quantum limit for a TEM_{00} coherent beam:

$$\Delta p = \frac{\sqrt{8}}{\pi} \frac{w_0}{\sqrt{N}}$$

The split detector method is not the optimal technique
(by 22%)

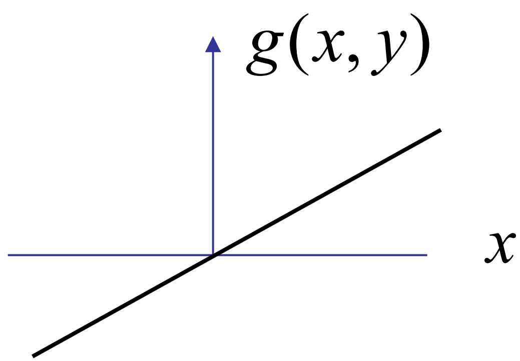
optimized technique n° 1



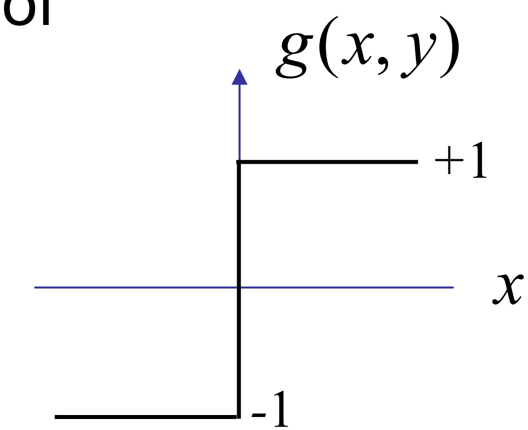
$$E(p) = \iint dx dy\, i(x, y, p) g(x, y)$$

optimized choice of $g(x, y)$ for a TEM_{00} beam:

$$\boxed{g(x, y) = x}$$

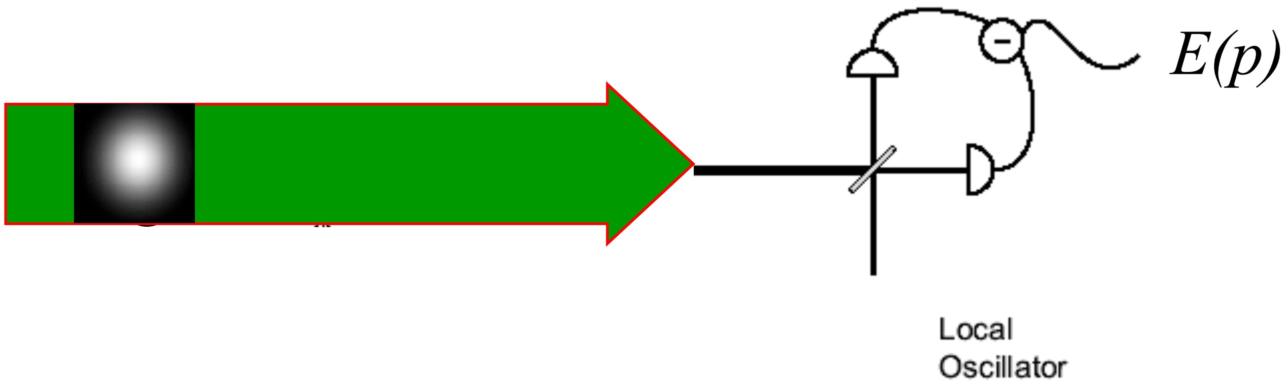


instead of

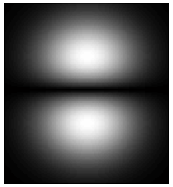


Standard Quantum Cramer Rao Bound reached

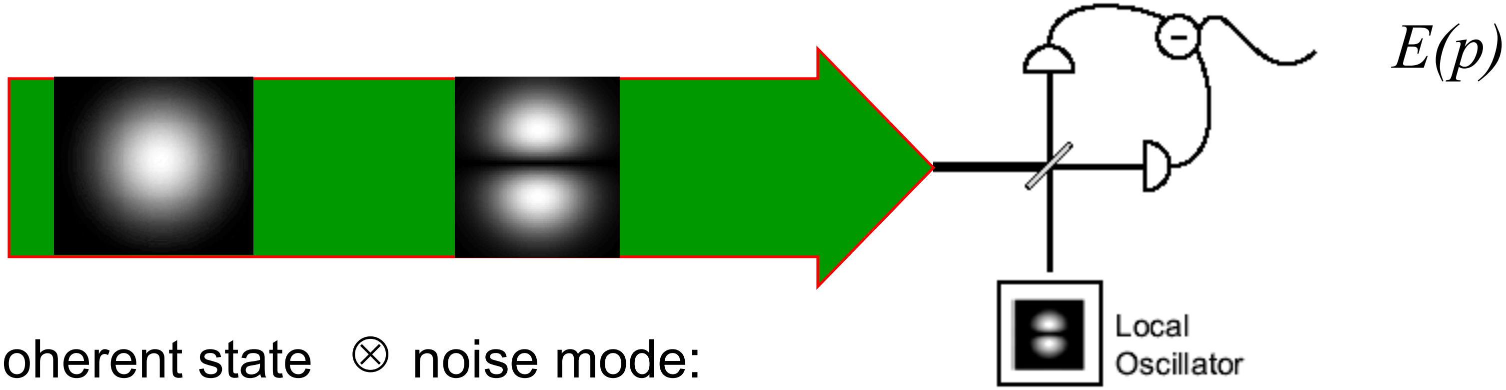
Optimized technique n° 2 : homodyne detection



Detection mode ? $\frac{\partial}{\partial y} TEM_{00} \propto TEM_{01}$

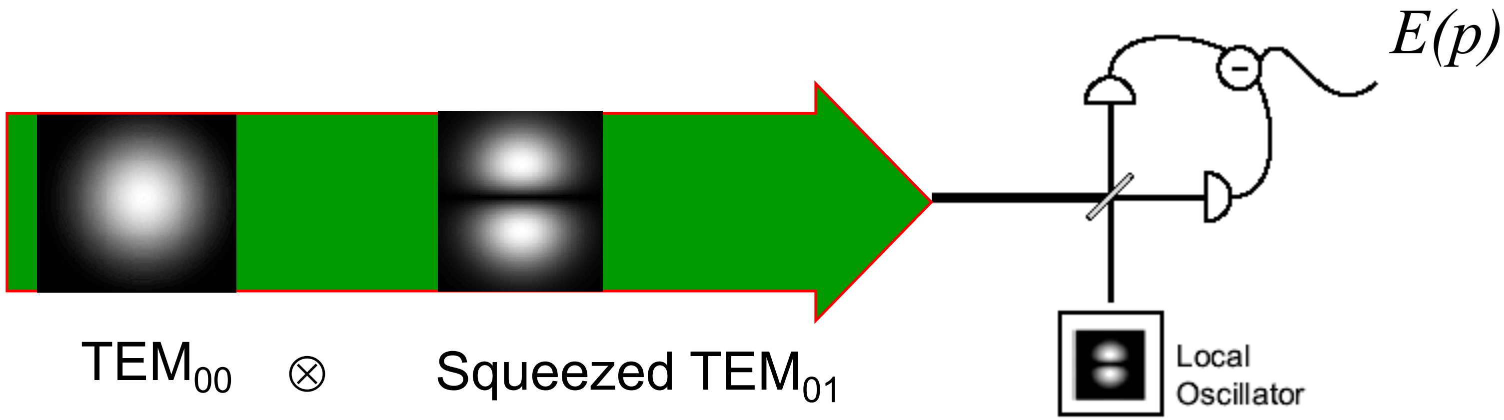


Beyond the standard quantum noise



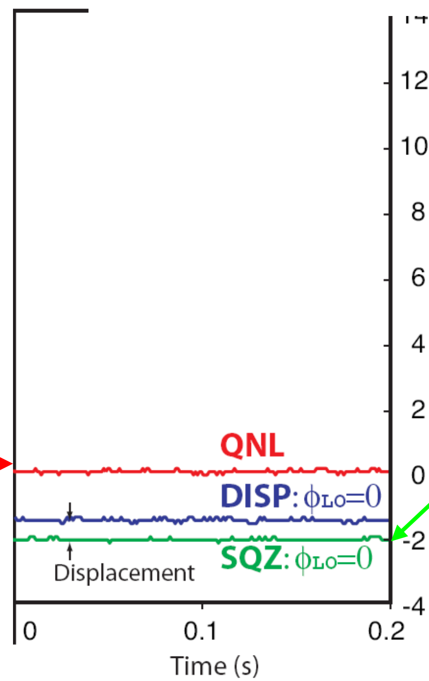
Coherent state $\otimes$ noise mode:
TEM_{10} in squeezed
vacuum state

Experimental implementation



M. Lassen et al.
Phys. Rev Letters
98, 083602 (2007)

Standard Cramer Rao bound



Reduced noise floor using squeezed TEM$_{10}$

Conclusion

considering not only the quantum state,
but the modes in which it exists, is a fruitful approach

mode basis change
-is rather easy to implement experimentally
-provides different points of view on the same quantum state

easy scalability

applications to quantum information processing
and quantum metrology

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