Atoms and photons

Chapter 1

J.M. Raimond

September 6, 2016

Introduction

The fundamental importance of the atom-field interaction problem

- Provides all information we have on the universe
- Provides the most precise theory so far: QED
- Provides the best tests of fundamental quantum physics

Introduction

The practical importance of the atom-field interaction problem

- Lasers
- Atomic clocks
- Cold atoms and BEC
- Quantum simulation

Outline of this course

Chapter 1: Interaction of atoms with a classical field

1. The harmonically bound electron: a surprisingly successful model
2. The Einstein coefficients

Outline of this course

Chapter 2: Quantized atom and classical field

1. Interaction Hamiltonian
2. Free atom and resonant field
3. Relaxing atom and resonant field
4. Optical Bloch equations
5. Applications of the optical Bloch equations

Outline of this course

Chapter 3: Field quantization

1. Field eigenmodes
2. Quantization
3. Field quantum states
4. Field relaxation

Outline of this course

Chapter 4: quantized matter and quantized field

1. Interaction Hamiltonian
2. Spontaneous emission
3. Photodetection
4. The dressed atom
5. Cavity Quantum electrodynamics

Bibliography

- Cohen-Tannoudji, Dupont-Roc and Grynberg, *An introduction to quantum electrodynamics* and *Photons and atoms*, Wiley, 1992
- Cohen-Tannoudji and Guery Odelin *Advances in atomic physics: an overview*, World Scientific 2012
- Schleich, *Quantum optics in phase space*, Wiley 2000
- Vogel, Welsch and Wallentowitz, *Quantum optics an introduction*, Wiley 2001
- Meystre and Sargent *Elements of quantum optics*, Springer 1999
- Barnett and Radmore *Methods in theoretical quantum optics*, OUP, 1997
- Scully and Zubairy *Quantum optics*, 1997
- Loudon *Quantum theory of light*, OUP 1983
- Haroche and Raimond *Exploring the quantum*, OUP 2006
- and, of course, the online lecture notes and slides handouts.

Online lecture notes

- www.cqed.org, following the menu items ‘teaching’, ‘Jean-Michel Raimond’
- http://www.lkb.upmc.fr/cqed/teachingjmr/

Outline

Outline

Outline

A classical model: the harmonically bound electron

The simplest classical model for an atom: a single charge (electron) bound to a force center by an harmonic potential.

- An early atomic theory model (Thomson's 'plum pudding')
- A good guide to identify relevant parameters by dimensional analysis
- Surprisingly accurate predictions

A classical model: the harmonically bound electron

Equations of motion

Dynamics

$$\frac{d^2\mathbf{r}}{dt^2} + \omega_0^2\mathbf{r} = 0 \qquad (1)$$

Solution

$$\mathbf{r} = \mathbf{r}_0 \exp(-i\omega_0 t) \qquad (2)$$

A classical model: the harmonically bound electron

Equations of motion

Damping: radiation reaction. Model emitted power by a viscous damping term in the equation of motion. A reasonable approximation for weak damping.

Larmor formula for radiated power

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = m\tau a^2 \tag{3}$$

where

$$\tau = \frac{1}{6\pi\epsilon_0}\frac{q^2}{mc^3} = 6.32\,10^{-24}\ \text{s} \tag{4}$$

linked to the classical radius of electron $r_e = \frac{q^2}{4\pi\epsilon_0 mc^2} = 3.\,10^{-15}$ m by $r_e = \frac{3}{2}c\tau$

A classical model: the harmonically bound electron

Equations of motion

Modified equation of motion

$$\frac{d^2\mathbf{r}}{dt^2} + \gamma\frac{d\mathbf{r}}{dt} + \omega_0^2\mathbf{r} = 0 \tag{5}$$

with

$$\gamma = \omega_0^2\tau \tag{6}$$

being the amplitude damping coefficient obtained by equalling the average dissipated energy to the average radiated power (the energy damping coefficient is obviously 2γ).

A classical model: the harmonically bound electron

Equations of motion

Order of magnitude estimate for γ:
$\omega_0 \approx R_y/\hbar$, where $R = mc^2\alpha^2/2$ is the Rydberg constant and

$$\alpha = \frac{q^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137} \qquad (7)$$

the fine structure constant. Then

$$\frac{\gamma}{\omega_0} = \omega_0\tau = \frac{R\tau}{\hbar} = \frac{R}{\hbar}\frac{1}{6\pi\epsilon_0}\frac{q^2}{mc^3} = \frac{\alpha^3}{3} \approx 1.3\,10^{-7} \qquad (8)$$

A classical model: the harmonically bound electron

Polarizability

Response to a classical oscillating field $E_0\mathbf{u}_z\exp(-i\omega t)$

Equation of motion

$$\frac{d^2\mathbf{r}}{dt^2}+\gamma\frac{d\mathbf{r}}{dt}+\omega_0^2\mathbf{r}=\frac{qE_0}{m}\mathbf{u}_z e^{-i\omega t} \quad (9)$$

Steady-state solution

Position: $\mathbf{r}=\mathbf{r}_0\exp(-i\omega t)$; Dipole: $\mathbf{d}=\mathbf{d}_0\exp(-i\omega t)$ with

$$\mathbf{d}_0=q\mathbf{r}_0=\epsilon_0\alpha_c E_0\mathbf{u}_z \quad (10)$$

where

$$\alpha_c=\frac{q^2}{m\epsilon_0}\frac{1}{\omega_0^2-\omega^2-i\gamma\omega} \quad (11)$$

A classical model: the harmonically bound electron

Diffusion

Total power diffused by the atom given by Larmor formula:

$$\mathcal{P} = \frac{1}{2} m\tau\omega^4 |r_0|^2 \tag{12}$$

or

$$\mathcal{P} = \frac{\epsilon_0}{12\pi c^3} |\alpha_c|^2 \omega^4 E_0^2 \tag{13}$$

Cross Section

Ratio of this power to the incident power per unit surface $\mathcal{P}_i = \epsilon_0 c E_0^2/2$:

$$\sigma_c = \frac{1}{6\pi} \left(\frac{\omega}{c}\right)^4 |\alpha_c|^2 = \frac{8\pi}{3} r_e^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2} \tag{14}$$

A classical model: the harmonically bound electron

The three diffusion regimes

Rayleigh diffusion for $\omega < \omega_0$ and $\omega_0 - \omega \gg \gamma$

$$\sigma_c = \frac{8\pi}{3} r_e^2 \frac{\omega^4}{\omega_0^4} \tag{15}$$

Blue sky: $\sigma_c \approx 10^{-30}$ m^2, $N = 10^{25}$ m^{-3}: the attenuation length is $L = 1/N\sigma_c \approx 100$ km

Thomson diffusion for $\omega > \omega_0$

$$\sigma_c = \frac{8\pi}{3} r_e^2 \ . \tag{16}$$

The resonant regime for $\omega \approx \omega_0$

$$\sigma_c = \frac{8\pi}{3} r_e^2 \frac{\omega_0^2}{4(\omega_0 - \omega)^2 + \gamma^2} \tag{17}$$

A classical model: the harmonically bound electron

Resonant diffusion

At exact resonance $\omega_0 = \omega$:

$$\sigma_c = \frac{8\pi}{3} r_e^2 \frac{\omega_0^2}{\gamma^2} \tag{18}$$

With

$$r_e \frac{\omega_0}{\gamma} = \frac{3}{2} c\tau \frac{1}{\omega_0 \tau} = \frac{3}{4\pi} \lambda_0 \tag{19}$$

where $\lambda_0 = 2\pi c/\omega_0$ is the wavelength. Hence

$$\sigma_c = \frac{3}{2\pi} \lambda_0^2 \tag{20}$$

This model doe not apply for high powers: saturation (about 1 mW/cm^2). A quantum effect. More on that in next Chapter.

A classical model: the harmonically bound electron

Propagation in matter

Apply the model to propagation in matter. Simplifiying hypothesis:

- Consider harmonic plane wave
- Linear response theory
- Dilute matter: no difference between local and global field

Equation of propagation

$$\Delta \mathbf{E} + \frac{\omega^2}{c^2}\epsilon_r \mathbf{E} = 0 \tag{21}$$

with $\epsilon_r = 1 + N\alpha_c$

Dispersion relation

$$k^2 = k_0^2 \epsilon_r \tag{22}$$

where $k_0 = \omega/c$

A classical model: the harmonically bound electron

Propagation in matter

Refraction index $n = \sqrt{\epsilon_r} = n' + in''$

$$n' = \frac{1}{\sqrt{2}}\sqrt{\epsilon'_r + \sqrt{\epsilon'^2_r + \epsilon''^2_r}} \quad \text{and} \quad n'' = \frac{\epsilon''_r}{\sqrt{2}}\frac{1}{\sqrt{\epsilon'_r + \sqrt{\epsilon'^2_r + \epsilon''^2_r}}} \tag{23}$$

Real part: refraction (ordinary index), Imaginary part: absorption.
Power released in matter $\frac{1}{2}\text{Re}\,\mathbf{j}_0 \cdot \mathbf{E}_0*$ where $\mathbf{j}_0 = -i\omega\mathbf{P}_0$.

$$\mathcal{E} = \frac{1}{2}\text{Re}\;(-i\omega\mathbf{P}_0 \cdot \mathbf{E}_0^*) \tag{24}$$

$$\mathcal{E} = \frac{1}{2}\epsilon_0\omega\chi''|E_0|^2 = \frac{1}{2}\epsilon_0\omega N\alpha''|E_0|^2 \tag{25}$$

A classical model: the harmonically bound electron

Propagation in matter

$$\mathcal{E} = \frac{1}{2}\epsilon_0\omega\chi''|E_0|^2 = \frac{1}{2}\epsilon_0\omega N\alpha''|E_0|^2 \quad (26)$$

Imaginary part of polarizability:

$$\alpha'' = \frac{q^2}{m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \quad (27)$$

Power released always positive, matter always absorbing. Laser needs a quantum ingredient

Einstein's coefficents

Introduction

A phenomenological description of energy exchanges between light and matter. A very simple description:

- Field only described by its spectral energy density u_ν. Numerical density of photons between ν and $\nu + d\nu$: $u_\nu/h\nu$. Total energy per unit volume: $u = \int u_\nu \, d\nu$
- Matter made of two-level atoms e above g energies E_e and E_g. No degeneracy. $(E_e - E_g)/h = \nu_0$. Transition wavelength $\lambda_0 = c/\nu_0$.
- Number (or density) of atoms in the two levels N_e and N_g, normalized to the total atom number (or density $\mathcal{N}$) so that $N_e + N_g = 1$.

Goal: obtain rate equations for the variations of N_e and u_ν. We shall consider in particular the radiation/matter thermal equilibrium at a temperature T. For that, three process come into play:

Einstein's coefficents

Three processs

Spontaneous emission

Deexcitation of e with a constant probability per unit time, $A_{eg} = \Gamma$.

$$\left.\frac{dN_e}{dt}\right)_{\text{spon}} = -A_{eg} N_e \tag{28}$$

Absorption

Transfer from g to e by absorption of photons. Rate proportional to the photon density (a cross-section approach).

$$\left.\frac{dN_e}{dt}\right)_{\text{abs}} = B_{ge} u_{\nu_0} N_g \tag{29}$$

Einstein's coefficents

Something is lacking

Absorption and spontaneous emission are not enough. At infinite temperature, all atoms in the upper state. Not the prediction of thermodynamics (50% in each state). Einstein adds a third process:

Stimulated emission

Transition from e to g and emission of a photon at a rate proportional to the photon density.

$$\left.\frac{dN_e}{dt}\right)_{\text{stim}} = -B_{eg} u_{\nu_0} N_e \qquad (30)$$

Einstein's rate equations

$$\frac{dN_e}{dt} = -A_{eg} N_e - B_{eg} u_{\nu_0} N_e + B_{ge} u_{\nu_0} N_g \qquad (31)$$

Einstein's coefficents

Relations between the three coefficients

At thermal equilibrium (temperature T)

$$\frac{N_e}{N_g} = e^{(E_g - E_e)/k_b T} = e^{-h\nu_0/k_b T} \tag{32}$$

k_b: Boltzmann constant. And (Planck's law)

$$u_{\nu_0} = \frac{8\pi h {\nu_0}^3}{c^3} \frac{1}{\exp(h\nu_0/k_b T) - 1} \tag{33}$$

Einstein's coefficents

Relations between the three coefficients

Steady state for $T \to \infty$ i.e. $u_{\nu_0} \to \infty$ and $N_e/N_g \to 1$. Neglect spontaneous emission.

$$B_{ge} = B_{eg} = B \tag{34}$$

Noting $A_{eg} = A$, steady state at a finite temperature T:

$$A + Bu_{\nu_0} = Bu_{\nu_0} e^{h\nu_0/k_b T} \tag{35}$$

Hence

$$u_{\nu_0} = \frac{A}{B} \frac{1}{\exp(h\nu_0/k_b T) - 1} \tag{36}$$

Comparing with Planck's law

$$\frac{A}{B} = \frac{8\pi h {\nu_0}^3}{c^3} = \frac{8\pi h}{\lambda_0^3} \tag{37}$$

Einstein's coefficents

Case of degenerate atomic levels

g_e and g_g degeneracies of energies E_e and E_g. At thermal equilibrium

$$N_e/N_g = (g_e/g_g)\exp(-h\nu_0/k_bT) \tag{38}$$

From the infinite temperature limit:

$$B_{ge}/B_{eg} = g_e/g_g \tag{39}$$

A purely algebraic complication, not to be considered any further here.

The Laser

Light Amplification

Stimulated emission: addition of energy to the incoming wave.
A simple situation: plane wave at frequency ν_0 on a thin slice of atoms.
Incoming power per unit surface $\mathcal{P}$, outgoing $\mathcal{P} + d\mathcal{P}$. Obviously:

$$d\mathcal{P} \propto \mathcal{P}(N_e - N_g) = \mathcal{P}\, D \quad (40)$$

where we define the population inversion density:

$$D = N_e - N_g \quad (41)$$

The power increases when $D > 0$: gain when population inversion

The Laser

Population inversion

Conditions to achieve $D > 0$

- No thermal equilibrium
- No two-level system (in the steady state)
- Three or four level system
- Case of a four level system (f: ground state, i intermediate, plus e and g:
  - Fast incoherent pumping from f to i
  - Fast relaxation from i to e
  - Stimulated emission from e to g
  - Extremely fast relaxation from g to f

The Laser

Principle

- Gain + feedback = oscillation
- A laser is composed of an amplifying medium (gain) and of an optical resonant cavity (feedback).
- When the gain exceeds the losses in the feedback, a self-sustained steady-state oscillation occurs.

The Laser

A simple model

Catpures the main physical ideas without any complication. Forget about all details and proportionality constants.

Variables

- Population inversion density D. If g strongly damped, $D = N_e$.
- Intra-cavity intensity I (photon density)

Evolution of intensity

$$\frac{dI}{dt} = -\kappa I + gID \tag{42}$$

κ: rate of internal or coupling cavity losses.

The Laser

A simple model

Evolution of population inversion

$$\frac{dD}{dt} = \Lambda - \Gamma D - gID \quad (43)$$

with

- Λ : pumping rate in the upper level e
- Γ : relaxation rate of e (spontaneous emission in modes other than the cavity one, other sources of atomic losses)

The Laser

Steady state

Laser off solution

- $I = 0$ always a solution
- $D = \Lambda/\Gamma$

Laser on solution

- $D = \kappa/g$

$$I = \frac{1}{\kappa}\left(\Lambda - \frac{\Gamma\kappa}{g}\right) \quad (44)$$

- Relevant if $I \geq 0$

$$\Lambda \geq \Lambda_t = \frac{\Gamma\kappa}{g} \quad (45)$$

- Threshold condition

The Laser

Steady state

Stability of the solutions:

- $\Lambda < \Lambda_t$: only solution $I = 0$
- $\Lambda \geq \Lambda_t$: two possible solutions, but $I = 0$ unstable

The Laser

Main properties of laser radiation

- Directive
- Intense
- Narrow band and extremely long coherence length or...
- Extremely short pulses (as)